A LOW-COMPLEXITY ADAPTIVE BLIND SUBSPACE CHANNEL ESTIMATION ALGORITHM

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ABSTRACT

A low-complexity adaptive blind subspace channel estimation algorithm is proposed for direct sequence spread spectrum CDMA systems. Compared with so-called hybrid adaptive channel estimation algorithms, where only the subspace estimation is carried out adaptively, the proposed algorithm is fully adaptive in that both subspace *and* channel estimates are updated recursively. The new algorithm is derived by exploiting common structural properties of plane rotation-based subspace trackers (e.g. Proteus, RO-FST, etc.). It is characterized by a low-complexity of implementation and numerical robustness over long periods of operation, an essential requirement for wireless radio applications. Moreover, we find in the case of a heavy loaded system that the proposed algorithm has better performance than the previous hybrid algorithms.

1. INTRODUCTION

Code Division Multiple Access (CDMA) with Direct Sequence Spread Spectrum (DS-SS) has proven to be an effective multiple access technique for modern wireless communication systems. Due to the multi-path nature of the wireless channel, CDMA signals experience a fading phenomenon, which severely degrades the performance of the receiver and thus limits the capacity of the CDMA system. Some diversity-combining schemes, such as RAKE receiver, space diversity, Space-Time Block Codes (STBC), etc., have been proposed to combat the fading problem. For all of these schemes, an accurate estimation of the channel response is necessary for the optimal combining.

Compared with the pilot-based channel estimation methods, it is more bandwidth efficient to implement blind channel estimation approaches. In the past decade, subspace-based blind channel estimation algorithms were at first proposed for frequency selective fading channel [1, 2] and later extended to dispersive channel [3], multiple antenna channel [4] and STBC channel [5]. A representative example of the above family is the batch algorithm by Liu and Xu [2], which first estimates orthonormal signal and noise subspaces via SVD on a data block (step 1) and then estimates the channel response by utilizing the orthogonal property of subspaces (step 2).

Liu and Xu's algorithm was originally proposed for time-invariant channels. In order to apply it for on-line estimation of time-variant channels, some hybrid (adaptive-batch) algorithms have been proposed that use subspace tracking to adaptively generate the subspaces; however, the batch method is still used in the second step and requires a considerable amount of computations [6]. Recently, a low-complexity, adaptive channel estimation algorithm based on the OPAST subspace tracker was proposed in [4]. However, OPAST only provides a basis of the signal subspace and cannot track the individual dominant eigenvectors and eigenvalues, which are needed in advanced subspaced-based multi-user detection. Furthermore, some numerical stability problems have been observed with OPAST when running it over a large number of time iterations.

In this paper, we propose a fully adaptive subspace-based channel estimation algorithm that overcomes the above limitations. The new algorithm is derived by exploiting common structural properties of plane rotation-based subspace trackers (e.g. [7, 8]), whose numerical robustness has been well established. The main advantage of the proposed algorithm is its low-complexity of implementation and numerical robustness over long periods of utilization, an essential requirement for practical operation in wireless radio applications. Moreover, we find in the case of a heavy loaded system that the proposed algorithm has better performance than the previous hybrid algorithms.

The paper is organized as follows. Section 2 introduces the system model of a synchronous CDMA system in frequency selective fading channel. Section 3 describes the proposed algorithm. Results of computer experiments are presented in Section 4. This is followed by a brief conclusion in Section 5.

2. SYSTEM MODEL

We develop our algorithm for a synchronous CDMA system operating in a frequency selective fading channel; however, it can be easily extended to asynchronous systems. In a DS-SS CDMA system, information symbols are modulated by pre-assigned signature waveforms of length L_c. For the qth user, the normalized signature waveform is represented by $\mathbf{c}_q^T = [c_{q,1}, \cdots, c_{q,L_c}]$. At time t, the spreaded transmitted signal of the q-th user $\mathbf{x}_q(t)$ may be represented in vector form as

$$\mathbf{x}_q(t) = A_q(t) b_q(t) \mathbf{c}_q \tag{1}$$

where $A_q(t)$ is the amplitude of the q-th user signal and $b_q(t)$ is the corresponding information bit. The frequency selective fading channel can be modelled as a time-variant FIR filter, namely:

$$\begin{aligned} \mathbf{y}_q(t) &= [y_{q,1}(t), \cdots, y_{q,L_c+L-1}(t)]^T \\ &= A_q(t) b_q(t) [\mathbf{c}_q \otimes \mathbf{h}_q(t)] \end{aligned} \tag{2}$$

