ENEE626. Final examination **Instructor: A. Barg**

Your paper is due by 5pm on **Friday December 17 2010** in my office, AV Williams Bldg. Rm 2361. If I am absent, please slide your paper under the door.

Please be concise in your exam paper!

1.(10pt) (a) Find a 7×4 right inverse A of the matrix G over $\mathbb{F}_2$:

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

(b) Using the matrix A find the message $\boldsymbol{m}$ if $\boldsymbol{m}G = 1000110$.

2. Let α be a primitive element of the field $\mathbb{F}_{16}$. Let $\mathscr{P} = (\alpha^4, \alpha^6, \alpha^7, \alpha^8, \alpha^9, \alpha^{10}, \alpha^{11}, \alpha^{12}, \alpha^{13}, \alpha^{14})$ be the defining set of an RS code $\mathcal{C}$ of dimension $k = 4$.

(a) (3pt) What are the parameters (n, k, d) of the code?

(b) (10pt) Decode the vector $\boldsymbol{y} = (\alpha, \alpha^{12}, \alpha^7, 0, \alpha^{13}, \alpha^6, \alpha^9, \alpha^7, \alpha^7, \alpha^2)$ with the code $\mathcal{C}$ (use any decoding algorithm you know; show the steps).

3. Let G be a binary $k \times n$ matrix whose entries are chosen from $\mathbb{F}_2$ independently with probabilities $P(1) = r, P(0) = 1 - r$. Let $\mathcal{C}$ be the code spanned by the rows of the matrix G and let $\mathscr{C}$ be the ensemble of random binary codes arising in this way. The number A_w of vectors of weight $w, w = 0, 1, \dots, n$ in $\mathcal{C}$ is a random variable.

(a) (10pt) Let $r = 1/2$. Find EA_w (begin by considering the codeword $\boldsymbol{x} = \boldsymbol{m}G$ obtained after you fix a binary message vector $\boldsymbol{m} \in \mathbb{F}_2^k$. Argue about the probability that $\mathrm{wt}(\boldsymbol{x}) = w$.).

(b) (8pt) Let $0 < r < 1$. Find EA_w (argue about the probability that $x_i = 1$ where $\boldsymbol{x} = (x_1, \dots, x_n)$ is as above in part (a). Note that this probability will depend on $\mathrm{wt}(\boldsymbol{m})$.).

4. In this problem you are asked to give proofs of some results stated in class.

(a) (10 pt) Find the 2nd generalized Hamming weight d_2 of the simplex code $\mathcal{S}_m[2^m - 1, m, 2^{m-1}]$.

(b) (8 pt) Find the third generalized Hamming weight d_3 of $\mathcal{S}_m$.

5. (10pt) Consider the network G with receivers R_1, R_2, R_3, R_4 shown in the figure.

What is the MinCut from S to R_1, R_2, R_3, R_4?

Find the smallest q such that there exists a linear multicast network coding scheme that can transmit 2 packets to the receivers. Construct the coding scheme by writing out the local and global encoding kernels for each node resp. edge of the network.