Relativity, Quantum Theory, and Theology

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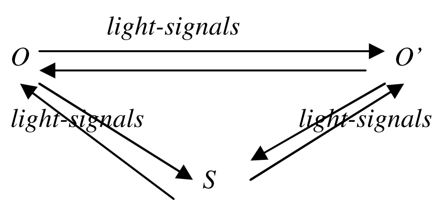
The purpose of this presentation is to give a concise description of some of the unique and essential aspects of the theories of Relativity and Quantum physics which suggest, in the author's opinion, compelling engagements with some issues in present-day philosophy and theology. The concepts presented are summarized in a 'top-down' fashion. Appendices are included giving the more technical details deriving such concepts 'from the bottom-up,' for the interested reader.

Relativity and the quantum theories are '*meta-physical*,' in the sense that their authors not only advanced new conceptual tools and models, but in turn *revised* the *way* physicists were formulating their theoretical models. The analogy is akin to that of a group of cartographers who not only set out drawing more accurate maps, but introduce fundamental *revisions* and *inventions* to the technique of map-drawing. The theorists' creative imagination and expertise irreversibly revised the research paradigms of twentieth (and twenty-first) century mathematical physics in ways not seen since the Copernican Revolution.

I. The Relativity Theories (*Albert Einstein: 1908-1918)*

Actually, 'relativity' is a bit of a misnomer. Originally published in German, Einstein used the word: *invariants* in his seminal papers. Much of Einstein's original thought was guided by the simple (but profound) question:

Suppose observers *O*, and *O'* are studying system *S*. Considering question 1., for there to be communication between *O* and *O' about S*, then all three must be mutually causally related. *At the very least* this requires mutual connection with light-signals among *O,O',S* :



Assuming *O,O',S* are connected in this way, by light-signals, *what features about S* remain *invariant under all possible degrees and types of relative motion among S,O,O'* ?

The above is asking something about the nature of *light*; reminding us at the same time that that light is what makes communication possible. So it's clarifying what makes physics possible: communication and information. Information-flow is conditioned by the medium of light. We're being asked to imagine what are the necessary conditions making some system physicists study (whether it's a star, an atom, or a baseball) appear *invariant*; given that, at the very least, we must interact with the system using light-signals.

So, at the outset, in these two questions, one detects:

- A ***relational world-view***. Einstein is speaking from a tradition articulated by Leibnitz: *All measurable and observable properties arise from* <u>*interactions*</u> *between observer and observed.* Newton disagreed with Leibnitz, as he felt that measurable properties are <u>intrinsic</u>

and <u>innate</u> to a system (and are the system's *primary qualities.*)[1] The relational view is *holist*, whereas the absolutist view is *atomist*, and *reductionist,* in an epistemological sense.

- A ***phenomenological method.*** Following the heels of a relational world-view, Einstein's second question addresses the *appearance* of 'objectivity.' All measurable properties are relational, and arise at the very least from the exchange of light signals. Therefore, we must dispense with (capital O) 'Objective' notions like 'thing-in-itself' replacing them with (little o) 'objective' notions like *intersubjective agreement on what appears invariant.* Physics deals only with phenomena. This position was articulated most conspicuously by Kant[2,3]

Physically, *light* is an indispensable concept. Using techniques of *thought-experiments* involving light as an operant concept enabled Einstein to formulate his brilliant and radical theories and conceptions. Before surveying his results, however, one must qualify: Einstein did not reveal to us the *fundamental nature* of light. Indeed, already Galileo seriously doubted that the new mathematical physics he was developing was capable of giving an account of any fundamental causes. The work on the study of light survives today in the field of *Quantum Electrodynamics* (QED). What Einstein did was to give *operational* meaning to some acts of measurement which (ideally) necessitate exchanges of light signals among observers.[4].

A.) Special Relativity

The theory of Special Relativity came about when Einstein tackled question 2. by restricting the type of motion among *O,O',S* as *inertial* (free of acceleration.) Mathematically, the stage was set. Maxwell's Equations, which unify electric and magnetic phenomena into the *electromagnetic continuum* (what light is part of) demonstrate that the speed of light is indeed a constant = *c*. (**Appendix I**) Moreover:

$$c = 3.00\times10^{8}\ {}^{\text{m}}/_{\text{sec}} = {}^{1}/_{\sqrt{\varepsilon_0\mu_0}}$$

where: $\varepsilon_0\mu_0$ are the electric permittivity and the magnetic permeability of empty space

Maxwell's Newtonian leanings, however, made him believe that the propagation of electromagnetic waves required an underlying medium. This medium was dubbed the *aether* and was accepted as some completely unobservable essence which acted as the medium for electromagnetic radiation to propagate in. The belief in the aether was mainstream in the nineteenth century and seemed natural enough. The three major reasons for the belief in the medium were (in progression of increasing degree of abstraction):

i.) Light travels in material media and indeed slows down (refraction). Therefore, light must travel in 'something' in the vacuum. This must be the aether.

ii.) Newton's physics was so successful in the nineteenth century that all theories advanced must be demonstrated to reduce to Newtonian concepts. Newton's mechanical laws

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[1] Newton, for example, affirmed that space and time have *absolute* properties and could conceive of a universe with one particle in it, Leibnitz couldn't. After all, the particle had to be *in relation* to something else for it to make any sense to talk about the particle's *location.*

[2] *Critique of Pure Reason, Prolegomena to any Future Metaphysic*

[3] Philosophically, Einstein's theories share much with Kant and Leibnitz. Not because Eisntein sought to extend or defend these philosophies, but rather because of the general philosophical climate of his German education and the nature of his probings, which naturally led him naturally to some similar conclusions. Personally, Einstein admired the work of Ernst Mach (articulate champion of positivism) but later became more 'Platonic' and found inspiration in Spinoza.

[4] By *operational* it is meant that one defines something purely along the lines of *observed properties* one derives in *controlled experiments*

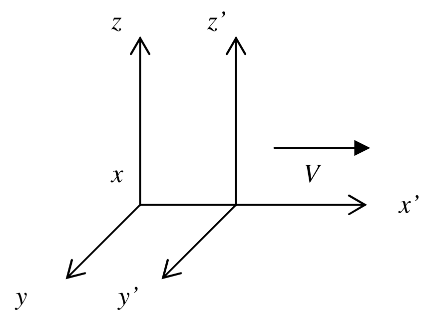
involving contact forces and momentum were hypothesized as the necessary model for any physical causal mechanism. The aether, therefore, was thought of as the medium comprising the microscopic hypothesized forces giving rise to electromagnetic waves.

iii.) Philosophically, as mentioned, Newton's theories were atomist and reductionist. The aether is the fundamentally necessary medium for the property of electromagnetic interaction. It matters little if this aether is in principle unobservable. Fundamental properties need no interaction with an observing agent.

Point ii) was the most significant reason why Maxwell and his contemporaries clung steadfastly to the aether concept. Maxwell's Equations must be reconciled with Newton's. Newton's equations presuppose absolute properties of space and time which are summarized in the transformation of motion properties known as the Galilean Transformations:

- Given two Inertial Reference Frames (i.e. two reference frames that aren't accelerating) (one at rest relative to the motion of the other) described by coordinates: (x,y,z,t) and (x',y',z',t') for the respective rest and moving frame. If the moving frame is travelling at a constant velocity V along the x-axis, then:

$x' = x - Vt \qquad y'=y \qquad z'=z \qquad t'=t$



This 'Galilean' picture of space, time, and relative motion offers (since the seventeenth century) a 'commonsensical' view in which:

- **The Euclidean geometric character of space and time :** (x,y,z); (x',y',z'): The coordinate system drawing from a particle's center of mass (its reference frame) is a 3D coordinate system presupposing Euclidean Geometry. The geometry of space is therefore Euclidean
- **Simultaneity:** $t = t'$: Time is independent of location (but not vice versa). The 'fact' that time ticks uniformly at all locations presupposes *absolute simultaneity.*
- **Simple addition of relative velocity:** $x' = x - Vt$. The relative location, and velocities between two reference frames is governed by the relative velocity V between them. Thus relative velocities add in simple way (for example, a man running 10 mph in a train car moving at 15 mph in the same direction would be moving at 25 mph relative to a person at rest in a train station.) Anyone who has suddenly stepped off a moving conveyor belt at an airport has experienced this relative addition of velocities!

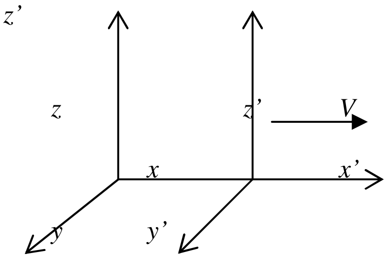
Galileo originally formulated his above conceptions of space, time, and motion to argue that as long as two systems aren't accelerating, the laws of physics should hold in the same manner in their respective reference frames. Laws of nature shouldn't 'prefer' any particular reference frame, they should be *uniform* or *invariant* across all inertial reference frames. Seems common-sensical enough. Newton's Laws of motion, for example, exhibit this property of *Galilean Invariance.*

The problem is that Maxwell's Equations, which so brilliantly describe the propagation of light and all forms of electromagnetic radiation do *not* obey Galilean transformations! This was a huge stumbling block. Does this imply radiation is a not a physical reality, or exhibits a 'bias' towards certain reference frame? Certainly not! Light seems real enough, as does any other form of radiation , and it certainly doesn't vanish when we switch coordinate systems willy-nilly. It was known in the experiments of Lorenz and Fitzgerald (ca. 1880) that electromagnetism is actually invariant under a different class of coordinate transformations known as the *Lorenz Contractions* (Derivation: **Appendix II**)*:*

- Given two IFRs (one at rest relative to the motion of the other) described by coordinates: *(x,y,z,t)* and *(x',y',z',t')* for the respective rest and moving frame. If the moving frame is travelling at a constant velocity *V* along the x-axis, then:

  $x' = \gamma(x - Vt)$ $\quad y'=y \quad z'=z \quad t'= \gamma(t - Vx/c^2) \quad z = z'$

  where: $\gamma = [1 - (V/c)^2]^{-1/2}$



Lorenz derived his contraction equations well before Einstein re-interpreted them. At the time, it was thought EM obeyed these unusual transformations because of the *aether* This (undetectable) aether was thought to 'bend' Galilean invariance like a lens might, for example, bend a light ray otherwise travelling truly straight.