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where $\mathbf{h}_q(t)$ is an $L \times 1$ discrete time-variant normalized channel impulse response vector and $\otimes$ denotes discrete convolution operation. Since the length of $\mathbf{y}_q(t)$ is $L_c + L - 1$ and the symbol duration is L_c chips, there exists an $(L-1)$-chip overlap at the receiver, or Inter-Symbol Interference (ISI), between $\mathbf{y}_q(t-1)$ and $\mathbf{y}_q(t)$. We may assume $L \ll L_c$ in the case that the time delay spread of the channel is much smaller than the symbol period [2]. To avoid the ambiguity caused by ISI, we will consider $\overline{\mathbf{y}}_q(t) = [y_{q,L}(t), \cdots, y_{q,L_c}(t)]^T$, the ISI-free section of $\mathbf{y}_q(t)$, in the rest of this paper. Denote $K \triangleq L_c - L + 1$ as the length of the vector $\overline{\mathbf{y}}_q(t)$. Then $\overline{\mathbf{y}}_q(t)$ may be expressed as

$$\overline{\mathbf{y}}_q(t) = A_q(t) b_q(t) \overline{C}_q \mathbf{h}_q(t) \tag{3}$$

where

$$\overline{C}_q \triangleq \underbrace{\begin{bmatrix} c_{q,L}(t) & \cdots & c_{q,1}(t) \\ c_{q,L+1}(t) & \cdots & c_{q,2}(t) \\ \vdots & \vdots & \vdots \\ c_{q,L_c}(t) & \cdots & c_{q,L_c-L+1}(t) \end{bmatrix}}_{K \times L} \tag{4}$$

Define the effective signature waveform of the q-th user as

$$\mathbf{w}_q(t) \triangleq \overline{C}_q \mathbf{h}_q(t). \tag{5}$$

Then the received noisy signal is expressed as

$$\begin{aligned} \mathbf{r}(t) &= \textstyle\sum_{q=1}^{P} \overline{\mathbf{y}}_q(t) + \mathbf{n}(t) \\ &= \textstyle\sum_{q=1}^{P} A_q(t) b_q(t) \mathbf{w}_q(t) + \mathbf{n}(t) \\ &= W(t) A(t) \mathbf{b}(t) + \mathbf{n}(t) \end{aligned} \tag{6}$$

where

$$\begin{aligned} W(t) &\triangleq [\mathbf{w}_1(t), \cdots, \mathbf{w}_P(t)] \\ A(t) &\triangleq \text{diag}[A_1(t), \cdots, A_P(t)] \\ \mathbf{b}(t) &\triangleq [b_1(t), \cdots, b_P(t)]^T \end{aligned}$$

and $\mathbf{n}(t)$ is a zero-mean white Gaussian noise vector.

3. ADAPTIVE SUBSPACE CHANNEL ESTIMATION

Subspace methods utilize the second-order statistics of the received signal $\mathbf{r}(t)$. Define $H(t) \triangleq W(t)A(t)$ in (6) and assume that the transmitted signals from different users are independent to each other. Then

$$R(t) = E[\mathbf{r}(t)\mathbf{r}(t)^H] = H(t)H(t)^H + \sigma^2 I_K \tag{7}$$

Apply EigenValue Decomposition (EVD) to the correlation matrix $R(t)$ and arrange the eigenvalues in a non-increasing order:

