Aspects of Dilute 2 Dimensional Bose Gas

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1 Theory of Homogeneous 2D Dilute Bose Gas

1.1 Emergence of XY Model and Superfluidity

In statistical physics, systems of low dimensions is often of interest since dimensionality can qualitatively alter the physics. It is well known that in 3D, a weakly interacting Bose gas undergoes Bose-Einstein condensation, after which a macroscopic number of particles occupy the ground state, forming a coherent condensate. Not quite surprisingly, this picture breaks down in two dimensions. The famous Mermin-Wagner-Hohenberg Theorem states that long wavelength thermal fluctuations destroy long range order in two dimensions, preventing formation of condensate. [1]

Though long range order is destroyed and condensation is rendered impossible, the behavior of 2D ideal(non-interacting) Bose gas is far from that of simple thermal gas. To show this it is convenient to exploit the X-Y model from modern condensed matter theory.The discussion here follows [2].

Using the language of path integral, the action of a d- dimensional ideal Bose gas is

$$S = \int d^d x dt [i \frac{1}{2}(\phi^* \partial_t \phi - \phi \partial_t \phi^*) - \frac{1}{2m} \partial_x \phi^* \partial_x \phi + \mu \phi^* \phi] \tag{1}$$

At low temperature one expects the low-energy excitations to play a dominant role, justifying the construction of a low-energy effective theory.

Rewrite the field as

$$\phi = \sqrt{\rho_0 + \delta\rho}\, e^{i\theta} \tag{2}$$

The low-energy excitations are described by θ field. And to derive the low-energy effective action we only need to integrate out in path integral the density fluctuation which is of high frequency.

$$\mathbf{Z} = \int \mathbf{D}\theta e^{i \int d^d x dt [-\frac{\rho_0}{2m}(\partial_x \theta)^2 + \frac{\rho}{2\mu}(\partial_t \theta)^2]} \tag{3}$$

In the derivation it is assumed that θ field changes slowly in spacetime.

This action corresponds precisely to X-Y model, which can be thought of as describing a magnet in which the direction of spin is constrained on a circle.

$$S_{eff} = \int d^d x dt [\frac{\rho}{2\mu} |\partial_t z|^2 - \frac{\rho_0}{2m} |\partial_x z|^2] \tag{4}$$

Since our aim is to heuristically discuss the system's behavior at finite temperature, an imaginary-time path integral should be performed, "integrating out" the time dependence in the above action, yielding

$$S_{eff} = \int d^d x \frac{R}{2T} (\partial_x \theta)^2 \tag{5}$$

in which $R = \frac{\phi_0^2}{m}$ is the phase rigidity. The correlation function for $d \neq 2$ is

$$< e^{in\theta(x)}e^{-in\theta(0)} >= e^{-C_d n^2 \frac{T}{R}|x|^{2-d}} \tag{6}$$

showing that for $d > 2$ the system possesses true long range order $\lim_{x\to\inf} < e^{in\theta(x)}e^{-in\theta(0)} >= 1$and for $d < 2$ the correlation is exponentially decaying.

In this sense, $d = 2$ is marginal and one obtains algebraic-long-range-order

$$< e^{in\theta(x)}e^{-in\theta(0)} >\propto |x|^{-n^2T/2\pi R} \tag{7}$$

Knowing the absence of true long-range order, whether superfluidity will be present becomes an interesting question. An elegant treatment of 2D dilute interacting Bose gas was given by Popov[**??**], who derived an analytic expression of superfluid density which will disappear at finite temperature, which he took to be the phase transition temperature.

Popov exploited the so-called t-matrix description of interaction and constructed a perturbation expansion which generalize the earlier field theoretical approach to interacting Bose gas at T=0 by Beliaev. It is worth mentioning that Popov's strategy is conceptually similar to modern renormalization group approach[4]. He first use an auxiliary momentum k_0 to separate the field into fast and slow modes. The former "sees" the latter mode as effective condensate. After applying different schemes of perturbation to the integration of fast and slow modes, the dependence on k_0 is removed and one arrived at expressions[4] for the density of non-superfluid component

$$\rho_n = \frac{\beta}{(2\pi)^2}\int d^k \frac{k^2}{2m} n_B(k)[1 + n_B(k)] \tag{8}$$

