The Multiplier

We introduce the concept of the multiplier in Chapter 11 of the text. There are develop a number of different multipliers for: investment spending (I), government spending (G), taxes (T), exports (X) and autonomous consumption (a). While we have not explicitly derived all of these multipliers, Hall and Lieberman do discuss how the size of the multiplier is affected by wealth (which affects the “a” in the consumption function, taxes and transfers payments as a function of income (equation (4) below), and imports (equation (8) below).

I am always asked: “Which multiplier should we use?” The answer is: It depends on the model you are working with – closed economy, lump-sum tax, income tax, open economy.

All of the multipliers we derive in this class are a sub-set of a general macro model for an open economy with taxes and imports depending on the level of national income (Y). This model is described by the following (8) equations:

(1) \( Y = C + I + G + (X - M) \) - equilibrium condition.
(2) \( C = a + b(Yd) \) - consumption equation.
(3) \( Yd = Y - T \) - disposable income.
(4) \( T = T_0 + t(Y) \) - tax equation.
(5) \( I = I_0 \) - investment.
(6) \( G = G_0 \) - government spending.
(7) \( X = X_0 \) - exports.
(8) \( M = mY \) - import equation.

Substitute:
(9) \( Y = a + b(Y - T) + I_0 + G_0 + X_0 - mY \)
(10) \( Y = a + b(Y - (T_0 + tY)) + I_0 + G_0 + X_0 - mY \)
(11) \( Y = a + b(Y - T_0 - tY) + I_0 + G_0 + X_0 - mY \)
(12) \( Y = a + bY - bT_0 - btY + I_0 + G_0 + X_0 - mY \)

Now solve this model for Y:

(13) \( Y - bY + btY + mY = a - bT_0 + I_0 + G_0 + X_0 \)

(14) \( Y = \frac{a - bT_0 + I_0 + G_0 + X_0}{1 - b + bt + m} \)
The government spending and lump-sum tax multipliers are:

\[
\frac{\Delta Y}{\Delta G} = \frac{1}{1 - b + bt + m} \quad \text{and} \quad \frac{\Delta Y}{\Delta T_0} = \frac{-b}{1 - b + bt + m}
\]

**No Foreign Sector and No Income Tax**

Now, let’s work backwards. What happens if there is no foreign sector (a closed economy) and taxes do not depend on income (T is lump-sum)? Equations (7) and (8) listed above disappear and the tax rate (t) disappears. \( X, t \) and \( m \) in Equations (14) and (15) are equal to zero:

(16)

\[
Y = \frac{a - bT_0 + I_0 + G_0}{1 - b}
\]

(17)

\[
\Delta Y = \frac{a - b\Delta T_0 + \Delta I_0 + \Delta G_0}{1 - b}
\]

We get the investment spending, government spending and tax multipliers from Chapters 11 and 12:

\[
\frac{\Delta Y}{\Delta I} = \frac{\Delta Y}{\Delta G} = \frac{1}{1 - b} \quad \text{and} \quad \frac{\Delta Y}{\Delta T} = \frac{-b}{1 - b}
\]