$$\begin{aligned} R(t) &= U(t)\Sigma(t)U(t)^H \\ &= \begin{bmatrix} U_s(t) & U_n(t) \end{bmatrix} \begin{bmatrix} \Sigma_s(t) & 0 \\ 0 & \Sigma_n(t) \end{bmatrix} \begin{bmatrix} U_s(t)^H \\ U_n(t)^H \end{bmatrix} \end{aligned} \tag{8}$$

where eigenvector matrices $U_s(t)$ and $U_n(t)$ have dimension $K \times P$ and $K \times (K-P)$, respectively. The columns of $U_s(t)$ span the signal subspace with dimension P, while those of $U_n(t)$ span its orthogonal complement, i.e. the noise subspace.

Traditional methods from numerical analysis for EVD of $R(t)$ have complexity $O(K^3)$ [9]. During the last decade, fast subspace tracking algorithms have been developed which provide a much cheaper way of adaptively computing the signal subspace eigenvalues and eigenvectors in only $O(PK)$. Among these, plane rotation-based algorithms are particularly attractive as they naturally maintain eigenvector orthonormality during the updating process and can operate over long periods of time without numerical instability (e.g. [7, 8]). Below, we develop a low-complexity adaptive blind channel estimation algorithm by exploiting common structural properties of plane rotation-based subspace trackers.

Unlike MUSIC-type algorithms, which use $U_n(t)$ as a null space to determine channel model parameters, the proposed algorithm begins from a different interpretation, namely [2]:

$$\mathbf{w}_q(t) \in Span(\overline{C}_q) \cap Span(U_s(t)) \tag{9}$$

The vector $\mathbf{w}_q(t)$ can be uniquely determined from the intersection of $Span(\overline{C}_q)$ and $Span(U_s(t))$ when the intersection space is rank-one, which implies $P + L \le K + 1$, or $P \le L_c - 2L + 2$.

A standard method for computing the intersection of two subspaces is given in [9]. At first, a QR decomposition is applied to $\overline{C}_q$, i.e. $\overline{C}_q = Q_q R_q$, where Q_q is a $K \times L$ orthonormal basis of $Span(\overline{C}_q)$ and R_q is an upper-triangular matrix. Thus

$$\mathbf{w}_q(t) = \overline{C}_q \mathbf{h}_q(t) = Q_q R_q \mathbf{h}_q(t) = Q_q \mathbf{h}'_q(t) \tag{10}$$

where $\mathbf{h}'_q(t) = R_q \mathbf{h}_q(t)$. Then $\mathbf{h}'_q(t)$ can be estimated as the dominant left singular vector of $Q_q^H U_s(t)$. An adaptive channel estimation algorithm based on this approach is presented under Algorithm 1. We refer to Algorithm 1 as an hybrid approach, for the signal subspace eigenvectors are updated adaptively with a subspace tracker but the subspace intersection (9) is computed using an exact (non-adaptive) Singular Value Decomposition (SVD).

Algorithm 1 Hybrid channel estimation

$U_s(t)$ is given by the subspace tracker
Initialization:
$[Q_q, R_q] =$ QR decomposition of $\overline{C}_q$
Recursion:
for $t = 1, 2, \ldots$ **do**
$[U, S, V] = SVD(Q_q^H U_s(t))$
$\mathbf{h}'_q(t) = U(:, 1)$
$\mathbf{w}_q(t) = Q_q \mathbf{h}'_q(t)$
end for

The complexity of this hybrid algorithm is $L(P+1)K + O(L^3)$ per time iteration: the first term is contributed by the matrix product computations while the second one is contributed by the SVD operation needed to calculate the dominant left singular vector. We note that the complexity of Algorithm 1 exceeds the $O(PK)$ figure characterizing fast subspace trackers.

