The Baumol-Tobin and the Tobin Mean-Variance Models of the Demand for Money

THE BAUMOL-TOBIN MODEL OF TRANSACTIONS DEMAND FOR MONEY

William Baumol and James Tobin independently developed similar demand for money models, which demonstrated that even money balances held for transactions purposes are sensitive to the level of interest rates. In developing their models, they considered a hypothetical individual who receives a payment once a period and spends it over the course of this period. In their model, money, which earns zero interest, is held only because it can be used to carry out transactions.

To refine this analysis, let’s say that Grant Smith receives $1,000 at the beginning of the month and spends it on transactions that occur at a constant rate during the course of the month. If Grant keeps the $1,000 in cash to carry out his transactions, his money balances follow the sawtooth pattern displayed in panel (a) of Figure 1. At the beginning of the month he has $1,000, and by the end of the month he has no cash left because he has spent it all. Over the course of the month, his holdings of money will, on average, be $500 (his holdings at the beginning of the month, $1,000, plus his holdings at the end of the month, $0, divided by 2).

At the beginning of the next month, Grant receives another $1,000 payment, which he holds as cash, and the same decline in money balances begins again. This process repeats monthly, and his average money balance during the course of the year is $500. Since his yearly nominal income is $12,000 and his holdings of money average $500, the velocity of money \( (V = \frac{PY}{M}) \) is $12,000/$500 = 24.

Suppose that as a result of taking a money and banking course, Grant realizes that he can improve his situation by not always holding cash. In January, then, he decides to hold part of his $1,000 in cash and puts part of it into an income-earning security such as bonds. At the beginning of each month, Grant keeps $500 in cash and uses the other $500 to buy a Treasury bond. As you can see in panel (b), he starts out each month with $500 of cash, and by the middle of the month, his cash balance has run down to zero. Because bonds cannot be used directly to carry out transactions, Grant must sell them and turn them into cash so that he can carry out the rest of the month’s transactions. At the middle of the month, then, Grant’s cash balance rises back up to $500. By the end of the month, the cash is gone. When he again receives his next $1,000 monthly

---

payment, he again divides it into $500 of cash and $500 of bonds, and the process continues. The net result of this process is that the average cash balance held during the month is $500/2 = $250—just half of what it was before. Velocity has doubled to $12,000/$250 = 48.

What has Grant Smith gained from his new strategy? He has earned interest on $500 of bonds that he held for half the month. If the interest rate is 1% per month, he has earned an additional $2.50 ($ = \frac{1}{2} \times $500 \times 1\%) per month.

Sounds like a pretty good deal, doesn’t it? In fact, if he had kept $333.33 in cash at the beginning of the month, he would have been able to hold $666.67 in bonds for the first third of the month. Then he could have sold $333.33 of bonds and held on to $333.34 of bonds for the next third of the month. Finally, two-thirds of the way through the month, he would have had to sell the remaining bonds to raise cash. The net result of this is that Grant would have earned $3.33 per month [$= (\frac{1}{3} \times $666.67 \times 1\%) \times (\frac{2}{3} \times $333.34 \times 1\%)]. This is an even better deal. His average cash holdings in this case would be $333.33/2 = $166.67. Clearly, the lower his average cash balance, the more interest he will earn.

As you might expect, there is a catch to all this. In buying bonds, Grant incurs transaction costs of two types. First, he must pay a straight brokerage fee for the buying and selling of the bonds. These fees increase when average cash balances are lower because Grant will be buying and selling bonds more often. Second, by holding less cash, he will have to take time to sell the bonds to get the cash. Because time is money, this must also be counted as part of the transaction costs.