Einstein boldly proposed, on the other hand, in a move reminiscient of Ockham's Razor[5], to jettison the concept of the necessity of an underlying aether and instead proposed that light may travel in a vacuum. This is formulated by the *Equivalence Principle of Special Relativity:*

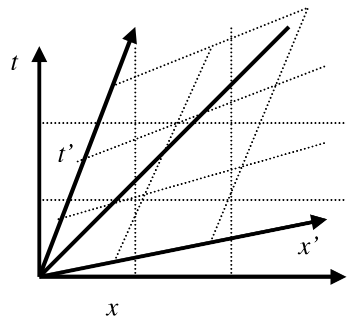
I.) **(SR)** The speed of electromagnetic propagation is an invariant and therefore independent of the motion of its source. The IFR describing a photon propagating in empty space is invariant under Lorenz transformations

Mathematically, one can recover Galilean transformations from the Lorenz transformations in the two limiting scenarios: 1) $V \ll c$ 2.) $c \to \infty$

Scenario 2.) ($c \to \infty$) was assumed in the centuries preceding Newton. Galileo himself conducted an experiment trying to measure the speed of light and concluded *c* was 'arbitrarily large' hence the propagation of light for all practical purposes was assumed to be virtually instantaneous. Conversely, when travelling ordinary speeds (scenario 1) $V \ll c$ *)* Galilean transformations apply to negligible error. In either case, one can well understand why Galilean relativity seem so intuitive (and Lorenz's do not) : *c* is terribly difficult to measure and most of the time we're travelling well below *c*.

Special Relativity extends *IFRs* to obey this broader class of Lorenz transformations. Accepting such transformations means we must give up many precious assumptions: simultaneity among the foremost. For observe $t'= \gamma(t - Vx/c^2)$ which implies time ticks *t'* at a different rate on the moving frame than on the rest frame: therefore absolute synchronization among all clocks at any given instant is impossible. Among other assumptions that must be abandoned is *absolute length scale*: length contracts as time dilates when one travels at appreciably high speeds *V*. Length contraction and time dilation has been experimentally confirmed in countless occasions[6]. A simple way to demonstrate these notions is to compare how *x',t'* compare with *x,t*, indicated in the schematic below:

[5] A method of argument put forth by William of Ockham (~13th cent): If two or more arguments arrive at the same conclusion, *always select the argument requiring the least number of steps to prove.*
[6] A famous example being the experiment where an atomic clock was placed on board a jumbo jet which circled the globe compared to an atomic clock remaining on the earth. The clock on board the jet was found to tick more slowly.



The thick diagonal defines the reciprocal of the speed of light: $^{1}/_{c}$. As the grid lines indicate, what's simultaneous in one frame isn't simultaneous in the other; an object of length $L = x_2 - x_1$ in the rest frame (x,t) won't have the same length *L'* in the moving frame and vice versa.

In summary, the major revolutionary concepts arising from Special Relativity are:

- **'Space' and 'Time' become: 'Spacetime.'** In Galilean Relativity, space can depend on time, but not vice versa. In Special Relativity, space and time enjoy a reciprocal relation. They are *unified* in a 4 D non-Euclidean 4D geometry known as Minkowski spacetime. Events describing IFRs in Minkowski spacetime are connected by Lorenz transformations the way events in 3D Euclidean space geometry were connected by Galilean transformations. Our conceptions of space and time are irreversibly altered and include:

  i.) Relativity of Simultaneity : The belief in absolute simultaneity (as formulated by Galilean transformations) is a vestige of the Ptolemaic conception of absolute rest:

  > In everyday life, the 'firm well-founded earth' for good reasons provides a body of evidence of (absolute rest). But who tells us that the earth stands still, or rather, what do we mean by it? The belief that simultaneity exists in the world is originally based on the fact that every person places the events he perceived in the moment of their perception. But this naïve belief lost its ground long ago through the discovery of the finite velocity of the propagation of light.[7]

  ii.) Time dilation/Length Contraction/Twin Paradox
  According to the Lorenz transformation, the length *L'* of a moving object decreases, and the rate of its time *t'* dilates, according to the factor $\gamma = [1 - (V/c)^2]^{-1/2}$ *as seen from the frame at rest.* For example, if I were travelling at 90%c, then my $\gamma = 2.29$. Which means, to an observe at rest relative to me, my height would shrink by a factor of 2.29 and my time would slow down by a factor of 2.29! In other words, to her, I would appear 2 feet 8 inches tall and live 2.29 times longer! (My aging would slow down by a factor of 2.29). Of course, from my reference frame, i would notice none of this happening to me. If I left on a journey for 10 years averaging 90%c, I'd return to a planet where 23 years have gone by (ignoring effects of acceleration).

  iii.) Undetectable/Unknowable Events
  The fact that *c* is finite, and that *every* physical interaction involves the transmittal of some kind of information means gives apriori limitations to what any observer can experience, i.e. causally interact with. This is illustrated in the simple diagram below:
  Focus on just one dimension of the observer's space. Then her IFR becomes the two dimensional diagram:

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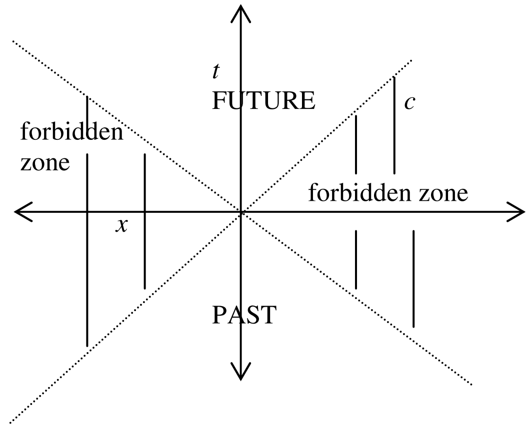
[7] Hermann Weil, *The Open World* (Ox Bow Press 1989) p.14 (reprinted from original Yale U Press Lecture publ `1932)



The origin is her present moment. Her future and past comprise all the events she has causally interacted with, which means they were connected, at the very least, by light signals. This picture is called the observer's *light-cone*. Events outside of this cone are events that cannot be causally connected to her at the origin, as it would require the exchange of information at speeds faster than light. We are limited by our light cones and cannot hence apriori know and experience everything.

- **'Mass' and 'Energy' become: 'Mass-energy.'**

In Minkowski spacetime, the laws of dynamics alter. A particle's energy and mass are reciprocally related, and can be mutually transformed into one another. This implies:

i.) The unity of matter and energy

The famous equation: $E = mc^2$ can be derived from first principles in Minkowski geometry (**See Appendix III)**

ii.) *c* is the ultimate speed limit

Any object with nonzero mass at rest m_0 has *relativistic mass* $m = \gamma m_0$. Because $\gamma \rightarrow \infty$ when the object's velocity $V \rightarrow c$, the object's mass therefore increases to infinity. Therefore it requires infinite energy to accelerate something with nonzero rest mass to the speed of light. Therefore we are strictly bound to travel at speeds below that of light. Photons can do it (travel at c) only because they have no rest mass.

- **Overall Assessment**

The unexpected results and conclusions derived from Special Relativity occurred when Einstein turned a question on its head. Instead of *examining* a physical situation mediated by light signals from some *imposed apriori geometry* (Euclidean), Einstein *began* with observers travelling at constant velocities and *derived* the spacetime geometry *which makes communication with light signals possible.* In other words, the *geometry* falls out of the physical situation. Consider then the geometry of Minkowski spacetime to be the 'geometry of causal connection' (communication). Since all we can do is communicate with each other if there is to be public and repeatable data, we adopt Minkowski geometry as the 'truly empirical' geometry. . Thus Special Relativity is profoundly *operational*: the conceptions of space and time and their mathematical relations arise *aposteriori* via physical interaction*, and aren't imposed apriori onto the phenomena*

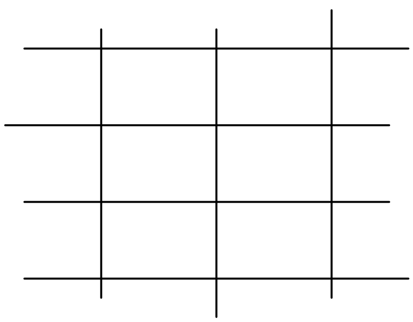
B.) General Relativity

The enormous success of SR caused Einstein to push the idea of *equivalence* to the utmost generality. Special relativity maintains that all *inertial* (i.e. *non-accelerating*) frames of reference are invariant under Lorenz Transformations. What about *accelerating* frames of reference? In a brilliant and bold maneuver, Einstein proposed his principle of *General Equivalence:*

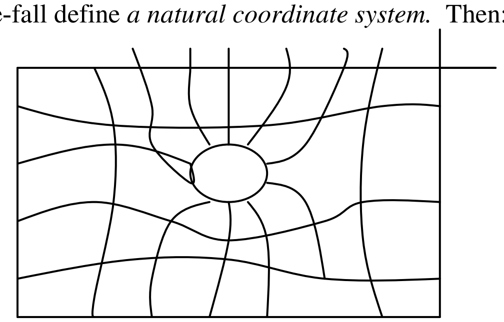
II.) **(GR)** Any system undergoing constant acceleration is equivalent (in description) *locally* to an object in free-fall in a gravitational field.

General Equivalence seems to say something so obvious it's almost embarrassing: locally, we know that a gravitational field can be described by the gravity constant *g* which happens to be 9.8 m/s^2. For example, Galileo already demonstrated that if one neglects air resistance, objects *close to the surface of the earth fall at a constant acceleration.* This is true independent of the objects' masses. The word 'local' is deliberate. For greater distances, the rate of acceleration due to gravity stops being constant. For example, a satellite falling toward the earth at an initial altitude of 100 km experiences a changing acceleration over long distances. *But* an object locally in free-fall is indistinguishable from an object constantly accelerating at a=9.8 m/s^2. We know an object in free-fall falls the same way regardless of its material properties whether it's an atom or an Aardvark, such a system accelerates uniformly in free fall.

But it's the *mathematical* consequence of **(GR)** that makes this (otherwise unremarkable) insight so powerful. Since objects fall in the same way regardless of their material properties, Einstein intended to show that gravity was indeed a phenomenon of such regularity and generality that it's better expressible as a *structural feature of spacetime* instead of a field embedded in a spacetime. How does one incorporate gravity into the structure of spacetime? Simple. Let the *geodesics* (i.e. trajectories traversed in their minimum local time, i.e. proper time) of an object in free-fall define *a natural coordinate system.* Then:



a.) No mass (gravity) present implies the geodesics traced by an IFR are straight.



b.) The geodesics traced by an object in free fall in the proximity of a gravitational source curve toward the source.

General Relativity converts gravity into a principle regulating (locally) the *geometry* of a particular region of space-time. In the above illustrations, the absence of gravity implies (in a.)) a *flat* space time: the geodesics of an IFR trace a straight line. Case b.) is what is meant by *curved* space time: the quantity of matter at the source 'tells spacetime how to curve' and conversely the relative curvature of spacetime bespeaks of the intensity of the local gravitational field. This is not mere word-games or re-labeling; one must remember that *photons* travel along these geodesics. Hence for all practical purposes space-time has a variable curvature. (Recall from Special Relativity that all structural and causal information is regulated by the propagation of light-signals.)

