Important Results - Complex Analysis

\[ \oint \frac{1}{z} \, dz = 2\pi i \text{ for } \oint \frac{1}{z^{n+1}} \, dz = 0, \text{ for } n \geq 1 \]

Furthermore, for simply connected domain:

\[ \int_{C_2} f(z) + f(z) - f(z) = 0 \]

Where \( p(z) = f(z) \)

\[ \frac{1}{z} \int_{C_2} f(z) = 0 \]

\[ \oint_{C_1} f(z) = \oint_{C_2} f(z) \]

\[ \oint_{C_2} f(z) = 0 \]

X = singular point

\( mm = \text{ branch cut} \)

\( xx = \text{ singular point} \)

\( \text{Paths} \)

\( \text{are equivalent} \)

\( C_{2}, C_{3} \)

\( C_{1} \text{ and } C_{3} \)

Simplify connected domain.

Similarly, connected domain in topology, a path preserved in complex analysis of paths, providing

\[ \int_{C_2} f(z) = 0 \]
\[ \lim_{t \to \infty} \int_{a}^{\infty} f(t) \, dt = \lim_{t \to \infty} \int_{a}^{t} f(t) \, dt. \]

[Diagram of a region in the plane, possibly a semicircle or a quadrant, with coordinates labeled.]

\[ 0 < y = 0 = \lim_{t \to \infty} \int_{a}^{t} f(t) \, dt \]

\[ \frac{1}{1 - \frac{1}{n}} + \frac{1}{a - 1} + \frac{1}{a - 2} + \ldots + \frac{1}{a - n} = 0 \]

Where \( z \) are singularities of \( a_n \) around 0.

\[ \text{Residue thm.} \]