Lecture Four

Simultaneous Move Games

Simultaneous Move Games: Outline

- Description and Defense of Simultaneous Move games -- simultaneous versus sequential.
- Pure strategies and mixed strategies.
- Dominant strategy equilibria.
- Best responses and Nash equilibria.
- Examples: Games with multiple equilibria.
- Games with no equilibria in pure strategies.
Simultaneous Move Games: Their Characteristics.

- While in sequential games, there is a specific order of play, a more important feature is that players observe what rivals have done in the past.
- In simultaneous move games, this feature is absent.
- Instead, all players select strategies without observing the choices of their rivals.

Simultaneous Move Games: Their Characteristics.

- The most “intuitive” explanation of this is that players choose at exactly the same time.
- A simple example is rock, scissors, paper. Both players make their choice at the same time.
- Because players do not know the other moves, these are called “games of imperfect information.”
Simultaneous Move Games:
Their Characteristics.

• Moving at exacty the same time, seems a little special.
• So a better story is that, the timing should not be taken literally.
• Instead, players move without being able to see the strategy choices of other players.
• Think of a voting “game”. Although, voters do not literally vote at the same time, whenever they vote, they do not know what all other voters have done.

Different Representations of Simultaneous Move Games.

• The most common way to represent (2 player) simultaneous move games is in a matrix form.
• One player selects a row at the same time as the other player selects a column.
• The “cell” that emerges is the outcome of the game.
• Traditionally, the first entry in a cell represents the payoff of the row player, the second entry is the payoff of the column player.
Rock, Paper, Scissors (A Zero-Sum Game).

<table>
<thead>
<tr>
<th></th>
<th>Rock</th>
<th>Paper</th>
<th>Scissors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock</td>
<td>0, 0</td>
<td>-1, 1</td>
<td>1, -1</td>
</tr>
<tr>
<td>Paper</td>
<td>1, -1</td>
<td>0, 0</td>
<td>-1, 1</td>
</tr>
<tr>
<td>Scissors</td>
<td>-1, 1</td>
<td>1, -1</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

Game Tree Representation

- Simultaneous move games can also be represented by game trees.
- Information sets are used to emphasis the simultaneity (or imperfect information.)
- With game trees is is easier to have games with more than two players. (Though it is still not simple.)
Telex vs. IBM and Microsoft, extensive form

Pure versus Mixed Strategies.

- Recall that there may be many strategies which are “Best Responses”
- “Pure strategies” result when players pick one and only one strategy from their BR.
- “Mixed Strategies” result when players randomize (not necessarily equally) over the strategies in their set of Best Responses.
Pure versus Mixed Strategies.

- In sequential games with perfect information, players will typically select pure strategies.
- This is not always the case in simultaneous games.
- For this lecture, we focus on Pure Strategies, though.
- The next lecture considers Mixed Strategies.

Dominant and Dominated Strategies.

- Suppose the strategy set of player $i$ is $S_i = \{s_1, s_2, \ldots, s_N\}$, so $i$ has $N$ pure strategies to choose from. The strategy set of player $j$ is $Z_j = \{z_1, z_2, \ldots, z_M\}$.
- Recall the definition of a dominant strategy.
- Strategy $s_i$ is a dominant strategy for player $i$ if $U_i(s_i, z_j) \geq U_i(s_k, z_j)$ for every $s_k$ and for every $z_j$. (And for at least one comparison, the inequality is strict ($>$) rather than weak ($\geq$).
Dominant and Dominated Strategies.

- If all of the comparisons are strict, then we say that $s_i$ strictly dominates the other strategies or is strictly dominant.
- Otherwise, it is weakly dominant.
- If player $i$ has strategies $s_i, s_k$ with the feature that $U_i(s_i, z_j) > U_i(s_k, z_j)$ for every $z_j$ then we say that $s_k$ is dominated by $s_j$.
- Note the difference between this condition and that for dominant strategies. In particular, $s_i$ may not be a dominant strategy for $s_k$ to be dominated.

Dominant and Dominated Strategies.

- Dominant and dominated strategies provide us with our first steps in analyzing simultaneous move games.
- Step 1: Identify any dominant strategies for all players (they may or may not exist)
- If each player has a dominant strategy, then we are done.
- We predict that players will play dominant strategies. If there are many dominant strategies, then any of them are possible.
Dominant and Dominated Strategies.

- Step 2: If there are no dominant strategies, identify all strategies which are dominated by some other strategies.
- Consider eliminating the dominated strategies (since players should not play dominated strategies)
- What is left is our candidates for the strategies that players will play.

Prisoners’ Dilemma

- Two (guilty) prisoners are interrogated separately.
- Each is told that the police have enough evidence to convict both to 3 years.
- Each is told if he confesses, he will get a break and a sentence of only 1 year while the other gets 25 years.
- If they both confess, they get 10 years.
- What do they do? Remember, they cannot talk to each other.
Prisoners’ Dilemma: A Game Where Both Players Have Dominant Strategies

<table>
<thead>
<tr>
<th>Prisoner 2</th>
<th>Confess</th>
<th>Deny</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confess</td>
<td>10yr,10yr</td>
<td>1yr,25yr</td>
</tr>
<tr>
<td>Deny</td>
<td>25yr,1yr</td>
<td>3yr,3yr</td>
</tr>
</tbody>
</table>

Prisoners’ Dilemma

- Observe that each player has a dominant strategy.
- If they each play their dominant strategy, they each get 10 years.
- Of course, 3 years is better than 10. Why can’t they just keep quiet?
- In some games, the structure can be very harmful for the players.
Nash Equilibrium i)

- In many games, players do not have dominant strategies.
- How can we predict or solve these games.
- Recall the notion of Best Responses.
- Given an environment, $e$ a strategy $s$ is a best response if $s$ gives a player a higher payoff than any other strategy.
- Formally, $U_i(s, e) \geq U_i(s', e)$ for every $s'$.

