ENEE324 Fall 2016. Problem set 8

Date due November 30, 2016

Please justify your answers

1. Given two independent RVs X, Y such that $M_X(s) = \exp(2e^s - 2)$ and $M_Y(s) = (\frac{3}{4}e^s + \frac{1}{4})^{10}$, find (a) $P(X+Y=3)$, (b) $P(XY=0)$, (c) $E(XY)$.

2. Let $X_i, i = 1, \dots, 20$ be independent Poisson RVs with expectation 1.

(a) Use the Markov inequality to bound above $P(\sum_{i=1}^{20} X_i > 25)$;

(b) Use the central limit theorem to find the approximate value of the probability in part (a).

3. Motorists in a town collectively own 10000 vehicles. A vehicle on average spends 240 gallons of fuel a year with a standard deviation of 800 gallons. Approximate the probability that the total yearly fuel consumption in the town exceeds 2,700,000 gallons.

4. Let X be a nonnegative RV with pdf $f_X(x)$. Put $Y_i = 1$ if $X > i$ and $Y_i = 0$ otherwise, $i = 1, 2, \dots$. Does the sequence Y_i converge in probability, and if yes, to which value? (Convergence in probability is covered in the textbook, p.271)

5. Packets arrive at a server at a Poisson rate λ per minute. Suppose that two packets arrived within the first minute. Find the probability that

(a) Both packets arrived during the first 20 seconds;

(b) At least one packet arrived during the first 20 seconds.

(c) Generally, given that the number of arrivals by time t in a Poisson process with rate λ equals n, find the distribution of the number of arrivals in an interval $[0; s]$ for a given $s < t$. (Hint: your task is to compute the conditional probability $P(N(s) = i | N(t) = n); 0 \le i \le n$, where $N(t)$ is the number of arrivals by time t. The surprising answer is that the distribution is Binomial$(n, s/t)$).

6. A transmitter sends 7 bits per second over a noisy channel. The channel introduces errors (i.e., flips the bits) at a Poisson rate of one per 60 transmissions. Find the probability that there was more than one one error in the span of 10 seconds.