Numerous experimental confirmations of GR have been conducted, perhaps the most famous being the *exact* resolution of the discrepancy of the perihelion shift of Mercury under the old (Newtonian)_ theory.

Despite its beauty, simplicity, generality, the mathematics of General Relativity (differential geometry and differential topology, developed in the 19th century) is considerably more formidable than that of Special Relativity, which prevents any kind of detailed summary. Nevertheless, the salient features are:

- **The Geometry of Spacetime is Dynamic, Locally Varying, and Depends on the Quantity of Matter Present**

General Relativity extends Special Relativity to include cases of frames of reference accelerating with respect to one another. This inevitably brings up the role played by gravity. However, instead of treating gravity as a 'force' embedded *apriori* in a Euclidean, Galilean space-time the way Newton did, Einstein's Equivalence Principle drags gravity into the discussion originated by Special Relativity. *What kind of geometry falls out of physical interactions mediated by light signals* *<u>in the presence of gravity</u> ?* Answer: A very complicated kind! Indeed this geometry is so complicated it requires much more sophisticated mathematics to describe. Suffice it to say here that the Minkowski spacetime of Special Relativity survives as a special case in this general geometry *when there is no matter present*. Hence, we say Minkowski Geometry describes a 'flat' spacetime. Only in regions between galaxies, then, where there's hardly any matter, would we expect to encounter locally a geometry that's Minkowskian.

i.) **Physically**, the consequences of General Relativity bore the most fruit in the study of astrophysics and stellar evolution. The fact that matter couples to spacetime gives rise to the exotic phenomena of Black Holes, Neutron Stars, White Dwarves. The results of General Relativity (which have experimentally confirmed to very high precision) are summarized in the table (**Appendix IV)** in the discussion on singularities

ii) **Philosophically,** General Relativity vindicates Leibnitz in his attempt to derive a relational cosmology. Newton and Leibnitz debated their respective absolutist and relational concepts of space and time. Newton won out primarily because he was able to account for *acceleration* in a simple manner in his absolutist framework. Leibnitz couldn't account for acceleration in his relational model. Leibnitz had a much more difficult problem to solve. Einstein's Equivalence Principle solved it, as Einstein shows how acceleration is equivalent to local curvature of spacetime, which is governed by the presence of the distribution of matter.

Einstein, later in his life, became interested in another problem in General Relativity that has philosophical ramifications. The *metric tensor* $g_{\mu\nu}$ is the mathematical entity that describes the local structure of the spacetime geometry (which in the case of General Relativity, becomes a function of the quantity of matter present.) Einstein was interested in *how* this metric $g_{\mu\nu}$ varies, what regulates its variations? A quantity known as the *Hilbert Action*. The Hilbert Action gives a dynamical law which governs the variation of the dynamic of space time geometry. In turn, a dynamical spacetime geometry regulates dynamical law. We see another incident of a reciprocity:

Hilbert Action →

Law (dynamic) *Geometry (dynamic)*

← Metric tensor

The line below is dashed to indicate that Einstein didn't completely solve this problem; the circle can't be entirely closed. Nevertheless, it provides an example of what in general Barbour writes, that: "nature is today understood to be a dynamic evolutionary process with a long history of emergent novelty, characterized throughout by chance and law. The natural order is ecological, interdependent, and multiveled."[8]

[8] *Religion and Science*, [Harper 1997] p.101

QUANTUM PHYSICS

Quantum mechanics' (QM) chief pioneers. include: Max Planck, Erwin Schroedinger, Neils Bohr, Paul Dirac, Werner Heisenberg. Quantum mechanics is part of an attempt of a (future) general theory of the microstructure of matter. In this respect, it shares little in common with the Relativity theories, as they themselves describe large-scale causal features of spacetime. Indeed, in ordinary (non-relativistic) QM spacetime geometry is again Euclidean and the QM's fundamental obey Galilean transformations, like the laws of Newton. However, ordinary QM is profoundly non-Newtonian as well. To indicate the complexity of the situation, Relativity (despite its vastly different geometric conceptions of spacetime) doesn't depart from basic conceptions of causal interactions and determinism shared by Newtonian physics. This is *not* true for quantum physics. Indeed, despite getting the Nobel Prize for his work on the photo-electric effect (a well known quantum mechanical phenomenon) Einstein parted company with many researchers who developed and refined QM to its present day subtlety and difficulty. Einstein thought his theories *refined* notions of causality and determinism from Newton's, yet the theorists working on QM were calling these very notions into serious question.

Attempts to develop a unified theory out of which the formalisms of quantum physics and Relativity could arise lie at the forefront of contemporary research. Notable versions include *String Theory* (originally developed at Princeton U) and David Finkelstein's research at Georgia Tech (*Quantum Relativity*). [9].

I will briefly summarize some of this terrain of the unique chimeras and challenges posed by present-day quantum physics. But first, a brief definition and disclaimer:

- **Definition:** *Kinematics* is the study of the possible evolution(s) of a physical system. *Dynamics* is the study of the *actual* evolution of a physical system. Kinematics and dynamics reflect the modal distinctions between possibility and actuality.
- **Disclaimer:** The position advocated here in the attempt of an interpretation of quantum physics is derived mostly from the author's understanding of the work of David Finkelstein, who advocates an 'operational' approach.[10]

1. The Mathematics

Quantum physics has a richer formalism than that of pre-quantum physics. In pre-quantum physics, the 'mode' or 'state' of a physical system is defined by a *point* in a *set* (the system's *phase space.*) The system's *observables* (i.e., the quantities one can measure, such as its energy, position, momentum, etc) are modelled by certain classes of *functions*.[11] On the other hand, in quantum physics, a system's mode is modelled by a *ray* ('statevector') in a linear space. And the system's *observables* are represented by *linear operators on the system's statevectors.* This subtle changeover into this richer mathematical formalism is what spells out many of quantum physics' unique consequences, some which seem quite bizarre from the standpoint of 'traditional' physics steeped in 'traditional' formalism. It's important to note here, however, that this change in formalism did not occur willy-nilly, but was the culmination of a long period of productive research between the times of quantum physics' 'early' (1900-1910) to its 'mature' stages (1930s-1940). Werner Heisenberg, Paul Dirac, and John von Neumann are considered the chief architects of this elegant formalism used today as a standard.

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[9] David Finkestein*Quantum Relativity: A Synthesis of the Ideas of Einstein and Heisenberg* [Springer Verlag 1996]. See also: http://www.physics.gatech.edu/people/faculty/dfinkelstein.html

[10] Acts and actions are interpreted as primary explanans, as opposed to 'states.' Much of the sources Finkelstein draws from include the writings of Heisenberg, and elements of AN Whitehead. See: *Quantum Relativity, Elementary Operation Physics* (2001), *Process Studies* (1997)

[11] They are known as *diffeomorphisms,* smooth, differentiable. It's important to not here that by 'pre-quantum,' this includes relativity.

- **Heisenberg's Uncertainties**

  With every advance there inevitably seems to be price to pay. Some cherished assumption(s) must be jettisoned. In the case of adopting this richer mathematical formalism, though it has greater descriptive power, inevitable *uncertainty relations* are also derived. Among the most well-known were those originally discovered by Heisenberg:

  $$\Delta p\,\Delta x \geq {}^{h}/_{4\pi} \qquad\qquad \Delta E \Delta t \geq {}^{h}/_{4\pi}$$

  What these inequalities say is that when seeking, for example, to determine position and momentum, the product of their dispersions is bounded below by the small (but not zero) quantity ${}^{h}/_{8\pi}$. That is to say, should one wish to fix *precisely* the system's positon (i.e. $\Delta x \rightarrow 0$) then the uncertainty in momentum Δp shall arbitrarily increase. Same is true in converse, as also in the case of time and energy. We can't have our cake and eat it too. Determining one variable to arbitrary precision means we must sacrifice the determination of information of its 'dual' (which in the case of time, is energy; in the case of position, is momentum)

  It's important to remember that these uncertainties arise *a priori* out of the formalism. When choosing such a mathematical formulation, one must inevitably contend with such matters.[12] To see how these uncertainty relations arise strictly from the formalism, see **Appendix V**

2. **Quantum Physics is Dynamically Atomistic**

This has been an operant assumption since its beginnings, when Max Planck assumed a *discrete* structure to the *action* of the electromagnetic spectrum. Planck originally coined the term 'quanta' to explain, what he thought, were such *indivisible units of action*, which occur in integer multiples f the 'atomic' unit:

$$h = 6.63 \times 10^{-34} \text{ Joules*sec}$$

…a quantity called (what else?) Planck's constant. The units of action are in the form of energy × time. *Action* was a quantity developed in late nineteenth century celestial mechanics, to analyze planetary orbital dynamics. This *discrete dynamics* incorporated into quantum physics accounts for, among other things, the observed discrete energy levels of atomic spectra, the photoelectric effect, the specific orientations of atomic and subatomic spin, etc.

It is often thought that quantum physics presumes a discrete structure to nature. This is a half-truth. As we'll see, *continua* are also part of quantum theory, and arise in a very bizarre fashion. What is true, however, is that quantum physics presupposes *discrete action* in its dynamics:

> The fundamental question of physics then is not, 'What are things made of?' It is 'What goes on here?' What we seek are the basic actions that go on in nature…We retain Francis Bacon's maxim: 'Dissect Nature' but we read it (or misread it) as the injunction to dissect dynamic history into least action, not some hypothetical static matter into atoms.[13]

3. **The Continuum In Quantum Kinematics**

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[12] From personal experience I recall the professor deriving 'Heisenberg Uncertainty relations' for the position and momentum of clouds in a graduate course I took in physical meteorology. Is this a quantum mechanical effect? Certainly not! It happens to be that in atmospheric dynamics one adopts a similar mathematical formalism: linear algebra and Fourier Analysis.;

[13] David Finkelstein, *Quantum Relativity,* pp. 23-24

As mentioned in 1., replacing a *point* with a *vector* when describing a mode of a system means we've got a mathematically richer structure on our hands. We can do more with *vectors* than we can with points. Points just sit there in a space, about all we can do with them is act on them with functions. Internally in a set, we can distinguish points and give a definition of what it means for points to be 'close to' each other. This was the mathematical situation before quantum physics.

