ENEE324, Home assignment 7. Date due April 27, 2026, 11:59pm EDT.

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Please upload your work as a **single PDF file** to ELMS (under the "Assignments" tab)

- Submissions on paper or by email will not be accepted.
- Please do not submit your solutions as multiple separate files (pictures of individual pages). Such submissions are difficult to grade and will not be accepted.

- Justification of solutions is required.
- Each problem is worth 10 points unless noted otherwise.

Problem 1. Let (X, Y) have joint pmf

$$p_{X,Y}(x, y) = c(x + y), \quad x, y \in \{0, 1, 2\}.$$

(1) Find the constant c.
(2) Find the marginal distributions of X and Y.
(3) Compute $E[X]$, $E[Y]$, and $E[XY]$.
(4) Are X and Y independent?

Problem 2. Let (X, Y) have joint pdf

$$f_{X,Y}(x, y) = 2, \quad 0 < y < x < 1,$$

and 0 otherwise.

(1) Find the marginal densities of X and Y.
(2) Compute $P(X + Y \leq 1)$.
(3) Find the conditional density $f_{X|Y}(x \mid y)$. Make sure to give your answer for all values of x and y.
(4) Compute $E[X \mid Y = 0.5]$.

Problem 3. A company assigns n tasks one by one. For each task, an employee is chosen independently and uniformly at random from m employees (so the same employee may receive multiple tasks).

Let X be the number of tasks assigned to employee 1, and let Y be the number of tasks assigned to employee 2.

(1) Find the joint pmf of (X, Y).
(2) Use the result of (1) to find the marginal distributions of X and Y: explicitly calculate

$$\sum_j P(X = i, Y = j)$$

(3) Compute $P(X = Y)$.
(4) Compute $E[X \mid Y = k]$ for $k = 0, 1, \ldots, n$.
(5) Are X and Y independent? Justify your answer.

Problem 4. Let $X \sim \text{Uniform}(0, 1)$ and, conditional on $X = x$, let Y be uniform on $(0, x)$.

(1) Find the joint pdf of (X, Y).
(2) Find the marginal density of Y.
(3) Compute $P(Y \leq 1/2)$.

Problem 5. A drone is flying over a circular park of radius 1 (in suitable units). Its position (X, Y) is uniformly distributed over the disk

$$\{(x, y) : x^2 + y^2 \leq 1\}.$$

(1) Find the joint pdf of (X, Y).
(2) Find the marginal density of X.
(3) Compute the expected squared distance of the drone from the center, i.e., $E[X^2 + Y^2]$. Hint: Polar coordinates.

Problem 6. A square-shaped field has side length 2. Two sprinklers are placed independently and uniformly at random along the bottom and left edges of the field. Let X be the distance from the origin along the bottom edge, and Y the distance along the left edge.

These two sprinklers define a triangular region with vertices $(0, 0)$, $(X, 0)$, and $(0, Y)$.

(1) Find the joint pdf of (X, Y).
(2) Let A be the area of the triangle. Compute $E[A]$.
(3) Compute $P(A \leq 1/8)$.