

ENEE324 Fall 2016. Problem set 6

Date due November 2, 2016

Please justify your answers

1. Let $X \sim \text{unif}(0, \pi/4)$. Find $E(\cos 2X)$, $E(\cos^2 X)$.
2. Consider an RV X with pdf $f_X(x) = \frac{1}{\pi(1+x^2)}$, $x \in \mathbb{R}$. Find the pdf of $Y = \arctan X$.
3. Let X have pdf $f_X(x) = \sqrt{\frac{2}{\pi}} e^{-2(x-1)^2}$, $x \in \mathbb{R}$. Find EX , $\text{Var}(X)$.
4. Let X, Y be two random points in the interval $(0, 1)$. Find $P(Y \leq X)$ and $P(X^2 + Y^2 \leq 1)$.
5. Given a joint pdf of two RVs X, Y :

$$f_{XY}(x, y) = \begin{cases} 8xy & \text{if } 0 \leq y \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the marginal pdfs of X and Y . Are X and Y independent?

6. Let $X \sim \exp(\lambda)$, $Y \sim \exp(\lambda)$ be independent RVs. Find $E(\max(X, Y))$. Find the pdf of $X/(X + Y)$.

7. Let X, Y be RVs and

$$f_{X|Y}(x|y) = \frac{x+y}{1+y} e^{-x}, \quad 0 < x < \infty, 0 < y < \infty.$$

Find $P(X < 1|Y = 3)$.

8. Select a random point (X, Y) from the triangle given by the vertices $(0, 0)$, $(0, 1)$, $(1, 0)$.
(a) Find $f_{XY}(x, y)$; (b) Find $f_{X|Y}(x|y)$; (c) Find $E(X|Y = y)$.

9. Let X, Y be two independent standard Gaussian RVs. Let $R = \sqrt{X^2 + Y^2}$, $\Theta = \arctan(Y/X)$. Are R and Θ independent?