One thing we cannot do with points is *add* them. It makes no sense. We can, however, *add* vectors. It's called *superposition.* Two modes of a physical system can be added together to produce a unique third. In quantum physics, this is called *coherent superposition.* Because two modes are described by vectors, their addition to produce a third mathematically can produce a *continuum* of possibilities (depending of what the values of the initial coefficients are). This is how the continuum occurs in quantum physics. One way to view this is to think of a system's modes as having a *continuous phase*. There's simply no counterpart to this in pre-quantum physics.[14]

What *is* coherent superposition? There's simply no way to understand it in pre-quantum physics, because it cannot even be expressed in its formalism. The observable consequences of coherent superposition however are apparent enough: they include (but are not exhausted by): coherent light (LASERS), superconductivity, Bose Condensations

- **Entanglement and Bell's Inequalities**

One particularly interesting application involving coherent superposition occurs when preparing a system in such a way that measured correlations between constituent observables violate the laws of classical probability and statistics. That is to say, their measured probability of being correlated greatly exceeds what the laws of randomness inherent in classical statistics and probability theory would say that they should be. These include modes which are *entangled*. Entangled modes involve superposition, but are more complicated than that because they involve coupled systems. Entanglement implies superposition, but not vice versa.

A particularly dramatic illustration of how systems prepared in a coherent superposition can violate the laws of statistical randomness were phrased as Bell's Inequalities (see **Appendix VI**). What do Bell's Inequalities show? That the classical world-picture is untenable. It appears, in other words, that just as Relativity overthrew Euclidean geometry as the final arbiter for physics, so do Bell's Inequalities overthrow the typical assumptions inherent in the logic behind probability theory and statistics. Or perhaps less dramatically stated, *how* we *ideally* model correlation (using ordinary statistical methods) is not *actually* how some systems are correlated. Nature doesn't necessarily conform to our expectations derived from our human logic. We already saw this in the discussion of relativity and geometry: just because Euclidean geometry seems intuitive and simple enough doesn't actually mean that Nature shall choose such a thing. Bell's Inequalities were experimentally confirmed in the 1990s by the work of Alain Aspect.

- **Is Coherent Superposition a Deeper Kind of Relativity?**

Paul Dirac originally proposed such a compelling idea, which he developed in his *transformation theory.*[15] The basic idea is when defining a system's mode vector one must also specify its 'experimental frame,' which is the basis of the vector. The fact that vectors can be arbitrarily added to produce any third can be *passively* understood as the transformation to any third *basis.* This has

[14] To soften the blow, some researchers and authors formulate coherent superposition in terms of probability theory (van Fraasen [1993]). Classical physics allows one to add disjunct probabilities. Quantum physics tells us on the other hand, to add *probability amplitudes.* Probability amplitudes are a sort of 'square root' of probability. Probability amplitudes arise when preparing a system S described by mode-vector $|S\rangle$ in a position-measurement basis $\{|x\rangle\}$. Then the associated probability amplitude is matrix element: $\langle x|S\rangle$, which in earlier and more elementary formulations is called the system's 'wavefunction.' Computing: $|\langle x|S\rangle|^2$ gives us the probability of finding system in mode S at location x.,

[15] See *QR*, Finkelstein, ch 4

remarkable resonances with Relativity, which implies there is no 'preferred' frame of reference for a moving object in space-time. But it's deeper, because an experimental frame is a *conceptual* frame of reference modeling *acts of measurement/information* we extract from a system. Such acts need not necessarily be local (like measuring position). For example, a momentum measurement is non-local. So an experimental frame is much more general than a frame of reference in space-time.

4. The 'Logic' Of Quantum Physics

In the 1950s John von Neumann and Garret Birkhoff published seminal papers in which they sought to tease out an underlying logical structure to the formalism of quantum physics. What they discovered has interesting consequences. Ordinary (classical, or pre-quantum) predicate logic obeys certain properties originally formulated by George Boole (in his Boolean Algebra). Among some of these properties are: (where $\equiv$ means 'logically equivalent')

- **The Commutative Law (for 'AND': $\wedge$, 'OR': $\vee$)**
  : For any two predicates p, q: $p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$

- **The Distributive Law (for 'AND': $\wedge$, 'OR': $\vee$)**
  :For any three predicates p, q, r*:* $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

The logic underlying quantum physics violates these two properties! What Birkhoff and von Neumann additionally showed, however, was that by allowing Planck's constant $h \rightarrow 0$ one can recover distributivity and commutativity (just as one recovers Galilean transformations from Lorenz Transformations allowing $c \rightarrow \infty$)

The field of 'Quantum Logic' has been the source of much philosophical haggling. Because there is yet no all-encompassing interpretation to quantum theory, so there are many conflicting interpretations of quantum logic. For all these seemingly abstruse wrangling, however, one should remember: the violation of distributivity by quantum physics can be experimentally demonstrated. In front of school-children, really! All it takes is three polarizers (as the phenomenon of polarized light is one demonstration of a quantum physical effect)[16].

5. Quantum Spontaneity/Complementarity

Quantum physics is a radically indeterministic theory. This is by far its most quintessential features, troubling to some, compelling to others. Relativity, like Newtonian physics, is deterministic: given an (ideally) complete knowledge of a *simple* system's initial conditions, its (unique) dynamical evolution be established: its future is then knowable, all things being equal. The qualifier: 'simple' is reserved for cases avoiding large-scale 'noise' or indeterminacies arising from complex systems. The point is, however, *this has nothing to do with quantum indeterminacy*! Quantum indeterminacy is much more radical than chaos theory, which is still a classical (therefore deterministic) theory dealing with approximation techniques one must inevitably use when dealing with complex systems when the math gets otherwise forbidding.[17]

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[16] The simple demonstration is described by Gary Zukav in his *Dancing Wu Li Masters*, a book formed out of the many conversations he had with David Finkelstein.

[17] Another way of putting it is that even though Newtonian mechanics is a deterministic theory, any system comprising the motion of three or more particles is unsolvable in the Newtonian framework. The indeterminacies introduced by the interaction of three or more particles make the math too complicated, therefore approximation techniques must be developed (like those found in Chaos Theory)

So, in a nutshell, classical or pre-quantum physics is deterministic *in principle.* The assumption of determinism is incorporated in its very formalism, in the very notion of specifying the state or mode of a system by a *point*. What can more deterministic than a point? (Why do you think they say: 'Get to the point!')

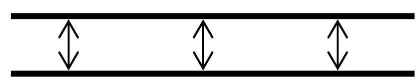
Quantum physics replaces points with 'arrows' or vectors as fundamental descriptors of a system's mode or state. As discussed in 2.,3. we observe that quantum theory has a discrete dynamics and continuous kinematics. This is the heart of its radical indeterminacy: at every moment, there is a *continuum of possible actions* for a given system, but the *actual evolution* such a system undergoes is discrete.

Possibility: a continuum
Actuality: discrete:

At any given instant, a system has a continuum of possible dynamic future(s) open to it, unpredictable apriori. The heart of the notion of spontaneity rests in the idea that possibility forms a greater set than actuality. This is what experience tells us, and it's reassuring the formalism of quantum theory confirms this.

There are many researchers who try to downplay or explain away the indeterminism of quantum physics. Rather than try to do so, we can turn the question on its head and ask ourselves how did classical physics become deterministic? From the standpoint of quantum physics we can illustrate this simply by considering a complex system like an aadvark or a galaxy. In both of these cases, the discrete dynamical character of such complex systems gets wiped out: the uniquely quantum mechanical effects get washed out and are unobservable. Hence we can perfectly well describe them with Newtonian mechanics (in the case of the aadvark) or relativistic mechanics (in the case of the galaxy.) In both Newtonian physics and Relativity, the dynamics becomes continuous. In other words, we have:

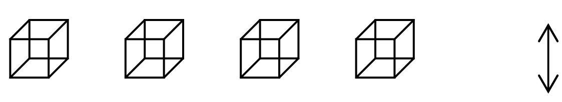
Possibility: a continuum
Actuality: a continuum:



Ironically, this case leads to determinism, because (as the arrows suggest), the set of possible outcomes can be put into 1-1 correspondence with the set of actual outcomes. Possibility is no bigger than actuality. Out of one actual outcome, only one possible future can arise. It's ironic to think that determinism arises from an approximation that classical physics makes, which quantum physics hopes to cure.

> Whence the curious concept of deterministic natural law? Presumably it developed from the experience that sometimes similar acts have dependably similar consequences. If one abstracts from, such unpredictables like lightning and love, one arrives at a pre-Socratic belief in a universal Logos governing all acts; a natural outgrowth of the experience of some lawfulness in nature and the desire for more. Then it takes one short step to identify with the Logos, to imagine we ourselves can hope to understand or control nature completely.[18]

What does it *mean* to deal with a system (like an electron or photon) small enough to exhibit quantum spontaneity? Consider the following illustration below:



[18] David Finkelstein, *Quantum Relativity,* p. 255

We have a collection of four *Necker Cubes.* What's maddening about them is they trigger circuits in the brain involving 3D perception. The optical illusion cannot be controlled, the orientation of each cube will appear to 'spontaneously' shift (either pointing UP [looking at 'bottom' of cubes] or pointing DOWN [looking at 'tops']). We can't control how we perceive their UP/DOWN orientations from moment to moment.

It is exactly the same when dealing with a quantum system: it has irreducibly spontaneous features. Determining the exact mode/state of an electron/photon means we irrevocably disturbed it. We'll irrevocably disturb each time we 'observe it,' which means interact it with a photon (itself a quantum physical entity.) Look at the Necker Cubes again. Consider a particular spontaneous orientation of one them as a 'measurement.' Is there any *necessary* connection between one configuration of orientations and the next? Of course not! Your brain is just cycling through the $2^4 = 16$ UP/DOWN ways of looking at the series as a 3D object.

The Necker Cubes also illustrate *Complementarity,* which follows the heels of the indeterminacy issue. Observables like position and momentum are said to be complementary, or dual. We saw such observables obey Heisenberg Uncertainty. Neils Bohr already pointed out this also means one can never simultaneously perform a position and a momentum measurement on a quantum system. The information of position destroys that of momentum, and vice versa, according to Heisenberg Uncertainty. We have *complementary* information: information about *aspects* of a system that's never complete.

What about the Necker Cubes? Our brain cannot perceive two orientations of a given cube at once. One orientation destroys the information of the previous. All the while, we *know* we're dealing with a 2D object. Our brain tricks us into thinking it's 3D. The orientations are *complementary*, and the *actual* cubes point neither up or down.

Many writers believe quantum complementarity implies a quantum mechanical system, like a photon, 'is' both a wave and a particle. This is inaccurate, and misleading. The 'wave-like' (non-local) properties result in *experiments* in scattering, where momentum (spectrum) is determined. On the other hand, the 'particle-like' aspects result in *localization* experiments, where position is determined. It's probably more accurate to say a photon, for example, 'is' *neither a wave nor a particle,* but *depending on the experiment* displays 'wave-like' OR 'particle-like' *aspects.* The Necker Cubes aren't oriented in either direction (they're 2D) but the brain makes an UP 'measurement' (orientation perception) or a DOWN 'measurement.'

