ENEE626. Midterm examination, 10/26/2005 **Instructor:** A. Barg

All answers should be accompanied with proofs or sufficient explanation. Intermediate calculations should be shown.

In the problems you may use any representation of $\mathbf{F}_{11}$ in the calculations. The *final answers* should be written using integers mod 11.

(a). (10pt). Prove that $\alpha = 2$ is a primitive element of $F = \mathbf{F}_{11}$.

(b). (10pts). Let C be a $[10, 6, d]$ RS code over F. Write out a parity-check H matrix of C.

(c). (20pt) Reduce H to a systematic form $H' = [I_4|A]$.

Help: $\begin{bmatrix}1 & 2 & 4 & 8\\ 1 & 4 & 5 & 9\\ 1 & 8 & 9 & 6\\ 1 & 5 & 3 & 4\end{bmatrix}^{-1} = \begin{bmatrix}2 & 1 & 8 & 1\\ 6 & 0 & 10 & 6\\ 2 & 5 & 0 & 4\\ 7 & 7 & 2 & 6\end{bmatrix}$ (mod 11). (If you find no use of this equality, ignore it.)

(d). (30pt) Write out a generator matrix of C in a systematic form

(Use caution: Lecture 2 applies to binary codes only)

(e). (10pt) Using H', find the codeword $\boldsymbol{c}_0$ that corresponds to the message symbols $(1, 1, 1, 1, 1, 1)$.

(f). (10pt) What is the polynomial f such that eval$(f) = \boldsymbol{c}_0$?

(g). (15pt) Is is true that $(c_0, c_1, \dots, c_9) \in C$ implies that $(c_9, c_1, c_2, \dots, c_8) \in C$?

(h). (20pt) Let $\boldsymbol{c} \in C$ be a vector of weight 5. Prove that if $\boldsymbol{c}' \in C$ is such that $\mathrm{supp}(\boldsymbol{c}) = \mathrm{supp}(\boldsymbol{c}')$ then $\boldsymbol{c}' = a\boldsymbol{c}$ where $a \in F\backslash\{0\}$ is some constant.

(i). (15pt) Using problem 8, compute directly, with proof, the number of vectors of weight 5 in C (your answer should be a number, not an expression).

(j). (30pt) Let $\boldsymbol{r} = (3, 0, 0, 10, 5, 4, 0, 6, 10, 0)$ be a received vector. Perform the Peterson-Gorenstein Zierler algorithm to determine the number of errors and to decode the vector.

(k). (10pt) Explain how to find the polynomial f such that eval(f) equals the decoded codeword from problem 10.

(l). (10pt) If time left, invent a nice problem and solve it.