ENEE324 Fall 2016. Problem set 3

Date due September 21, 2016

Please justify your answers

1. 2^n players enter a tennis tournament. In the first round, 2^{n-1} pairs are formed, and the winner of each pair advances to the next round, etc., until the winner emerges. Two players are chosen at random. What is the probability that they meet in the first or the second round? What is the probability that they meet in the final or semi-final? What is the probability that they do not meet?

2. In a urn there are 5 red and 10 blue balls. We remove two randomly chosen balls without learning their color. Next we choose a random ball R and find it to be blue. What is the probability that both removed balls were blue?

3. You are playing a lottery in which you choose 6 natural numbers out of the numbers in the set $N = (1, 2, \ldots, 49)$ without replacement. A set of 6 random numbers out of N is picked after that. Calculate the probability that you guess exactly k out 6 numbers for $k = 3, 4, 5, 6$.

4. We select a random integer from the set $\{1, 2, \ldots, 1000000\}$. What is the probability that it contains the digit 5?

5. An elevator with 6 passengers leaves the floor 1 and stops at each of the floors 2,3,4. Each of the passengers is equally likely to leave at any of the three floors. What is the probability that at each floor at least one passenger leaves?

6. Use a combinatorial argument to justify the following identity:

$$\sum_{i=0}^{r} \binom{m}{i}\binom{n}{r-i} = \binom{m+n}{r}.$$

Assume that $r \leq m, r \leq n$, where r, n, m are natural numbers.