> Quantum physics is kinematically non-local and dynamically local. Quanta only act where they are, but most initial and final actions…do not determine where they are.[19]

EPILOGUE AND ASSESSMENT

> Devout physicists today erect effigies of Mother Nature molded mainly on mathematics over an armature of experiment….We regard (this trend) as part of a continuous evolution of what constitutes an explanation, under the selective pressure of experiences further and further removed from those of our ancestors. When our innate language organ evolved, it did not have to cope with relativistic light speeds and quantum superposition. Oral cosmology in the language of the ancients seems archaic and futile today.[20]

Given the ground we just covered, how does assess the topics discussed in a theological and/or philosophical context? Adopting either a 'Conflict' or an 'Independence' position between science and theology, would, in the light of the above statement, lead one down to static positions of dogmatism or neo-orthododoxy. A *Dialogue* position seems best.

Based on the topics discussed, I offer the following points for dialogue:

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[19] ibid., Finkelstein, p. 372

[20] David Finkelstein & Wm Kallfelz, 'Organism and Physics,' *Process Studies* (Vol 26/3-4 Winter 1998)

- **Presuppositions and Limit Questions**

  Both relativity and quantum physics seek to dispense with *apriori* assumptions to the extent that their formalisms allow. In the case of Relativity, this involved bracketing imposed assumptions made concerning the geometry of space-time. In the case of quantum physics, this involved bracketing many assumptions: some logical, statistical, metaphysical, or epistemological (depending on the way the researcher chooses to interpret this continuously-evolving paradigm in perpetual 'crisis.')

  Relativity and quantum physics, however, share at least one epistemological point: Our very *acts* of measurement/observation preclude us from access to all information potentially available. In the case of relativity, this arose in the discussion of the observer's light cone. In the case of quantum theory, this point became apparent in the discussion on spontaneity/complementarity. Both physical theories then imply an *active, embodied* knower inextricably embedded in concentric dynamical system(s) of increasing order of refinement and complexity. We interact with this tapestry we're inextricably interwoven in and experience these interactions consciously as *phenomena.*

  Relativity and quantum physics, therefore, seem to 'save the phenomena' by confronting us to be more honest with ourselves. The phenomena we seek to represent and model in our mathematical physics are really just part of a terminal stage of a long dynamical chain of feed-forward and feed-back *interactions* arising from an embodied, human cognition. As Kant wrote, and as cognitive neuroscientists and philosophers of mind continuously discover, there is no simple or passive relationship between the 'knower' and the 'known.' The metaphor of mind as 'mirror' seems irretrievably naïve. Mind actively engages itself in the world. Most of these interactions occur behind the curtain, hidden from the dim light of consciousness. Mind is an *interactive instrument* in the world, not a mirror. Inevitably, interaction precludes and disturbs possibilities, by and through its very nature.

  Paradoxically, though, confronting such limitations gives us also greater freedom. There are phenomena discovered, and applications waiting to come to fruition, in the context of such theories which were inconceivable in the Newtonian world-view. Newton was no friend of the Cartesian physical tradition. Yet Newton's age was profoundly influenced by Cartesian thought, pitting separate 'substances' of mind over against 'matter' which seem to imply an observer at once omniscient and impotent.[21] The metaphysical implications of quantum theory and relativity, on the other hand, seem rather to imply a *plurality* of *aspects*, rather than a *dualism of substances*. Mind/matter/energy/time/space/… are *aspects* of Nature; which the theoretical physicist strives to show their implied common origin. This is what is meant by unification: the physicist seeks not to *conflate* aspects with one another; but rather like a Renaissance painter attempts to show how the infinite plurality of 'perspectives' (energy measurements/ position measurements/ etc..) indeed derive from some simple source. This 'source,' in the end, may only be partially, indirectly, and provisionally known. (After all, mind is an aspect itself, and the *representations* it constructs, largely unconsciously, cannot be confused with the actual entities it seeks to represent).

- **Methodological Paralells**

  Husserl developed the notion of 'epoche:' through analyzing ambiguous perceptual situations *via suspending as many apriori assumptions as possible,* one may perhaps gain valuable insight into the *way* the mind erects its representations.[22] Relativity and quantum theory present us with similar 'epoches' confronting us with the *way* our tradition in physics reared us to think. Most of this 'thinking' takes place on levels unquestioned. But relativity and quantum physics holds up a mirror to ourselves; we *must* confront certain basic questions and it would be difficult to imagine how we cannot.

[21] David Finkelstein, private conversation

[22] For example, consider the case of the Necker Cubes. Another famous example: consider situations when the mind confuses subjective with objective motion (like when sitting in a train at rest in a train station and watching another train begin to accelerate as it departs).

There is plenty written about what quantum physics and relativity tells us about 'the world.' But perhaps what is most crucial is what these theories tell us about ourselves, and the *way* we engage the world in our scientific tradition. Since Galileo, the tradition in mathematical physics has taught us to isolate aspects of Nature stable enough to earn the honorific 'observable,' and quantify and correlate such observables with methods in precise mathematical sciences, for purposes of prediction, of which explanation is often subordinate to.

Descartes' Damacles' sword cuts both ways, however. A passive 1-1 correspondence is drawn between mind and matter, rendering perception passive and relatively unproblematic. The Cartesian/Modernist tradition may have taught the physicist that the correlation between the mathematical entity one invents and the (theory-laden) quantifiable 'observable' is ideally 1-1, and at the very least, natural and unproblematic. To be sure, physics is a very complex art as well as a science, and a complex tapestry of 'bridge conditions,' background theories, and ad-hoc approximations tie experimental data with a theory's models and abstract relations. Yet in this process, the physicist seldom thinks of *himself or herself* in this process. The physicist is in dialogue with the apparatus, the formalism. Where is the physicist?

But in relativity and in quantum physics we witness a formalism at the inception of 'self-consciousness.' Relativity reminds us that the geometry of space-time measurements is derived from *light signals*; i.e., *information*. The formalism of quantum physics takes this one step further by representing *acts of measurement* directly *in* its formalism.[23] What this means is that the theoretical 'software' has become rich enough to include aspects of the subject conducting the experiment. Like a higher-level programming language, the formal system can 'talk about itself' to a certain extent. This seems to have happened on its own accord: the every evolutionary pressure wrought forth by the problems of ever-increasing subtlety in the field has put us in this situation today.

What, therefore, we are confronted with is the notion that *just as there is no simple, passive relationship between mind and the 'external' world, there is no simple, passive relationship between mind and its innermost workings.* That seems obvious enough. Psychology since the time of Freud has been reminding us of what by now must sound like a cliché. However, the obvious is sometime easy to miss. The physicist Wolfgang Pauli collaborated with the depth psychologist Carl Jung.[24] Why? Precisely because Pauli felt that the development of quantum physics poised the physicist toward a unique opportunity to study more closely the *archetypal patterns* through which the mind engages in when doing mathematical physics. By definition, Jung's theory of archetypes are unconscious, and therefore may only be analyzed by indirect means. By the same token, when the mind correlates observable data with mathematical structures, this is by and large an *unreflective process*. Much of the skills involved in mathematical modeling obviously are inherited from education, but it is also true that many of these activities are *unconscious.* Unconscious and unreflected assumptions are what precisely undergird research paradigms.

Quantum physics, however, may place the physicist in a unique position. While investigating subtle processes in the microworld, basic methodological issues regarding modelling such processes lend themselves to closer critical scrutiny in ways unparalleled since Galileo. What comprise of the basic methodological structures the mathematical physicist imposes on the manifold of theory-laden data in an effort to pattern them in ever-more coherent and encompassing ways? How can the formalism of quantum physics indirectly or directly evince a broader critical understanding of such structures?

The following are necessary aspects I believe comprise the syntax and semantics[25] of mathematical physics. The list, of course, is partial and provisional:

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[23] Recall that a measurable is a linear operator. An experimental act is typically modeled: as an eigenvalue equation. The type of experiment one chooses to perform is represented as the *basis* of the mode vector.
[24] *Man and His Symbols*
[25] (correlating mathematical entities with observable properties of a physical system)

I.) **Discrete/Continuum:** A fundamental hypothesis in mathematics[26] says sets can come in only two varieties: discrete and continuous. Perhaps this is so because our mind can only formally abstract in these two fundamental modes.

II.) **Symmetry:** We recognize symmetries in concrete and abstract senses. It is the theoretical physicists' guiding principle when investigating new formalisms.

III.) **Group Theoretic Simplicity/Stability** A technical notion first advanced by the mathematician Segal,[27] reveals that the "simplest[28]" theories are also the most "stable." Stability could be best understood here as the notion comprising what would happen when one chooses to vary the theories' fundamental parameters, i.e. how quickly would the theories terms veer into absurdity or incoherence.

IV.) **Reciprocity:** David Finkelstein defines this notion precisely in terms of a 'two-way coupling:'[29] entities such as time and space mutually transform into one another in the case of special relativity, for example. In quantum theory we see a reciprocity between system and observer: the observer acts on the system, and the system acts back on the observer.

Such above notions inadvertently play themselves out as the field of theoretical physics continues to evolve. Researchers like David Finkelstein consciously use them as operant principles in their work.

- **What Does This All Have To Do With Theology?**

Continuing in this 'Dialogue' engagement between science and religion, we are confronted with an urgent question. The philosophical ramifications of the topics discussed make for strong case that Modernism is irretrievable. Most thinkers consider the implications of relativity and quantum theory, 'post-modern,' as they do for a variety of other 'hard' and 'soft' scientific disciplines.[30]

But we must confront another question: are these implications *pre-Modern*? Certainly not literally so, but what about in philosophical and theological spirit? At the height of the Late Middle Ages, eclectic scholars like Nicholas of Cusa[31] (1405-1485) held to theological positions with remarkably resonant notions[32]:

I.) Two-term logic applies only to the finite categories of reason

II.) Beyond two-term logic, the intellect *apprehends without comprehending* (in an asymptotic manner) the mysteries best represented mathematically by the notion of *infinity.*

III.) Reason can never 'comprehend' the world. Though like a series of inscribed polygons in a circle, reason can asymptotically converge to the circle in a manner that is both infinitely close and infinitely far. Total comprehension, like absolute truth, is a secret known only by God. When the reason strives after absolute truth, it is inevitably seized in paradox.

IV.) Reason is ultimately *based on perspective.* Human reason 'escapes one perspective to be imprisoned by another.' The Mind of God alone encompasses all perspectives. By definition, the mind of God is unknowable.

V.) Mathematics is Reason's exemplary activity, for it is in this freely constructive exercise that the human mind best imitates God's free, creative activity. The mathematical symbols are arbitrarily 'true' module their closed, symbolic world they reference. However, the relation between mathematics is *not* strongly referential. Mathematics at best are part of the reason's noble attempt to 'illustrate' its attempts at comprehension of an infinite world that can at best disclose itself dimly to the reason and intellect's dim eye.