Here, we propose a new complexity reduction technique for the recursive update of $\mathbf{w}_q(t)$ by utilizing the special updating form of plane rotation-based subspace/EVD trackers, including PROTEUS-2 [7], RO-FST [8], etc. The common feature of these trackers is that the signal subspace eigenvectors are updated as

$$\begin{bmatrix} U_s(t) & \mathbf{u}_n^a(t) \end{bmatrix} = \begin{bmatrix} U_s(t-1) & \mathbf{u}_n^b(t) \end{bmatrix} \prod_i G_i \tag{11}$$

where $\mathbf{u}_n^b(t) = \frac{(I_K - U_s(t-1)U_s(t-1)^H) r(t)}{||(I_K - U_s(t-1)U_s(t-1)^H) r(t)||}$ is defined as the noise eigenvector before the update, $\mathbf{u}_n^a(t)$ is the corresponding eigenvector after update, and $\{G_i\}$ is a sequence of $O(P)$ plane (or Givens) rotations which depend on the specific subspace/EVD tracker being used. Define $G \triangleq \prod_i G_i$ and then $GG^H = I_K$.

Note that the dominant left singular vector of $Q_q^H U_s(t)$ is identical to the dominant eigenvector of matrix

$$D_q(t) \triangleq Q_q^H U_s(t) U_s(t)^H Q_q$$

By using the special form in (11), we may develop a simple recursive update for $D_q(t)$:

$$\begin{aligned} D_q(t) &= Q_q^H \left[\begin{array}{cc} U_s(t) & \mathbf{u}_n^a(t) \end{array} \right] \left[\begin{array}{c} U_s(t)^H \\ \mathbf{u}_n^a(t)^H \end{array} \right] Q_q \\ &\quad -Q_q^H \mathbf{u}_n^a(t)\mathbf{u}_n^a(t)^H Q_q \\ &= Q_q^H \left[\begin{array}{cc} U_s(t-1) & \mathbf{u}_n^b(t) \end{array} \right] GG^H \left[\begin{array}{c} U_s(t-1)^H \\ \mathbf{u}_n^b(t)^H \end{array} \right] Q_q \\ &\quad -Q_q^H \mathbf{u}_n^a(t)\mathbf{u}_n^a(t)^H Q_q \\ &= Q_q^H U_s(t-1) U_s(t-1)^H Q_q + Q_q^H \mathbf{u}_n^b(t)\mathbf{u}_n^b(t)^H Q_q \\ &\quad -Q_q^H \mathbf{u}_n^a(t)\mathbf{u}_n^a(t)^H Q_q \\ &= D_q(t-1) + \mathbf{v}_q^b(t)\mathbf{v}_q^b(t)^H - \mathbf{v}_q^a(t)\mathbf{v}_q^a(t)^H \end{aligned} \tag{12}$$

where $\mathbf{v}_q^b(t) \triangleq Q_q^H \mathbf{u}_n^b(t)$ and $\mathbf{v}_q^a(t) \triangleq Q_q^H \mathbf{u}_n^a(t)$. By using (12), complexity of updating $Q_q^H U_s(t) U_s(t)^H Q_q$ is reduced from $O(LPK)$ to $2LK + L^2$.

A further reduction in complexity can be achieved if we consider the SVD operation in the hybrid algorithm. Indeed, we note that only the dominant left singular vector is required for channel estimation. The power method is an iterative approach for searching the dominant eigenvector with low complexity [9]. In the present application, where the true channel estimate is varying slowly over time, a single iteration of the power method can be applied at each time step to update $\mathbf{h}'_q(t)$. Specifically:

$$\begin{aligned} \mathbf{h}'_q(t) &= D_q(t)\mathbf{h}'_q(t-1) \\ \mathbf{h}'_q(t) &= \mathbf{h}'_q(t)/||\mathbf{h}'_q(t)|| \end{aligned} \tag{13}$$

In a stationary environment, the power method will converge to the dominant eigenvector of $D_q \equiv D_q(t)$ when the initialization is not orthogonal to the eigenvector [9]. Based on (12) and (13), a fully adaptive subspace channel estimation algorithm is developed as Algorithm 2.