Grant faces a trade-off. If he holds very little cash, he can earn a lot of interest on bonds, but he will incur greater transaction costs. If the interest rate is high, the benefits of holding bonds will be high relative to the transaction costs, and he will hold more bonds and less cash. Conversely, if interest rates are low, the transaction costs involved

---

**FIGURE 1 Cash Balances in the Baumol-Tobin Model**

In panel (a), the $1,000 payment at the beginning of the month is held entirely in cash and is spent at a constant rate until it is exhausted by the end of the month. In panel (b), half of the monthly payment is put into cash and the other half into bonds. At the middle of the month, cash balances reach zero and bonds must be sold to bring balances up to $500. By the end of the month, cash balances again dwindle to zero.
in holding a lot of bonds may outweigh the interest payments, and Grant would then be better off holding more cash and fewer bonds.

The conclusion of the Baumol-Tobin analysis may be stated as follows: As interest rates increase, the amount of cash held for transactions purposes will decline, which in turn means that velocity will increase as interest rates increase.\footnote{Similar reasoning leads to the conclusion that as brokerage fees increase, the demand for transactions money balances increases as well. When these fees rise, the benefits from holding transactions money balances increase because by holding these balances, an individual will not have to sell bonds as often, thereby avoiding these higher brokerage costs. The greater benefits to holding money balances relative to the opportunity cost of interest forgone, then, lead to a higher demand for transactions balances.} Put another way, the transactions component of the demand for money is negatively related to the level of interest rates.

The basic idea in the Baumol-Tobin analysis is that there is an opportunity cost of holding money—the interest that can be earned on other assets. There is also a benefit to holding money—the avoidance of transaction costs. When interest rates increase, people will try to economize on their holdings of money for transactions purposes, because the opportunity cost of holding money has increased. By using simple models, Baumol and Tobin revealed something that we might not otherwise have seen: that the transactions demand for money, and not just the speculative demand, will be sensitive to interest rates. The Baumol-Tobin analysis presents a nice demonstration of the value of economic modeling.

The idea that as interest rates increase, the opportunity cost of holding money increases so that the demand for money falls, can be stated equivalently with the terminology of expected returns used in Chapter 5. As interest rates increase, the expected return on the other asset, bonds, increases, causing the relative expected return on money to fall, thereby lowering the demand for money. These two explanations are in fact identical, because as we saw in Chapter 5, changes in the opportunity cost of an asset are just a description of what is happening to the relative expected return. Baumol and Tobin used opportunity cost terminology in their work on the transactions demand for money, and that is why we use this terminology here.

**Mathematical Treatment of the Baumol-Tobin Model**

Here we explore the mathematics that underlie the model. The assumptions of the model are as follows:

1. An individual receives income of $T_0$ at the beginning of every period.
2. An individual spends this income at a constant rate, so at the end of the period, all income $T_0$ has been spent.
3. There are only two assets—cash and bonds. Cash earns a nominal return of zero, and bonds earn an interest rate $i$.
4. Every time an individual buys or sells bonds to raise cash, a fixed brokerage fee of $b$ is incurred.

Let us denote the amount of cash that the individual raises for each purchase or sale of bonds as $C$, and $n = \text{the number of times the individual conducts a transaction in bonds}$. As we saw in Figure 1 in the chapter, where $T_0 = 1,000$, $C = 500$, and $n = 2$:

$$n = \frac{T_0}{C}$$

Because the brokerage cost of each bond transaction is $b$, the total brokerage costs for a period are

$$nb = \frac{bT_0}{C}$$
Not only are there brokerage costs, but there is also an opportunity cost to holding cash rather than bonds. This opportunity cost is the bond interest rate \( i \) times average cash balances held during the period, which, from the discussion in the chapter, we know is equal to \( C/2 \). The opportunity cost is then:

\[
\frac{IC}{2}
\]

Combining these two costs, we have the total costs for an individual equal to:

\[
\text{COSTS} = \frac{bT_0}{C} + \frac{IC}{2}
\]

The individual wants to minimize costs by choosing the appropriate level of \( C \). This is accomplished by taking the derivative of costs with respect to \( C \) and setting it to zero.\(^3\) That is

\[
\frac{d}{dC} \text{COSTS} = -\frac{bT_0}{C^2} + \frac{i}{2} = 0
\]