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[26] The Continuum Hypothesis: There are no sets with cardinal numbers between $\aleph_0$ and $\aleph_1$

[27] Finkelstein (2001-2002), correspondence and conversation (Quantum Structures Workshop, Ga Tech.)

[28] Precisely, that their relativity groups *ideally* contain no invariant subgroups.

[29] correspondence and conversation (Quantum Structures Workshop, Ga Tech.)

[30] Frederik Ferre gives the criteria for what constitutes a postmodern science in his *Knowing and Value.*

[31] Jasper Hopkins (U Minnesota) is the nation's leading Cusa scholar

[32] *De Docta Ignorantia*, (trnsl J Hopkins -Minnesota Press 1981)

It's easy to make superficial comparisons. To prevent this, one must remember that Cusa articulated his theology and philosophy in a neoPlatonic and Aristotelean conception radically different form our contemporary world-view.[33]

Nevertheless, in spite of the different historical contexts, one can detect *methodological parallels.* Points I.) - IV.) seem rather concordant with the notions of light cone (reason is perspectival), complementarity (in efforts to completely comprehend, one is seized in paradox) Quantum Logic (logic applies to the empirical realm of reason), and the generally 'open' character of the horizons of Nature implied in these theories and by Cusa.

I believe this is no accident. Cusa was considered one of the most articulate proponents of the *via negativa*; (apophatic theology) which was tradition of theology and mysticism of early origin.[34] The tradition is complex, and is essentially united in the conviction that reason's activity in the world is limited: in many matters natural and divine reason is confronted *ineffable* and in many cases, *unknowable* realms and regions, that are nevertheless meaningful and significant. Nicholas of Cusa's 'negative theology' prompted him to utter his perhaps most succinct and poignantly:

> You my God would not satisfy the Intellect if You were wholly comprehensible. Though You would not satisfy the Intellect either were You wholly incomprehensible. Therefore You are neither entirely comprehensible nor entirely incomprehensible; and the Intellect comprehends You by not comprehending.[35]

One is tempted to see a trace of Heisenberg Uncertainty here, insofar as 'comprehension' is qualified along the lines of its opposite. Inevitable sacrifices must be made as the intellect attempts to arrive at any degree of illumination. Heidegger, for example, describes the world as 'luminous, but opaque' to the light of reason. The rationalist on the other hand might view the world as 'luminous and transparent.'[36]

In summary, the active, embodied view of the mind embedded in the world, as implied by the relativity and quantum theories, lend support to constructive dialogue with strains of neoPlatonic negative theology. This tradition flourished in the late Middle Ages. Whether twenty-first century science become constructive-postmodern or deconstructive, postmodern[37] one may envision a fruitful dialogue with the new physics and aspects of negative theology.

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[33] To name just one example: In neoPlatonic 'psychology,' reason (Ratio) was distinguished from Intellect. The intellect is the soul's transcendental faculty enabling 'apprehension' of ineffable mysteries, that go beyond the finite. Reason, on the other hand, preoccupied itself solely with the finite (world of knowable phenomena).

[34] Scholars debate its inception into Christianity. The writings of the Pseudo Dionysius the Areopagite indicate its origins go back to at least the fifth century.

[35] DDI (In Kallfelz, p. 15) The strongly paradoxical language by Cusa is not mere poetry, and reflects a complex dialectic and epistemology.

[36] When I asked John Polkinghorne to elaborate on his disdain toward the via negativa during Emory University's commemorative Science/Religion Seminar (Sp 1999) he explained his conviction that the universe is 'transparent to analysis.' This is succinct present-day endorsement of the via positiva.

[37] Ferre's distinctions (Introduction - *Knowing and Value*)

Appendix I- Maxwell's Equations In Free Space Give Electromagnetic Plane Wave Solutions, Travelling at *c*

In the case of a vacuum, Maxwell's Equations (in differential form) simplify to:

$$\nabla\bullet\boldsymbol{E} = 0 \qquad \nabla\bullet\boldsymbol{B} = 0 \qquad \nabla\times\boldsymbol{E} = -{}^{1}/_{c}\,{}^{\partial}/_{\partial t}\boldsymbol{B} \qquad \nabla\times\boldsymbol{B} = {}^{1}/_{c}\,{}^{\partial}/_{\partial t}\boldsymbol{E}$$

Lemma: For any smooth function $\boldsymbol{\Phi}: R^3 \to R^3$

$$\nabla\times\nabla\times\boldsymbol{\Phi} = \nabla(\nabla\bullet\boldsymbol{\Phi}) - \nabla^2\boldsymbol{\Phi}$$

Proof: Make use of the Levi-Civita Identity: $\epsilon_{ijk}\epsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$

$$\nabla\times\nabla\times\boldsymbol{\Phi} = \epsilon_{ijk}\epsilon_{klm}\partial_j\partial_l\Phi_m \quad = (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})\partial_j\partial_l\Phi_m = \delta_{il}\delta_{jm}\partial_j\partial_l\Phi_m - \delta_{im}\delta_{jl}\partial_j\partial_l\Phi_m = \partial_i\partial_j\Phi_j - \partial_j\partial_j\Phi_i$$
$$= \partial_i(\nabla\bullet\boldsymbol{\Phi}) - \nabla^2\Phi_i = \nabla(\nabla\bullet\boldsymbol{\Phi}) - \nabla^2\boldsymbol{\Phi}$$

Hence: $\nabla\times\nabla\times\boldsymbol{E} = \nabla(\nabla\bullet\boldsymbol{E}) - \nabla^2\boldsymbol{E} = -{}^{1}/_{c}\,{}^{\partial}/_{\partial t}\nabla\times\boldsymbol{B} = -{}^{1}/_{c^2}\partial^2_t\boldsymbol{E}$

Simplifying right hand side, we're left with: $\nabla^2\boldsymbol{E} = {}^{1}/_{c^2}\partial^2_t\boldsymbol{E}$

(The equation describing plane waves propagating with speed *c*!)

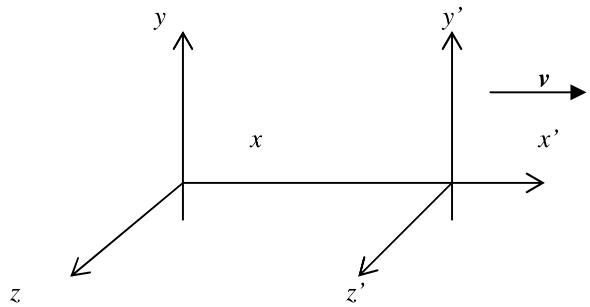
Similar results hold for magnetic field ***B***

Appendix II-Derivation of the Lorenz Contractions

As much as Einstein gets the credit for revising our notions of what remains invariant under motion, he was not the first to pose such kinds of questions. Aristotle was (as far as we know). Galileo, however, was the first to provide a quantitative account of the problem of motion (regarding what specifically remains unchanged by it.) Galileo's account differed radically from Aristotle's. For example, Aristotle believed that the laws of motion had a different form for bodies in motion versus bodies at rest (which is why for instance he placed the Earth at the center of the cosmos and made it fixed--he thought if the earth were moving, it would dissipate its atmosphere.) Galileo disagreed which made him in many just as radical as Einstein was for the early twentieth century. Galileo had one envision being in a small room on board a ship, sailing at a constant velocity. He demonstrated that as far as the observer was concerned -- nothing changes in his or her frame of reference. Our bodies feel and respond to *accelerations*, not (constant) velocities. The same is true for anything in our frame of reference moving at constant velocity. Galileo asks one to envision a fish-bowl, some butterflies in the ship's room, a bird in a cage, etc. As long as the ship was sailing at a constant velocity, everything in the room would behave as if the room were at rest. Such a notion of a frame of reference travelling at a constant velocity is known as an *intertial frame of reference (or IFR).*

- By now, the above observations by Galileo may seem so trite and 'commonsensical.' This is true only for historic reasons -- physics, as well as our education, has progressed for several centuries since Galileo. We have assimilated Galileo almost unconsciously in our thinking, confusing it with 'common sense,' (as Mediaevals felt Aristotle typified the essence of 'common sense.') Well as we're about to see, our 'common sense' certainly isn't all that reliable! Or perhaps better stated, 'common sense' only is reliable in certain conditions and special cases (namely those corresponding to 'ordinary' human experience.) Before discussing Einstein let's describe Galilean Relativity:

  Consider the two IFRs below. For mathematical simplicity, assume the primed system (with coordinates: (t',x',y',z')) is moving relative to the unprimed system along its *x*-axis, with velocity: *v:*



Then according to Galileo, the IFR at rest and the IFR in motion transform according the following rules:

$x'=x - vt$	$x=x'+vt$
$y'=y$	$y=y'$
$z'=z$	$z=z'$
$t'=t$	$t=t'$

…Which of course may seem commonsensical and obvious. For example, suppose I was sitting in a train and at $t=t'=0$ I leaned out the window and said good-by to you. Suppose the train moves at v=5 m/s in the x-direction. Suppose, further, that at $t=t'=0sec$, $x=x'=0$ (our origins coincided initially). Suppose you continue to stand there at the train station without moving, while I look our of the window as the train moves. Then, at t=50 *sec,* in your IFR your position would still be: $x = 0m$. Whereas from my moving IFR, your position would be: $x' = 0 - 5m/s*50sec = -250\ m$, our you'd be 250 m behind me. Similarly, in my moving IFR ***my*** position (as long as I sat there and didn't walk around in the train) would read: $x' = 0m$ whereas ***your*** measurement of ***my*** position measured from where you are standing at the railway station would read: $x = x' + vt = 0 + 5(50) = +250$ m, or I would be 250 m ahead of you. In both instances, as $t = t'$, we'd never think that somehow my clock was ticking at a different rate than your clock because of the train's motion.