Algorithm 2 Adaptive channel estimation

$U_s(t)$ is given by the subspace tracker
Initialization:
$[Q_q, R_q] =$ QR decomposition of $(\overline{C}_q)$
$\mathbf{h}'_q(0)$: an arbitrary vector
$D_q(0) = Q_q^H U_s(0) U_s(0)^H Q_q$
Recursion:
for $t = 1, 2, \ldots$ **do**
$\mathbf{v}_q^b(t) = Q_q^H \mathbf{u}_n^b(t)$
$\mathbf{v}_q^a(t) = Q_q^H \mathbf{u}_n^a(t)$
$D_q(t) = D_q(t-1) + \mathbf{v}_q^b(t)\mathbf{v}_q^b(t)^H - \mathbf{v}_q^a(t)\mathbf{v}_q^a(t)^H$
$\mathbf{h}'_q(t) = D_q(t)\mathbf{h}'(t-1)$
$\mathbf{h}'_q(t) = \mathbf{h}'_q(t)/||\mathbf{h}'_q(t)||$
$\mathbf{w}_q(t) = Q_q \mathbf{h}'_q(t)$
end for

The complexity of this algorithm at each iteration is $3LK + 2L^2 + 2L$, which is an order of magnitude lower than that of the hybrid algorithm. A comparison of the complexity for both algorithms is presented in Table 1 for reference.

Table 1. Complexity Comparison in the Second Step

Hybrid Algorithm	$L(P+1)K + O(L^3)$
Adaptive Algorithm	$3LK + O(L^2)$

Table 2. Average Failure Time of OPAST

Forgetting factor α	Average failure time
0.99	3038
0.995	6293
0.9975	12337

4. COMPUTER EXPERIMENTS

Computer experiments have been conducted to compare the performance of the proposed adaptive algorithm and the hybrid algorithm in time-variant channels. The performance of the OPAST based scheme in [4] is also discussed briefly. All simulations assume a down-link CDMA system with $L_c = 32$ and $L = 4$. In the down-link environment, all the users share the same channel: $\mathbf{h}_q(t) = \mathbf{h}(t)$, for $q = 1, \ldots, P$. The signature waveforms are randomly generated. All the users have equal-power. The channel is assumed to be a first-order AR model $\mathbf{h}(t) = \beta\mathbf{h}(t-1) + (1-\beta)\mathbf{f}(t)$, where $\mathbf{f}(t)$ is an i.i.d complex white Gaussian source. Parameter β is used to control the rate of change of the radio channel. The plane rotations based EVD tracker used in the simulations is PROTEUS-2, with forgetting factor $\alpha = 0.995$. The performance measure is the relative mean square error of $\mathbf{w}_1$, that is:

$$MSE = E[\frac{\|\Delta(\mathbf{w}_1(t))\|^2}{\|\mathbf{w}_1(t)\|}] \tag{14}$$

Here we use a time average over $T = 10^4$ iterations to approximate the expectation $E[\cdot]$.

Three simulation experiments were conducted to test the performance in the case of different number of users, different Signal/Noise Ratio (SNR), and different rate of change of the AR channel, respectively. The corresponding results are presented in Fig. 1 to 3, respectively. In all cases, the performance of the fully adaptive algorithm is comparable or superior to that of the hybrid scheme. For the same set of experiments, the OPAST-based scheme [4] failed as a result of numerical instability: the average number of time iterations before divergence (10 run average) is reported in Table 2. We note from Fig. 1 that in the case of a heavy loaded system, i.e. large P, the proposed adaptive algorithm performs better than the hybrid algorithm. Fig. 4 (top) compares the time evolution of the square error produced by both algorithms during a single run in the case $P = 26$; Fig. 4 (bottom) shows the time evolution of the first two dominant eigenvalues of $Q_1^H U_s(t) U_s(t)^H Q_1$ during the same experiment. In general, we find that the proposed adaptive algorithm is more robust than the hybrid algorithm when the second eigenvalue is close to 1.