and superfluid component

$$\rho_s = \frac{m\mu}{4\pi}(lnE_0/\mu - 1) - \frac{1}{(2\pi)^2}\int \frac{d^2k}{2m} k^2 n_B k \times (1/E_k - \beta[n_B(k) + 1]) \tag{9}$$

where $n_B(k) = (e^{\beta E_k} - 1)$ and E_0 is the natural high energy cutoff. The chemical potential is given by

$$\rho = \frac{m\mu}{4\pi}(lnE_0/\mu - 1) - \frac{1}{(2\pi)^2}\int \frac{d^2k}{2m} k^2 n_B(k) \tag{10}$$

1.2 Vortices and BKT Phase Transition

Popov's approach, although elegant and systematic, suffers from several drawbacks. Firstly, it was shown by Fisher and Hohenberg [6] that the diluteness condition assumed by Popov is $lnln(1/na^2) >> 1$, which is unrealizable in experiments. Secondly,as shown in [1] the critical region of this problem is large ($\propto 1/lnln(1/na^2)$), implying that Popov's mean field approach is unreliable in this region. In fact, it is in this region where new physics is lurking, which is associated with the excitations of quantum vortices that were taken to be of minor influence by Popov.

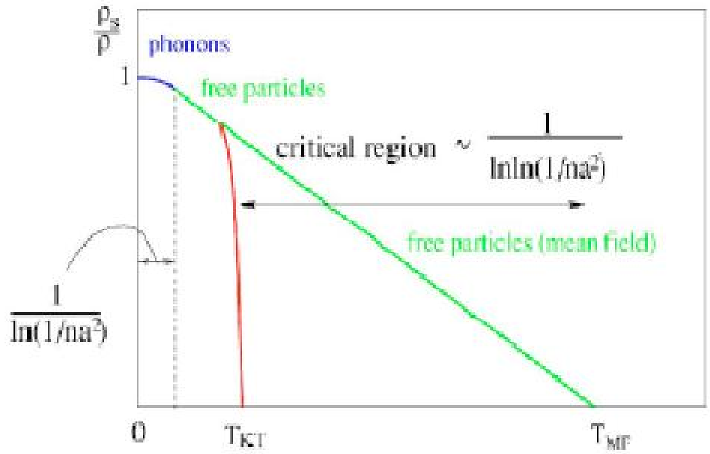

Figure 1: Schematic phase diagram of a uniform Bose gas

In the context of dilute Bose gas discussed here, vortices are excitations that lead the field to change by a multiple of 2π around the cores. The influence of such vortices on Bose gas was first revealed in the discussing the critical properties of 2D XY model by Berenzskii[7], Kosterlitz and Thouless[8]. They showed that vortex excitations will lead to phase transition at finite temperature and change the algebraic long range order into short range order.

For 2D dilute Bose gas, this means that the gas will undergo a phase transition at temperature lower than the vanishing point of superfluidity[1]. Such phase transition is characterized by the proliferation of single vortices, *i.e.* phase defects.

1.3 Characteristics of BKT Phase Transition

The physics of BKT phase transition is closely connected with vortices. While at temperature higher than BKT phase transition, a system with isolated vortices is favorable, below the transition point vortices can only exist as vortex-antivortex pairs. The 2D X-Y model can be mapped onto two dimensional Coulomb gas[9], implying that below T_{BKT} vortices can be treated as bound charges and across T_{BKT} these paired charges dissociate and form vortex plasma above T_{BKT}.

Another feature associated with BKT phase transition in homogeneous Bose gas is universal superfluid density jump,a universal relation of superfluid density at phase transition point derived by Nelson and Kosterlitz [4][**??**].

$$\frac{m^2 k_B T_{BKT}}{\hbar^2 \rho_s(T_{BKT})} = \pi/2 \tag{11}$$

2 2D Bose Gas in Experiments

Since the invention of laser cooling and experimental realization of Bose-Einstein Condensation(BEC), physicists have been employing cold atomic gases as an convenient tool of modeling quantum many-body systems[**??**]. To experimentally realize a 2D boson system, a optical trap sufficiently steep is created in the direction perpendicular to the plane so that the motion along that direction is "frozen out", *i.e.* essentially only occupying the lowest quantum state. But since excitation is still allowable, quasi-2D is a better description for such systems. As we will discuss below, new questions arise concerning the behavior of quasi-2D bose gas in these experimental setups.