- The Galilean transformations work perfectly fine so long as $v << c$ (where $c = 3.0\times 10^8\ m/s$ is the speed of light measured in a vacuum). Another way of phrasing this is to say Galileo is right if the speed of light is instantaneous (or infinite, as was thought in Galileo's day.) Einstein, however, argued, that if two or more people sitting on two or more IFRs are ever going to communicate, they ultimately need light-signals to do so. In fact, every measurement, and therefore every transmission of information, is mediated by light. There's no getting around this fact. Einstein postulated, based on what was known about radiation and electromagnetism at the time, that:

  The speed of light is measured as a finite constant all IFRs. (SRP)

  This Special Relativity Postulate (*SRP*), which has been to this date experimentally confirmed in a variety of ways (ranging from the subatomic to the interstellar) has profound consequences. Think about what it says: no matter how fast I travel, I *always* measure the speed of light to be the same in my frame of reference. Why? Because it takes light to make a measurement in the first place. And locally, I am always at rest with respect to light. Therefore I have no other option but to measure light to propagate at c.! This was the basis of Einstien's argument.[38]

- Mathematically, the above postulate translates into the following statement:

[38] I am summarizing its broad points. The subtleties of Einstein's arguments can be found in a variety of sources, among the more interesting (and readable) include Michael Shallis' *On Time* (Free Press 1983)

$$c^2t^2 - x^2 - y^2 - z^2 = c^2t'^2 - x'^2 - y'^2 - z'^2 \qquad \text{(I)}$$

which says concisely that the difference between the measured time (squared) and location of an event (squared) in the moving frame has the same value as the difference between the measured time (squared) and location of an event (squared) in the frame at rest. This is a consequence of the SRP. Consider, furthermore, that a person in the frame at rest turned on a flashlight. After t seconds, the tip of the beam would be at location (x,y,z) and had traveled a distance: ct. In other words: $ct = (x^2 + y^2 + z^2)^{1/2}$. In the moving frame, the observer would view the tip of the beam at position (x',y',z') after t' seconds…the beam in the moving frame would have traveled a distance $ct' = (x'^2 + y'^2 + z'^2)^{1/2}$

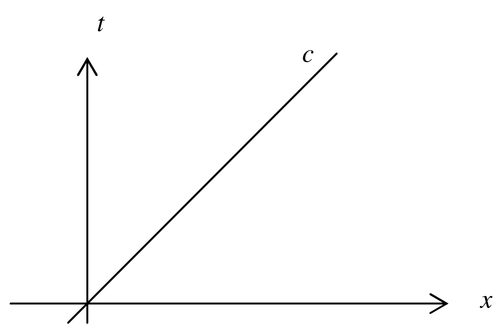
For convenience, we can square both statements, which read, hence:

$$c^2t^2 = x^2 + y^2 + z^2 \qquad c^2t'^2 = x'^2 + y'^2 + z'^2 \qquad \text{(II.)}$$

- In the light of equation (I) which results from the SRP let's re-examine how the formulae for the coordinates in the two IFRs transform. Will we recover Galileo's formulae? No, not in general. Galileo's formulae are recovered only in a special, limiting case. Here's what we need to do:

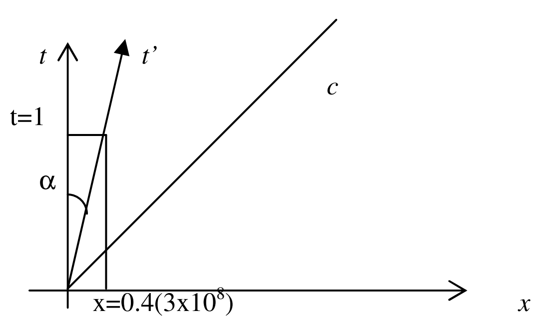
1. For simplicity, let's focus on (x,t) and (x',t') alone, which we can do as long as we continue to make the velocity $\boldsymbol{v}$ of the moving IFR parallel to the x-axis.

2. Let's compare the axes of the moving IFR with those of the IFR at rest. For the IFR at rest, Special Relativity implies:



The diagonal represents the finite speed of light in this IFR. If the speed of light were infinite, the slope of the line representing c would equal 0. (**Note**: for convention, we've placed the x axis at the horizontal and the t axis at the vertical.) Since the speed of light is finite (and equal to 3.0×10^8 m/s) for simplicity in the diagram we can set $c = 1$ (which makes the slope = 1[39].)

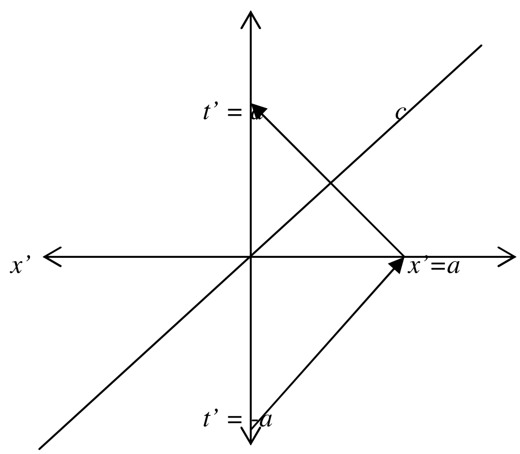
Now consider a moving IFR. The t' axis seen from the rest frame would look like:



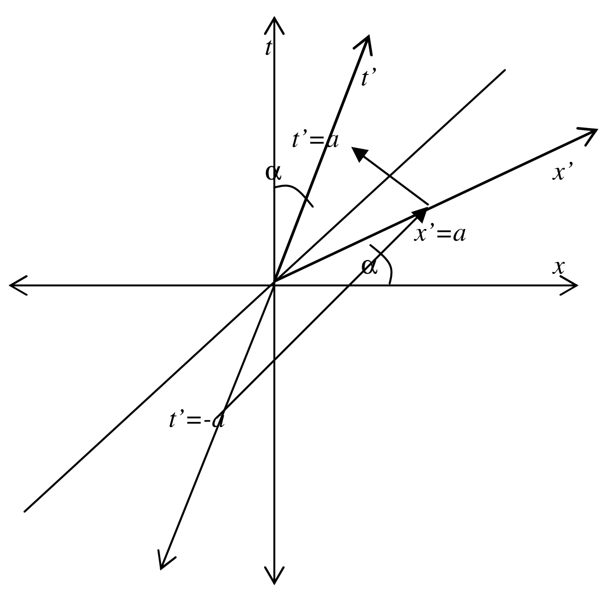
[39] Setting finite constants = 1 is a procedure known as 'normalization,'…it amounts to choosing a convenient system of units to make the arithmetic simple.

where: $\tan\alpha = v$. This *t'* axis is known as the ***world-line*** of the moving IFR. The way to understand this construction is to imagine someone at rest in the origin in the moving system (i.e. someone sitting on the *t'*-axis, where *x'=0*). That person would be receding away from the origin of the IFR at rest (where *x=0*) with velocity *v*. For example, if *v=0.4c*, then after *t = 1* sec then the person at rest in the moving IFR is located at $0.4(3.0 \times 10^8) = 1.2 \times 10^7$ m away from the origin of the IFR at rest. (See above figure.)

3. Now, what about the orientation of the *x'* axis? This is where we take explicit advantage of the SRP (that the speed of light is constant in all IFRs.) Let's imagine we're on the moving IFR. Suppose at *t'* = *-a*, we send a light signal to a mirror, stationed at *x = a*. Then, when *t'=0*, the light beam strikes the mirror. The beam will be reflected back and reach us at *t'=a*. See diagram:



Now, let's transform back into the IFR at rest. Where will *x'* be located? Answer: ***We need to locate x' such that the distance between t'=-a to t'=t=0 and the distance between t=t'=0 and t=a are equal.*** See diagram below:



For this to be possible, the location of x' must be a mirror image of the t' axis about the line c (see above illustration.)

So the equation of the line representing the t' axis ($x'=0$) is: $t = ({}^1/_v)x$ or: $x - vt = 0$
The equation of the line representing the x' axis ($t'=0$) is: $t = vx$ or: $t - vx = 0$

4. Now, we have to think of the most general way in which the rest frame (x,t) and the moving frame (x',t') transform. Mathematically, the most general way the moving system, for example, can transform with respect to the system at rest, is by the rule:

$$x' = Ax + Bt \qquad \text{(IV.)}$$
$$t' = Cx + Dt$$

…where A,B,C,D are undetermined parameters. Our task is to solve for these parameters.

According to our previous remark, we know that: $x'=0$ implies: $x = vt$
$t'=0$ implies: $t = vx$

Plugging in those conditions to the above transformations:

$$0 = Avt + Bt$$
$$0 = Cx + Dvx$$

which implies: ${}^B/_A = {}^C/_D = -v$

therefore we may write: $x' = A(x - vt)$
$t' = D(t - vx)$

Now, as shown above, we know the events: $(x'=0, t'=a)$ $(x=a\sin\alpha, t=a\cos\alpha)$ and $(x'=a, t'=0)$ $(x=a\cos\alpha, t=a\sin\alpha)$ are connected by a light-ray:

$$0 = A(a\sin\alpha - va\cos\alpha) \qquad a = D(a\cos\alpha - va\sin\alpha)$$
$$a = A(a\cos\alpha - - a\sin\alpha) \qquad 0 = D(a\sin\alpha - va\cos\alpha)$$

which implies: $A = D$

Last of all, consider: $c^2t^2 - x^2 = c^2t'^2 - x'^2$ (I.)
Setting $c = 1$ we have: $t^2 - x^2 = t'^2 - x'^2$
Inserting to solve for our last constant A:

$$t^2 - x^2 = A^2[(t - vx)^2 - (x - vt)^2]$$
$$= A^2[(t^2 - 2vxt + v^2x^2) - (x^2 - 2vxt + v^2t^2]$$
$$= A^2[(t^2 - x^2) - v^2(t^2 - x^2)]$$
$$t^2 - x^2 = A^2(t^2 - x^2)(1 - v^2)$$
$$\Rightarrow A^2 = (1 - v^2)^{-1}$$
$$\therefore A = \pm(1 - v^2)^{-1/2}$$

Select the positive term: $\mathbf{A = (1 - v^2)^{-1/2}}$

Therefore, we have specified the transformations, having solved for A,B,C,D :

$$x' = \mathbf{(1 - v^2)^{-1/2}}[x - vt] \qquad t' = \mathbf{(1 - v^2)^{-1/2}}[t - vx]$$

In this entire procedure, we have set $c=1$. Inserting c explicitly, the transformations become:

$$x' = [1 - (^{v}/_{c})^2]^{-1/2}(x - vt) \qquad t' = [1 - (^{v}/_{c})^2]^{-1/2}(t - vx/c^2)$$

The term: $[1 - (^{v}/_{c})^2]^{-1/2}$ is known as the *gamma-factor* (denoted: γ) and acts as a parameter governing special relativistic effects. We can re-write the transformation equations as:

$$x' = \gamma(x - vt) \qquad t' = \gamma(t - vx/c^2)$$

These transformations are known as the **Lorentz Transformations,** distinguishing them from the Galilean Transformations discussed earlier.

- How do the Lorentz Transformations contrast with the Galilean transformations? To answer this question, consider the two equivalent cases: 1) $v << c$ or 2) $c \to \infty$ Both of these cases are equivalent, because we're assuming the speeds is much less than the speed of light, so for all practical purposes the speed of light might as well be infinite. Now in this case observe that:

  $\gamma \to 1, \qquad vx/c^2 \to 0$

so our transformations reduce to: $x' = x - vt \qquad t' = t$

or we recover the Galilean transformations! Hence: *Special relativity reduces to Galilean relativity for speed much less than c.*

- When **do** special relativistic effects take place to noticeable effect? To answer this let's take a look at the gamma-factor. Suppose we're travelling at half the speed of light: $v=0.5c$ then:

  $\gamma(0.5c) = [1 - (^{0.5c}/_{c})^2]^{-1/2} = 1.15$

  …not much different from 1!

