Detailed concepts and methods list for Midterm2:
(subject to upgrade)

General:

- given any complex function or complex integrand \( f(z) \), ALWAYS first identify all singularities and branch cuts
- given any ODE, ALWAYS begin by scaling the equation near singular points and infinity, and find the zero order asymptotic behaviour in these regions

Detail:

- singular points, classification, implication of Fuch’s theorem
- asymptotic solution of ODE near singular points
- existence of power series solutions at regular singular points corroborated with local asymptotic behaviors
- non-existence of power series solutions at irregular singular points corroborated with WKB solution and essential singularity at these points
- the latter often includes behaviour at infinity
- “scaling” leads to ansatz for regular or singular perturbation methods
- self-consistency of solution as the test on rigor of ansatz
- \( \int dk \exp(kx) f(k) \) as general integral representation, useful for highest power of \( x \) smaller than order of ODE
- integral solutions for appropriate choice of contours and end points
- example: \( \exp(-k^3) \rightarrow 0 \) in 3 regions of complex \( k \) space at infinity
- example: \( \exp(1/k) \rightarrow 0 \) as \( \text{Re}(k) \rightarrow 0 \) from the negative side
- independent contours
- rules for asymptotics (posted separately)