2.1 Harmonically Trapped 2D Bose Gas: BKT or BEC?

[11]shows that in contrast to the homogeneous case, BEC is expected to be present at a non-interacting 2D Bose gas which is harmonically confined in the plane.

This result is of importance since experimentally the atomic gas is always trapped in the plane. Combining this and our previous knowledge about BKT phase, one is tempted to pose a challenging question: when a dilute quasi-2D Bose gas is transformed from normal to superfluid phase by lowering the temperature, will the system undergo a Bose-Einstein condensation, or BKT phase transition?

In the past several years a series of experiments were done in this aspect [5], [13],[14] with decisive evidence showing that at least for the parameters chosen in experiment, the transition to superfluid was BKT type.

To validate that type of transition requires identifying signatures of BKT picture, mainly:

- Proliferation of Vortices

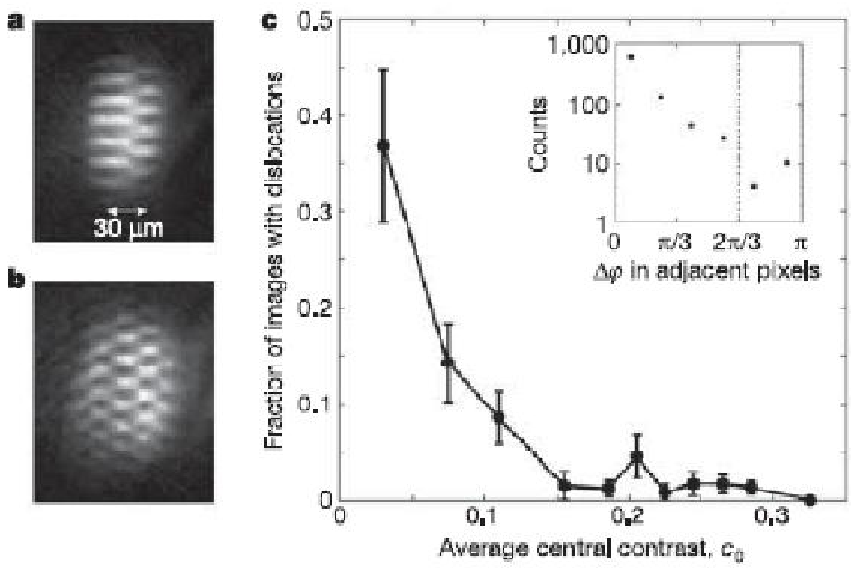

Figure 2: Proliferation of free vortices at high temperature. Temperature decreases along the positive direction of horizontal axis

