9.1
Try a Frobenius series type solution for Legendre functions* in powers of \( s = (1+x) \). Start from the complete Legendre equation for arbitrary \( \nu \). Rewrite this equation in the \( s \) variable and then try the series solution.

(a) Show that only one solution is obtainable. Check that this is consistent with the asymptotic behaviour of Legendre functions as \( s \to 0 \) (or \( x \to -1 \)).

(b) Show that the one good series solution may diverge at \( x=1 \) (a simple ratio test suffices). (In fact, this solution does, in general, diverge at \( x=1 \).)

(c) Show that the series terminates if \( \nu = n = \text{integer} \). Thus, for integer \( \nu \), one solution is well-behaved at both \( x=\pm 1 \). Find these integer solutions for \( n=0, 1, \text{and } 2 \), rewriting them in the \( x \) variable.

(d) *Compare (just browse) what you have found with what is asked for in AWH (7th Ed) Chapter 8 Problem 3.1.

9.2
(a) Try the Frobenius method to find solutions if \( y(x) \) satisfies the ODE \( y'' = y/x^4 \). Assume \( a_0 \) to be non-zero in commencing your trial solution. How many solutions can be found for this form of a series expansion?

(b) Find the leading order asymptotic behavior of this equation as \( x \to 0 \). Note that the Laurent expansions of the asymptotic solutions about \( x = 0 \) have a radius of convergence which does not include \( x = 0 \); and, in any case, the Laurent expansion does not have the same form as the starting ansatz of the Frobenius series.

9.3
The function \( \phi(x) \) satisfies

\[
(d/dx)[x d\phi/dx] - \phi/x = 0, \quad \phi(1) = 1, \quad \phi(\infty) = 0.
\]

Note that the domain excludes \( x=0 \), hence division by \( x \) is ok.

1.1 Find the solution by directly solving the ODE
1.2 Find the Green function for this problem, sketch a plot vs \( x' \)
1.3 Find \( \phi(x) \) using the Green function and compare

[1D Green function notes are posted.]