Solving for \( C \) yields the optimal level of \( C \):

\[
C = \sqrt{\frac{2bT_0}{i}}
\]

Because money demand \( M^d \) is the average desired holding of cash balances \( C/2 \),

\[
M^d = \frac{1}{2} \sqrt{\frac{2bT_0}{i}} = \sqrt{\frac{bT_0}{2i}}
\]

This is the famous square root rule.\(^4\) It has these implications for the demand for money:

1. The transactions demand for money is negatively related to the interest rate \( i \).
2. The transactions demand for money is positively related to income, but there are economies of scale in money holdings—that is, the demand for money rises less than proportionally with income. For example, if \( T_0 \) quadruples in Equation 1, the demand for money only doubles.

\(^3\)To minimize costs, the second derivative must be greater than zero. We find that it is, because

\[
\frac{d^2 \text{COSTS}}{dC^2} = -\frac{2}{C} \left( -\frac{bT_0}{C^3} \right) = \frac{2bT_0}{C^2} > 0
\]

\(^4\)An alternative way to get Equation 1 is to have the individual maximize profits, which equal the interest on bonds minus the brokerage costs. The average holding of bonds over a period is just

\[
\frac{T_0}{2} - \frac{C}{2}
\]

Thus profits are

\[
\text{PROFITS} = \frac{1}{2} (T_0 - C) - \frac{bT_0}{C}
\]

Then

\[
\frac{d\text{PROFITS}}{dC} = -\frac{i}{2} + \frac{bT_0}{C^2} = 0
\]

This equation yields the same square root rule as Equation 1.
3. A lowering of the brokerage costs due to technological improvements would decrease the demand for money.

4. There is no money illusion in the demand for money. If the price level doubles, $T_0$ and $b$ will double. Equation 1 then indicates that $M$ will double as well. Thus the demand for real money balances remains unchanged, which makes sense because neither the interest rate nor real income has changed.

**Tobin Mean-Variance Model**

Tobin's mean-variance analysis of money demand is just an application of the basic ideas in the theory of portfolio choice. Tobin assumes that the utility that people derive from their assets is positively related to the expected return on their portfolio of assets and is negatively related to the riskiness of this portfolio as represented by the variance (or standard deviation) of its returns. This framework implies that an individual has indifference curves that can be drawn as in Figure 2. Notice that these indifference curves slope upward because an individual is willing to accept more risk if offered a higher expected return. In addition, as we go to higher indifference curves, utility is higher, because for the same level of risk, the expected return is higher.

Tobin looks at the choice of holding money, which earns a certain zero return, or bonds, whose return can be stated as:

$$R_B = i + g$$

where

- $i =$ interest rate on the bond
- $g =$ capital gain

**Figure 2**

*Indifference Curves in a Mean-Variance Model*

The indifference curves are upward-sloping, and higher indifference curves indicate that utility is higher. In other words, $U_3 > U_2 > U_1$. 

- Higher Utility
- Expected Return $\mu$
- Standard Deviation of Returns $\sigma$
Appendix 1 to Chapter 22  \( \text{The Baumol-Tobin and the Tobin Mean-Variance Models} \)

Tobin also assumes that the expected capital gain is zero\(^5\) and its variance is \( \sigma_g^2 \). That is,

\[
E(g) = 0 \quad \text{and so} \quad E(R_b) = i + 0 = i \\
Var(g) = E[g - E(g)]^2 = E(g^2) = \sigma_g^2
\]

where

\[ E = \text{expectation of the variable inside the parentheses} \]
\[ Var = \text{variance of the variable inside the parentheses} \]

If \( A \) is the fraction of the portfolio put into bonds (0 ≤ \( A \) ≤ 1) and 1 - \( A \) is the fraction of the portfolio held as money, the return \( R \) on the portfolio can be written as:

\[
R = AR_b + (1 - A)(0) = AR_b = A(i + g)
\]

