2. Since \( R \) is Noetherian, the zero-ideal \((0)\) in \( R \) has a minimal primary decomposition \( q_1 \cap \cdots \cap q_n \), where \( q_j \) is \( p_j \)-primary and the \( p_j \)'s are the prime ideals associated to \((0)\). By a theorem proved in class, each \( p_j \) is of the form \((0 : x_j) = \text{Ann} x_j\) for some \( x_j \in R \), and thus consists of zero-divisors. In fact, the set of zero-divisors in \( R \) is precisely the union of the \( p_j \)'s, by Proposition 4.17 in A-M. Thus \( S \) is the intersection of the complements of the \( p_j \)'s.

Now the prime ideals of \( S^{-1}R \) are precisely the \( S^{-1}p \), with \( p \) a prime ideal of \( R \) not meeting \( S \). Not meeting \( S \) means \( p \subseteq \bigcup_j p_j \). By the “Prime avoidance” theorem proved in class, that means \( p \subseteq p_j \) for some \( j \). If \( S^{-1}p \) is maximal, then in fact \( p \) must be equal to \( p_j \) for some \( j \), and so there are only finitely many maximal ideals in \( S^{-1}R \).

If \( R \) is not Noetherian, this argument falls apart. In fact, if \( I \) is an infinite set and if \( R = \prod_{i \in I} \mathbb{F}_2 \), then \( R \) is a Boolean ring (each element is its own square, since this holds in \( \mathbb{F}_2 \) and passes to products). Furthermore, the only element of \( S \) is 1, since an element of \( R \) that is not the identity must have a coordinate 0 at some point \( i \in I \), and then is a zero-divisor (its product with the element \( e_i \) that is 1 at this value of \( i \) and 0 everywhere else is 0). So \( S^{-1}R = R \), which is not Noetherian. Indeed, if \( i_1, i_2, \cdots \) is an infinite sequence in \( I \), then

\[
(0) \subsetneq (e_{i_1}) \subsetneq (e_{i_1}, e_{i_2}) \subsetneq (e_{i_1}, e_{i_2}, e_{i_3}) \subsetneq \cdots
\]

in an infinite non-terminating ascending chain of ideals. The kernel of evaluation at \( i \in I \), \( p_i : \prod_{i \in I} \mathbb{F}_2 \to \mathbb{F}_2 \), is a maximal ideal \( p_i \) that contains \( 1 - e_i \) but not \( 1 - e_j \) for any \( j \neq i \), so \( R \) has infinitely many maximal ideals.