Nash Equilibrium ii)

- Now, suppose we think of the particular environment, $e$, as a strategy choice of the rival player(s). For example, $e=\text{right}$ or $e'=\text{left}$.
- In this case, $s(e)$ is the player 1’s best response to player 2’s strategy choice $e$.
- Similarly, we could think of player 2’s choice of strategy $e$ given “environment” $s$.
- If $e$ is a best response for 2, $U_2(e, s) \geq U_i(e', s)$ for every $e'$.
- In this case, $e(s)$ is the player 2’s best response to player 1’s strategy choice $s$. 
Nash Equilibrium iii)

- A **Nash Equilibrium** is a pair of strategies \((s,e)\) with the feature that for player 1, \(s\) is a best response given \(e\) and for player 2, \(e\) is a best response given \(s\).
- Prove for yourself, that using Rollback or Backward Induction, the outcome has the feature that every player plays a best response to the other player(s) strategy.

Nash Equilibrium iv)

- Prove for yourself, that in games where players have dominant strategies, playing a dominant strategy is a best response to the other player(s) strategy.
- When there are more than 2 players, a Nash Equilibrium is a profile of \(n\) strategies, each of which is a best response to the other \(n-1\) strategies.
Nash Equilibrium v)

- Finding Nash Equilibria can be difficult.
- When there are dominated or dominant strategies the process is simpler.
- Sometimes, we can use calculus as we will see in market games.
- Sometimes the best we can do is cell by cell inspection.
- See the next game, “Chicken”

Example: Chicken

- James and Dean engage in the following foolish and self-destructive game. (Do NOT try this at home!)
- They drive their cars at each other. The first to swerve loses. If neither swerve, they crash. If both swerve, then it’s a draw.
- The “payoffs” to this game are shown in the next bimatrix game.
Nash Equilibrium in the Game of Chicken.

Example: Chicken

- Note that neither player has a dominant strategy.
- What is Dean’s BR if James chooses “Swerve”?
- What is James BR if Dean chooses “Straight”?
- What is a Nash Equilibrium of this game?
Example: Chicken

- Consider the same reasoning starting with the assumption that Dean chooses “Swerve”
- This points out that there can be multiple Nash Equilibria.

2 Pure Strategy Nash Equilibrium in the Game of Chicken.

- Step 2 on dominated strategies suggests eliminating these strategies since it seems natural that players will never choose them.
- Although this is correct, a question arises as to which strategies to eliminate and when.
- The next example shows that there are multiple solutions to this process and the solutions that emerge depend on the order that elimination occurs.

Finding Nash Equilibria: Elimination of Dominated Strategies: Example

- there are 2 players, A and B. Each may buy 0, 1, or 2 lottery tickets for $5 each.
- There is a prize of $10. If each buy the same positive number of tickets they split the prize.
- If one buys more tickets than the other, he wins the prize.
- if no tickets are bought, no prize is awarded.
Elimination of Dominated Strategies: Not always a unique outcome.

Finding Nash Equilibria: Elimination of Dominated Strategies: Example

- Notice that strategy 2 is dominated by 1 for both players. Suppose we eliminate it for both.
- Now notice that strategy 0 is dominated by 1 for both.
- If we eliminate 0, we get a unique outcome, (1,1).
Finding Nash Equilibria: Elimination of Dominated Strategies: Example

- However, (0,1) and (1,0) are also Nash equilibria. (Prove it for yourself.)
- Suppose we did a different type of elimination.
- Focus on Player A first. Since we know Strategy 2 is dominated for him, eliminate it.
- Now focus on Player B. With Strategy 2 gone from A’s choice set, both 0 and 2 are dominated for B. Remove them.
- Now, for A, either 0 or 1 are best responses. Thus, we could have (0,1) as the outcome.

Finding Nash Equilibria: Elimination of Dominated Strategies: Example

- The moral is that while elimination of dominated strategies is valuable, we have to be careful in drawing firm conclusions from it.
- Obviously, we could have started with Player B first and reached (1,0) as outcomes.
- The order in which elimination occurs can affect where we end up.
Games with No Nash Equilibria in Pure Strategies.

- Consider the tennis example given by Dixit and Skeath.
- Seles is returning a shot to Hingis and is deciding between crosscourt and down the line.
- When Seles hits DL, and Hingis guesses right, Seles wins 50% of the time, when Seles hits DL and Hingis guesses wrong, Seles wins 80% of the time, etc.

Seles Tries to Make Hingis Guess: A game with no Pure Strategy Equilibrium.

<table>
<thead>
<tr>
<th></th>
<th>DL</th>
<th>CC</th>
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<tbody>
<tr>
<td><strong>Seles</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DL</td>
<td>50,50</td>
<td>80,20</td>
</tr>
<tr>
<td>CC</td>
<td>90,10</td>
<td>20,80</td>
</tr>
</tbody>
</table>
Games with No Nash Equilibria in Pure Strategies.

- Suppose that Hingis thinks Seles is going DL, then she wants to guard against DL.
- But if Seles thinks Hingis is guarding DL, she want to go CC.
- Now if Hingis thinks Seles is going CC, then she wants to guard against CC.
- But if Seles thinks Hingis is guarding CC, she want to go DL.
- Here, we have a cycle, no cell is a BR for each player against the other.

Games with No Nash Equilibria in Pure Strategies: Conclusion

- This makes sense. If one player thought she knew what the other was doing, then the other player would always want to change her strategy.
- But does this mean there is no solution?
- Not necessarily, players are going to want to fool each other and they can only do this if they are unpredictable.
- This leads us to Mixed Strategies.