Then the mean and variance of the return on the portfolio, denoted respectively as \( \mu \) and \( \sigma^2 \), can be calculated as follows:

\[
\mu = E(R) = E(AR_b) = AE(R_b) = Ai \\
\sigma^2 = E(R - \mu)^2 = E[A(i + g) - Ai]^2 = E(Ag)^2 = A^2E(g^2) = A^2\sigma_g^2
\]

Taking the square root of both sides of the equation directly above and solving for \( A \) yields:

\[
A = \frac{1}{\sigma_g} \sigma
\]

Substituting for \( A \) in the equation \( \mu = Ai \) using the preceding equation gives us:

\[
\mu = \frac{i}{\sigma_g} \sigma
\]

Equation 3 is known as the opportunity locus because it tells us the combinations of \( \mu \) and \( \sigma \) that are feasible for the individual. This equation is written in a form in which the \( \mu \) variable corresponds to the Y axis and the \( \sigma \) variable to the X axis. The opportunity locus is a straight line going through the origin with a slope of \( i/\sigma_g \). It is drawn in the top half of Figure 3 along with the indifference curves from Figure 2.

The highest indifference curve that can be reached is at point B, the tangency of the indifference curve and the opportunity locus. This point determines the optimal level of risk \( \sigma^* \) in the figure. As Equation 2 indicates, the optimal level of \( A \), \( A^* \), is:

\[
A^* = \frac{\sigma^*}{\sigma_g}
\]

This equation is solved in the bottom half of Figure 3. Equation 2 for \( A \) is a straight line through the origin with a slope of \( 1/\sigma_g \). Given \( \sigma^* \), the value of \( A \) read off this line is the optimal value \( A^* \). Notice that the bottom part of the figure is drawn so that as we move down, \( A \) is increasing.

Now let’s ask ourselves what happens when the interest rate increases from \( i_1 \) to \( i_2 \). This situation is shown in Figure 4. Because \( \sigma_g \) is unchanged, the Equation 2 line in the bottom half of the figure does not change. However, the slope of the opportunity locus does increase as \( i \) increases. Thus the opportunity locus rotates up and we move to point C at the tangency

\(^5\)This assumption is not critical to the results. If \( E(g) \neq 0 \), it can be added to the interest term \( i \), and the analysis proceeds as indicated.
of the new opportunity locus and the indifference curve. As you can see, the optimal level of risk increases from $\sigma_1^*$ and $\sigma_2^*$, and the optimal fraction of the portfolio in bonds rises from $A_1^*$ to $A_2^*$. The result is that as the interest rate on bonds rises, the demand for money falls; that is, $1 - A$, the fraction of the portfolio held as money, declines.\(^6\)

Tobin's model then yields the same result as Keynes's analysis of the speculative demand for money: It is negatively related to the level of interest rates. This model, however, makes two important points that Keynes's model does not:

1. Individuals diversify their portfolios and hold money and bonds at the same time.
2. Even if the expected return on bonds is greater than the expected return on money, individuals will still hold money as a store of wealth because its return is more certain.

\(^6\)The indifference curves have been drawn so that the usual result is obtained that as $i$ goes up, $A^*$ goes up as well. However, there is a subtle issue of income versus substitution effects. If, as people get wealthier, they are willing to bear less risk, and if this income effect is larger than the substitution effect, then it is possible to get the opposite result that as $i$ increases, $A^*$ declines. This set of conditions is unlikely, which is why the figure is drawn so that the usual result is obtained. For a discussion of income versus substitution effects, see David Laidler, The Demand for Money: Theories and Evidence, 4th ed. (New York: HarperCollins, 1993).
FIGURE 4
Optimal Choice of the Fraction of the Portfolio in Bonds as the Interest Rate Rises

The interest rate on bonds rises from \( i_1 \) to \( i_2 \), rotating the opportunity locus upward. The highest indifference curve is now at point \( C \), where it is tangent to the new opportunity locus. The optimal level of risk rises from \( \sigma_1 \) to \( \sigma_2 \), and then Equation 2, in the bottom half of the figure, shows that the optimal fraction of the portfolio in bonds rises from \( A_1^* \) to \( A_2^* \).