ENEE324-03 Spring 2019. Problem set 4

Date due March 7, 2019

Problem 1. Consider an RV (random variable) with PDF

$$f(x) = \begin{cases} c(1-x^2) - 1 & -1 < x < 1 \\ 0 & \text{o/w} \end{cases}$$

(a) Find c; (b) Find the CDF $F_X(x)$ (make sure to give the answer for all real x) (c) Find $EX, \text{Var}(x)$.

Problem 2. Textbook, p.72, Problem 8

Problem 3. Textbook, p.72, Problem 10.

Problem 4. Textbook, p.73, Problem 14.

Problem 5. Let X be an exponential RV that represents the length of a phone call to Customer Service. Suppose that the average length of the call is 10 minutes.

(a) What is the probability that a specific call lasts 5 minutes?

(b) You join the queue to speak to Customer Service, and at that moment there is another caller speaking with them. What is the probability that that caller will not finish the call 4 minutes later? (you are asked to compute $P(X \ge t+4 | X \ge t)$ for some (unknown) time t).

Problem 6. Let X and Y be independent RVs, and let $U = \min(X, Y), V = \max(X, Y)$.

(a) Show that the CDFs of U and V are

$$F_U(x) = 1 - (1 - F_X(x))(1 - F_Y(x)), \quad F_V(x) = F_X(x)F_Y(x).$$

(b) Let $X \sim \text{Exp}(1), Y \sim \text{Exp}(1)$ be independent exponential RVs. Show that $U \sim \text{Exp}(2)$. Find the mean and variance of V.

Problem 7. Suppose the CDF of an RV X is given by

$$F(z) = \begin{cases} 0 & z < 0 \\ \frac{z}{4} & 0 \le z < 1 \\ \frac{1}{2} + \frac{z-1}{4} & 1 \le z < 2 \\ \frac{11}{12} & 2 \le z < 3 \\ 1 & z \ge 3. \end{cases}$$

Find (a) $P(X = m), m = 1, 2, 3$; (b) $P(\frac{1}{2} < X < \frac{3}{2})$; (c) EX and EX^2.