- Emergence of Quasi Long Range Order

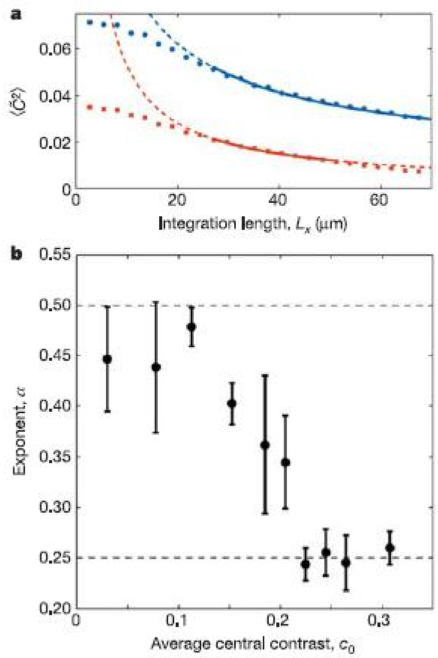

Figure 3: Emergence of quasi-long-range order in a 2D gas

- Phase Transition Conditions Different from BEC type

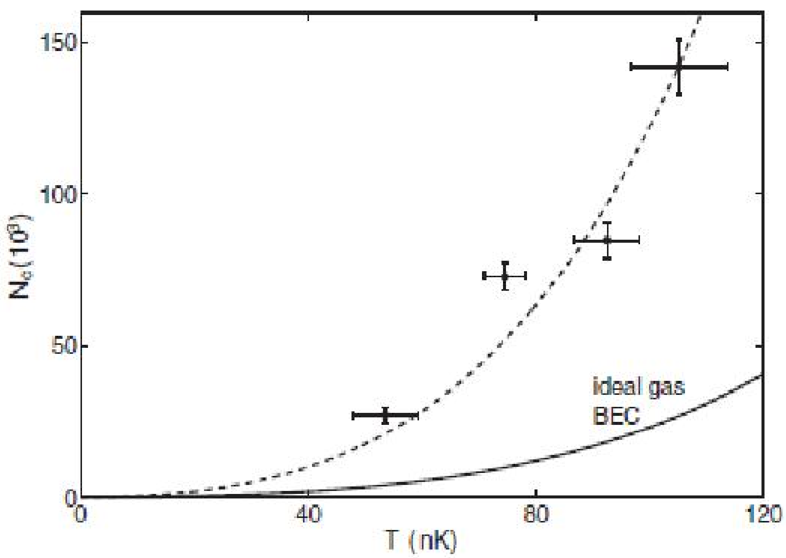

Figure 4: Schematic phase diagram of a uniform Bose gas

Although the above experiments pinned down the presence of BKT phase below a critical temperature, they suffer from the same problem, that the residual heat produced when the optical lattice is turned on prevents the system from achieving temperature low enough to observe BEC, as in formula [**??**]. This defect was partly remedied by theoretical work of [12], in which *abinitio* classical field simulation showed quantitative evidence of a vortex-free condensate phase and and its transition to BKT phase manifested by vortex-antivortex pairs at higher temperature.

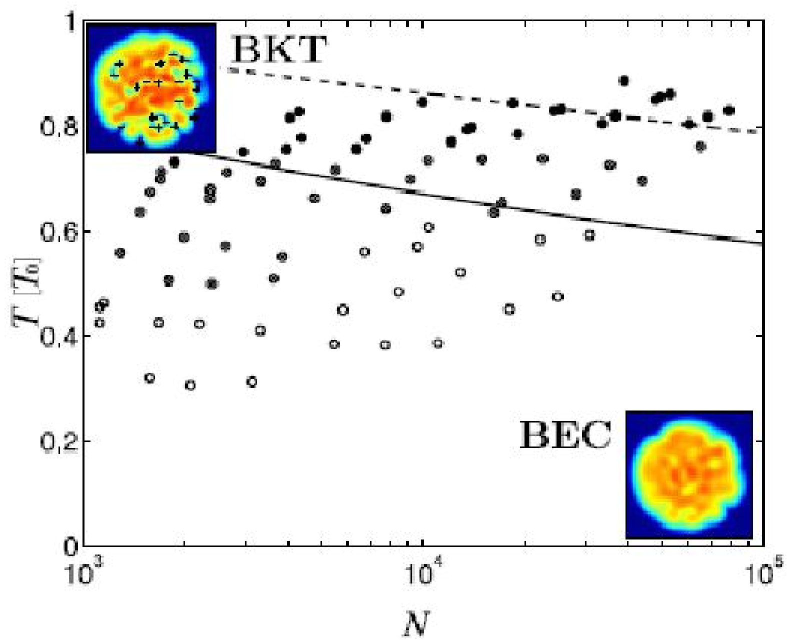

Figure 5: Low temperature phase diagram for quasi-2D Bose gas

To summarize, the problem concerning low temperature phase of experimentally producible quasi-2D Bose gas is essentially resolved. Since the gas is confined in a harmonic trap, as temperature is continually lowered the gas will first undergo a BKT phase transition into a superfluid characterized by vortex-antivortex pairs,and then into a vortex-free Bose-Einstein condensate.

2.2 Quasi-Condensate

Another feature of BKT phase that is not scrutinized by experiments mentioned sofar is the appearance of quasicondensate. As explained earlier, long wavelength phase fluctuations change true long-range order into algebraic long-range order.

When a weakly interacting 2D gas is in its superfluid state well below the fluctuation region, the correlation lengths of phase and density, R_c and r_c respectively, are well separated in scale($R_c >> r_c$)[15]. One can thus characterize quasicondensate by the amplitude of one-particle density matrix n_0 ($\rho(r) \approx n_0$,for $r_c << r << R_c$).