5. CONCLUSION

A low-complexity adaptive blind subspace channel estimation algorithm was proposed for direct sequence spread spectrum CDMA systems. Compared with so-called hybrid adaptive channel estimation algorithms, where only the subspace estimation is carried out adaptively, the proposed algorithm is fully adaptive in that both

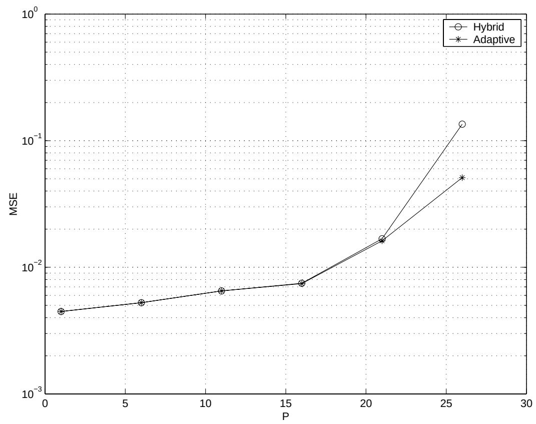

Fig. 1. MSE vs. P, SNR=10dB, β=0.995

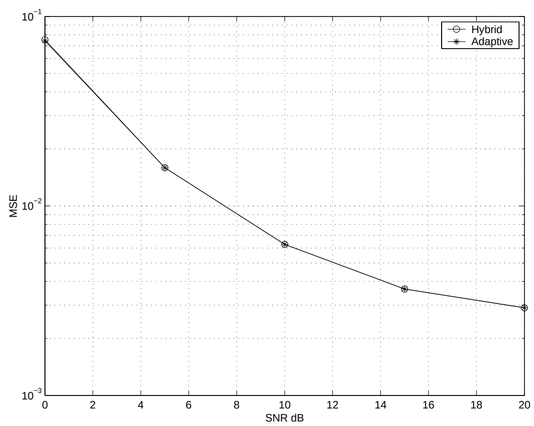

Fig. 2. MSE vs. SNR, P=10, β=0.995

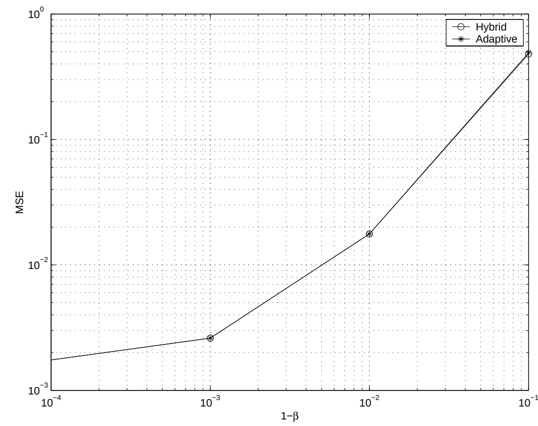

Fig. 3. MSE vs. $(1-\beta)$, P=10, SNR=10dB

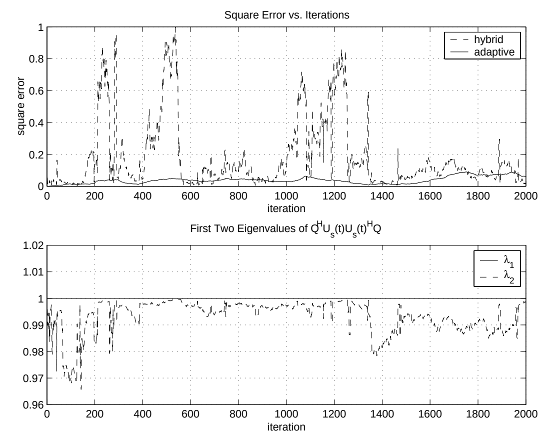

Fig. 4. Performance comparison, P=26

subspace *and* channel estimates are updated recursively. The new algorithm is derived by exploiting the special updating form of plane rotation-based subspace trackers, whose numerical robustness has been well established. The main advantages of the proposed algorithm are its low-complexity of implementation and numerical ability to operate over long periods of time, an essential requirement for wireless radio applications. Moreover, we find in the case of a heavy loaded system that the proposed algorithm has better performance than the previous hybrid algorithms.

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