Solutions to some homework problems

**Problem 8.** A hypothetical polymer chain of 100 segments of length \( b = 3 \) Å has the root-mean-square (RMS) end-to-end distance of 100 Å. Use these data to answer the following questions:

(A) Does this chain behave as an ideal freely-jointed chain? To answer this question, compare its RMS end-to-end distance with that for a freely-jointed chain.

The expected RMS end-to-end distance for an ideal freely jointed chain of this length is \( b\sqrt{N} = 30 \) Å. The “real” RMS (=100 Å) of this chain is greater than this number, therefore, it does not behave as an ideal chain.

(B) Calculate the number of Kuhn’s statistical segments in the chain and the Kuhn’s statistical segment length.

Flory’s characteristic ratio: \( C_\infty = \frac{<L^2>}{Nb^2} = \frac{100^2 A^2}{100 \times 3^2 A^2} \approx 11.1 \). The number of Kuhn’s statistical segments \( N_e = N / C_\infty = 9 \); the length of the Kuhn’s statistical segment is \( b_e = b \times C_\infty = 33.3 \) Å.

**Problem 9.** The genome of T2 bacteriophage is \( 1.7 \times 10^5 \) nucleotides long. Assume you have a linear piece of DNA of this length, and this DNA adopts a random coil conformation. The Kuhn’s statistical segment length for DNA is 120 nm, and the base-pair spacing along the DNA is 0.34 nm. Use these data to answer the following questions:

(A) Determine the root-mean-square end-to-end distance for a random coil that a DNA of this length can form.

First, let’s determine Flory’s characteristic ratio from these data: \( C_\infty = \frac{b_e}{b} = \frac{120}{0.34} \approx 353 \). Now we can calculate the RMS distance either as \( \sqrt{<L^2>} = \sqrt{Nb^2C_\infty} = \left(1.7 \times 10^5 \cdot (0.34nm)^3\right)^{1/2} \) or by first calculating \( N_e \) (see the previous problem) and then using the equation for RMS of an ideal chain of \( N_e \) virtual segments of length \( b_e=120 \) nm each. Either way, the result should be: \( \sqrt{<L^2>} = 2.6 \times 10^3 \) nm.

(B) Is the size of this coil larger or smaller than what one could expect for an ideal freely-jointed random coil of the same contour length? Determine the factor that relates the sizes of the two coils.

The factor that relates the sizes of the two coils is \( \text{RMS}_{\text{real}}/\text{RMS}_{\text{ideal}} = \sqrt{C_\infty} \approx 18.8 \). (Note that in Kuhn’s model the contour length is preserved.) Thus, this coil is bigger than an ideal coil.