Locally the properties of quasicondensate are the same as real condensate. Therefore the m-particle local correlator K_m will be reduced by $1/m!$ compared to its normal value.

$$K_m = n^{-m} < [\Phi^\dagger(\vec{0}, 0)]^m [\Phi(\vec{0}, 0]^m > \tag{12}$$

Experimentally related is the measurement of spin-polarized hydrogen on a helium surface [**??**]. In its setup, the rate of three-body recombination is proportional to three-particle local correlator, which should drop by a factor of 6 in a quasicondensate. Interesting enough, experimentally the recombination rate starts to decrease well before the BKT transition point, suggesting formation of quasi-condensate or suppression of density fluctuation well before transition. This result is later supported by Monte Carlo study done by Kagan *et al.* [15].

Recently an intriguing result associated with quasicondensate in BKT phase was reported in[17]. Using absorption imaging after TOF(Time of Flight), a non-superfluid condensate was observed([6]), together with a superfluid component and a thermal one.

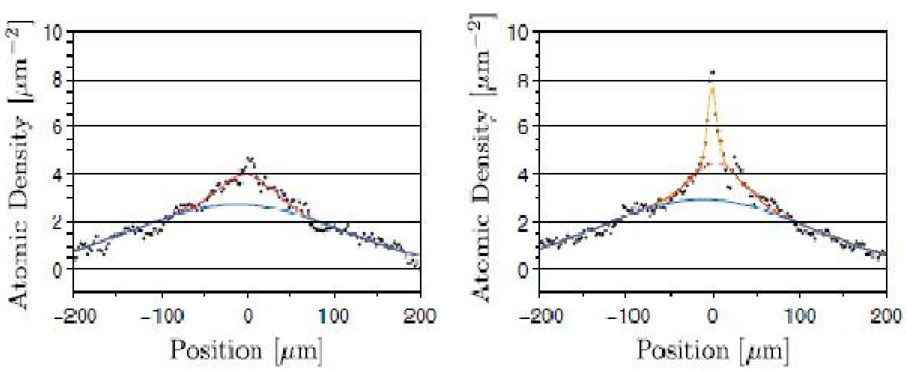

Figure 6: Cross Section of Density Profile after 10ms TOF. left: $n < n_{BKT}$ right:$n > n_{BKT}$

References

[1] Mermin, N. D., and H. Wagner, 1966, Phys. Rev. Lett. 22, 1133.

[2] Xiao-Gang Wen, Quantum Field Theory of Many-Body Systems, Oxford Graduate Texts, Oxford University Press, 2004

[3] Popov, V. N., 1983, Functional Integrals in Quantum Field Theory and Statistical Physics (Reidel, Dordrecht, Holland).

[4] Anna Posazhennikova, Colloquium: Weakly interacting, dilute Bose gases in 2D,Rev. Mod. Phys. Vol78, Oct-Dec, 2006

[5] Z. Hadzibabic et al., Nature (London) 441, 1118 (2006).

[6] Fisher, D. S., and P. C. Hohenberg, 1988, Phys. Rev. B 37, 4936

[7] Berezinskii, V. L., 1971, Sov. Phys. JETP 34, 610.

[8] Kosterlitz, J. M., and D. J. Thouless, 1973, J. Phys. C 6, 1181.

[9] Minnhagen, Petter, 1987, Rev. Mod. Phys. 59, 1001.

[10] I. Bloch, J. Dalibard, and W. Zwerger , arXiv.org:0704.3011, 2007.

[11] V. S. Bagnato and D. Kleppner., Phys. Rev. A. 44(11):7439-7441, 51 (1991).

[12] T. P. Simula and P. B. Blakie Phys. Rev. Lett. 96, 020404 (2006)

[13] P. Kruger et al Phys. Rev. Lett. 99, 040402 (2007)

[14] Stock et al. PRL 95, 190403 (2005)

[15] Kagan et al. PHYSICAL REVIEW A, VOLUME 61, 043608

[16] A. I. Safonov et al. Phys. Rev. Lett. 81, 4545 (1998)

[17] P. Clade et al. PRL 102, 170401 (2009)