ENEE324, Home assignment 1. Date due September 13, 2025, 11:59pm EDT.

Instructor: `Alexander Barg`

Please upload your work as a **single PDF file** to ELMS (under the "Assignments" tab)

- Submissions on paper or by email will not be accepted.
- Please do not submit your solutions as multiple separate files (pictures of individual pages). Such submissions are difficult to grade and will not be accepted.

- Justification of solutions is required.
- Each problem is worth 10 points.

Problem 1. A license plate consists of 3 letters followed by 3 digits (e.g., ABC123).

(a) How many different license plates are possible if letters and digits may repeat?

(b) How many if no letter or digit may repeat?

(a) Each of 26 letters can appear in the first 3 positions; each digit in the second

$$(26)^3 \cdot (10)^3 = 17{,}576{,}000$$

(b) $26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 = 11{,}232{,}000$

Problem 2. You roll two distinguishable dice.

(a) How many outcomes are there in total?

(b) How many outcomes give a sum of 7?

(c) What is the probability of rolling at least one six?

(a) $6 \times 6 = 36$

(b) $\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$ Ans. : 6

(c) $\dfrac{\#\{\text{outcomes with one or two 6's}\}}{36} = \dfrac{11}{36}$

$(6,1),(6,2)\ldots,(6,6)$
$(1,6)(2,6)\ldots,(5,6)$

Problem 3. You place 6 identical balls into 4 distinct boxes.

(a) How many possible placements are there?

(b) How many if no box is empty?

(a) Rephrasing, we would like to write $6 = B_1 + B_2 + B_3 + B_4$, where

$B_i \geq 0$ is # Balls in Box i. Since the Boxes are distinct, for instance,

1 2 2 1 and 2 1 2 1 are two distinct placements.

This is the same as $(B_1+1)+(B_2+1)+(B_3+1)+(B_4+1) = 10$.

• • ✓ • • ✓ • ✓ • • • • •

Place 3 ✓ in the 9 gaps arbitrarily. The above example gives

$2+2+1+5 = 10$, meaning that $B_1 = 1$, $B_2 = 1$, $B_3 = 0$, $B_4 = 4$

Altogether $\binom{9}{3} = \frac{9 \cdot 8 \cdot 7}{6} = 3 \cdot 4 \cdot 7 = 84$ //

(b) As above, but without adding 1 to B_i's : $\binom{5}{3} = 10$
(no boxes are empty)

Problem 4. A committee of 5 people is to be chosen from 12 men and 8 women.

(a) How many committees are possible?

(b) How many committees contain at least 2 women?

(c) What is the probability a randomly chosen committee has at least 2 women?

(a) $$\sum_{i=0}^{5} \binom{12}{i}\binom{8}{5-i} = \binom{20}{5} = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16}{120} = 19 \cdot 3 \cdot 17 \cdot 16 = 15504$$

(b) $$\sum_{i=2}^{5} \binom{8}{i}\binom{12}{5-i} = \binom{8}{2}\binom{12}{3} + \binom{8}{3}\binom{12}{2} + \binom{8}{4}\binom{12}{1} + \binom{8}{5}$$
$$= 10752$$

(c) $$\frac{10752}{15504} \cong 0.69$$

Problem 5. A class, such as ENEE324, has 2 prerequisites, courses C1 and C2. Of the 100 students in class, 50 have taken C1 (call them set A), 40 have taken C2 (call them set B), and 20 have taken both courses C1 and C2.

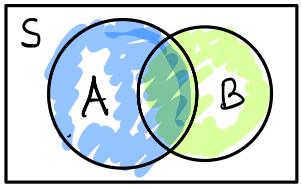
1. Draw the Venn diagram for this example for sets A and B. Please label all the regions in the sample space S with their numerical values.

2. How many students have not taken course C2?

3. How many students have taken C2 but not C1?

4. How many students have taken neither of the two prerequisites?

5. Using the set notation (such as A, B, their unions, intersections or complements), write the expression for the set of students who have not taken either of the two prerequisites OR who have taken only C2, but not C1. Simplify as much as possible.

1. 

$|A| = 50$

$|B| = 40$

$|A \cap B| = 20$

S = all 100 students

2. $|B^c| = 100 - 40 = 60$

3. $|B \setminus (A \cap B)| = 40 - 20 = 20$

4. $|(A \cup B)^c| = |S| - |A| - |B \setminus A| = 100 - 50 - 20 = 30$

5. $(A^c \cap B^c) \cup (B \setminus A) = (B^c \cap A^c) \cup (\underbrace{B \cap A^c}_{= B\setminus A}) = (\underbrace{B^c \cup B}_{= S}) \cap A^c = S \cap A^c = A^c$

Problem 6. (Textbook, Problem 56, p.37) For each part, decide whether the blank should be filled in with =, <, or >, and give a clear explanation. In (a) and (b), the order does not matter.

(a) (# ways to choose 5 people out of 10) ____ (# ways to choose 6 people out of 10)

(b) (# ways to break 10 people into 2 teams of 5) ____ (# to break 10 people into a team of 6 and a team of 4)

(c) (probability that all 3 people in a group of 3 were born on Jan.1) ____ (probability that in a group of 3 people, their birthdays are January 1, 2, and 3)

(d) In a game of coins, there are two players. A single coin is tossed repeatedly until HH (two consecutive Heads) occurs or TH (Tails followed immediately by Heads). P1 wins if and only if HH occurs before TH. Is the probability that P1 wins >, =, or < than 1/2? The answer is not as obvious as it initially looks.

(a) $\binom{10}{5} > \binom{10}{6} = \binom{10}{4}$ since choosing 4, we still have choices to add the 5^{th} person

(b) Once team T_1 is chosen, this also fixes T_2. We obtain a pair (T_1, T_2); for each such choice, the choice (T_2, T_1) gives the same result (order of the teams does not matter); we obtain $\frac{1}{2}\binom{10}{5} \underline{\underline{<}} \binom{10}{6}$

(c) Possible outcomes

Jan 1
P_1, P_2, P_3

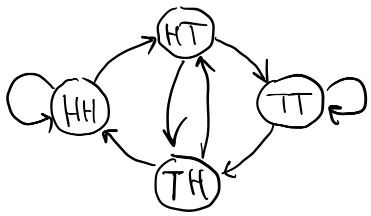
Jan 1	Jan 2	Jan 3
P_1	P_2	P_3
P_2	P_1	P_3
P_2	P_3	P_1
P_3	P_2	P_1
P_3	P_1	P_2
P_1	P_3	P_2

The prob. of each of the above outcomes is $\frac{1}{365^3}$

$\therefore \; P(\text{Jan } 1) = \frac{1}{6} P(\text{Jan } 1, 2, 3)$

So the answer is $<$.

(d) $<$



If the game ends after 2 rounds, then the players have equal chances.

Suppose the game goes into the third round. In this case the last 2 tosses were HT or TT. From the above diagram it is clear that in this case TH will occur before HH.