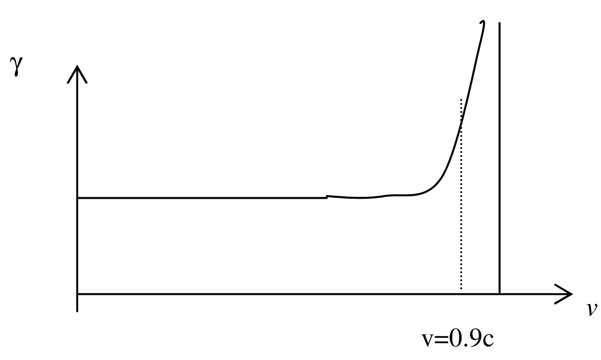
  How about if we're travelling at $v=0.75c$?

  $\gamma(0.5c) = [1 - (^{0.75c}/_{c})^2]^{-1/2} = 1.15$

  …and at $v=0.9c$?

  $\gamma(0.9c) = [1 - (^{0.9c}/_{c})^2]^{-1/2} = 2.29$

Examining the behavior of γ with respect to v we see that significant realtivistic effects don't kick in until v is significantly close to c…we have to travel to almost 75% before we notice any of the weird effects associated with the theory of special relativity



Notice that when v becomes c, $\gamma \to \infty$. This implies (as we'll see later in the course) that nothing with any rest mass can ever travel the speed of light (photons have zero rest mass) for it would require infinite energy.

Appendix III Velocity, Momentum in Minkowski Spacetime: A Derivation of $E = mc^2$

- **Relativistic Velocity**

Suppose someone is travelling in the *x'* direction (in the moving frame) with velocity *u'* (example: Captain Kirk running along the bridge of the Enterprise to avoid Klingon sneak attack). How does *u'* transform in a frame locally at rest? (In other words, if I'm a Vulcan watching Captain Kirk from my planet and I know the speed of the Enterprise relative to my planet is *v*, how fast will I measure Captain Kirk's speed *u* before I helplessly see him clobbered for the nth time by those wily Klingons?)

Answer: $u' = dx'/dt' = \gamma \, {}^{d}/_{dt'}(x - vt) = \gamma\,({}^{dx}/_{dt'} - v\,{}^{dt}/_{dt'}) = \gamma\,({}^{dx}/_{dt}\,{}^{dt}/_{dt'} - v\,{}^{dt}/_{dt'})$

(pulling γ out since it's a constant, and applying the Chain Rule to ${}^{dx}/_{dt'}$ in the last step)

However, ${}^{dx}/_{dt} = u$ (the speed as measured in the rest frame, i.e. Planet Vulcan)

Hence: $u' = \gamma(u\,{}^{dt}/_{dt'} - v\,{}^{dt}/_{dt'}) = \gamma(u - v)\,{}^{dt}/_{dt'}$

Now let's try to figure out ${}^{dt}/_{dt'}$ which describes the relative rate of time passing on Vulcan with respect to time passing on board the Enterprise. Since we're all Vulcans (yes, Spock?) hanging out on our home planet we'd really be more interested in ${}^{dt'}/_{dt}$ (the relative rate of change of time on board that rust bucket Enterprise measureed with respect to our proper time of our own home planet. Hence: ${}^{dt}/_{dt'} = {}^{1}/\,{}^{dt'}/_{dt}$

'Yes, I concur' says Spock, 'this is an innately superior approach[40]'

Okay anyway:

$${}^{dt'}/_{dt} = {}^{d}/_{dt}\gamma\,(t - vx/c^2) = \gamma\,{}^{d}/_{dt}(t - vx/c^2) = \gamma(1 - v/c^2\,{}^{dx}/_{dt}) = \gamma(1 - vu/c^2)$$

So therefore:

$$u' = \gamma(u - v)/{}^{dt}/_{dt'} = \frac{\gamma(u - v)}{\gamma(1 - vu/c^2)} = \frac{(u - v)}{(1 - vu/c^2)}$$

Which is the equation of velocity transformation between IFRs moving at realtivistic speeds. Observe that is for some reason in the struggle with the Klingons[41] Kirk threw the parking brake for the Enterprise, slowing it down to a crawl, so that : $v \ll c$. Then the denominator term:

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[40] Does anyone remember asking what Spock thought? Hello? Spock, you're out of order! Keep your pious platitudes to yourself! my God, I'm getting emotional, that's not supposed to happen…

$1 - vu/c^2 \approx 1$, so then: $u' = u - v$

(which is the transformation of relative velocities in a moving and fixed frames for the Galilean case, which you've seen in ch 2)

- **Relativistic Momentum and Energy**

When a particle is moving at a very fast speed u, we have to include its own gamma factor:

$$\gamma_u = [1 - (u/c)^2]^{-1/2}$$

Hence its *relativistic momentum* becomes:

$p = mu = \gamma_u m_0 u$ where m_0 is the particle's *rest mass*.

This is something new: as a particle gathers speed, its mass increases! We call this the particle's *relativistic mass:* $m = \gamma_u m_0$ In fact, it's easy to see from the gamma factor that as $u \to c$, $m \to \infty$ So it doesn't matter what kind of diet Captain Kirk is on, as long as he has measurable rest mass, there ain't no warp 5 or 4 or even 1 for that matter according to Special Relativity, unless the Enterprise has an infinite antimatter fuel payload.

Hence we must define relativistic force more generally using Newton's original definition:

$$F = \frac{d}{dt}p$$

- **Relativistic Energy:**

Expressed precisely (as an integral), the Work-Energy Theorem says:

$$W = \Delta KE = \int F dx = \int \frac{d}{dt}p\, dx = \int \frac{dp}{du}\frac{du}{dx}\frac{dx}{dt}\, dx$$

where we're using the trick of the chain rule in the last expression in order to pick an easy dummy variable of integration. Note that the dx's cancel, and that $dx/dt = u$, so that our integral can be expressed in terms of relativistic speed:

$$W = \Delta KE = \int \frac{dp}{du}\, u\, du$$

Let's first compute: dp/du

$$\frac{dp}{du} = \frac{d}{du}(\gamma_u m_0 u) = m_0 \frac{d}{du}(\gamma_u u) = m_0 \frac{d}{du}\{[1 - (u/c)^2]^{-1/2} u\}$$

[41] What I don't understand about early Star Trek is that with all of that sophisticated weaponry on board (photon cannons and maser watchamacallits that can blast a small galaxy to bits) why does Kirk always end up in some sort of wrestling match or fisticuffs? Better ask Spock…

$$= m_0 \frac{[1 - (u/c)^2]^{1/2} - u(1/2)[1 - (u/c)^2]^{-1/2}(-2u/c^2)}{[1 - (u/c)^2]}$$

(using quotient rule and chain rule)

Simplifying:

$$= m_0 \frac{[1 - (u/c)^2]^{1/2} + u[1 - (u/c)^2]^{-1/2}(u/c^2)}{[1 - (u/c)^2]}$$

$$= m_0 [1 - (u/c)^2]^{-1/2} \frac{\{[1 - (u/c)^2] + (u/c)^2\}}{[1 - (u/c)^2]}$$

(factoring out the $[1 - (u/c)^2]^{-1/2}$ term in the numerator)

$= m_0[1 - (u/c)^2]^{-3/2}$ (after cancelling the appropriate stuff in the numerator)

Hence: $W = \Delta KE = \int dp/du \, u du = m_0 \int [1 - (u/c)^2]^{-3/2} \, u du$

Of course, for the above integral to be 'phyisical' it must be *definite*. Consider the act of accelerating the object from rest (lower limit) u=0 to some specified final speed U

As you can see the above integral is easily dealt with by making the following substitution:

$w(u) = [1 - (u/c)^2]^{-}$ Hence: $dw = -2(u/c^2)du$ so: $udu = -(c^2/2)dw$

and for the limits of integration we have: $w(0) = 1$ $w(U) = [1 - (U/c)^2]$

So: $W = \Delta KE = -m_0c^2/2 \int_1^{[1 - (U/c)2]} w^{-3/2} \, dw = (-m_0c^2/2)\text{-}2w^{-1/2} \Big|_1^{[1 - (U/c)2]}$

$= m_0c^2[1 - (U/c)^2]^{-1/2} - m_0c^2$

The first term we recognize as $\gamma_u \, m_0 \, c^2$

The second term $- m_0c^2$ seems peculiar. Let's rewrite the top expression in a slightly different manner:

$\Delta KE + m_0c^2 = \gamma_u \, m_0 \, c^2$

On the right hand side, we saying that the *total energy E* of the particle is the sum of its kinetic term and its 'rest' energy term m_0c^2 Hence even when a particle isn't moving, it has energy extractable from its rest mass! (this is what has led to nuclear physics)

Now on the right hand side we recognize $\gamma_u \, m_0 = m$, i.e., the *relativistic mass* of a particle

Hence we can simplify the above equation by writing:

$$E = mc^2$$

which of course one of Einstein's profoundest derivations in his Theory of Special Relativity.

In words, relativistic energy of a particle is proportional to it relativistic mass (proportionality constant is the square of the speed of light.) But even more fundamentally, the above derivation tells us that mass and energy are *equivalent*. Special Relativity has unified mass and energy the way electromagnetism has unified electricity with magnetism, the way Newton has unified celestial physics with terrestrial physics, etc… Physicists get excited about unity because (ironically) it makes theories 'simpler' insofar as they require fewer fundamental explananda (explanatory categories) and also seems to indicate that the theory must has some ruich and descriptive power since we encounter a world of unified phenomena out there.

Appendix IV: Results from General Relativity: Gravitational Singularities ('Black Holes')

Schwarzschild (ca 1890) already observed that as one shrinks a spherical object to arbitrarily small radius, its gravitational potential shall increase to arbitrarily high intensity. Such "singularities" were also given a more rigorous treatment in Einstein's Theory of General Relativity (1914) but until the work of Hawking and Finkelstein in the 1960s, such singularities were thought to be mere mathematical artefacts, having no physical basis.

A singularity is theorized to form from intense gravitational pressures from the most massive continuous bodies known to exist in the observable universe: stars[42]. Most stars won't become black holes, however. Basically, stars undergo a particular 'life cycle' demarcated and determined by what their primary elements are residing at their core necessary for fusion. The 'lightest' element, Hydrogen, gradually will deplete itself, and the primary element for fusion then gradually switches itself to Helium, until that gets used up, etc…The heavier elements 'burn' less efficiently (give off less energy in fusion reactions). So at a certain point, once a star's core is composed of much heavier elements such as Fe, K, Pt, etc., their fusion reactions are so weak that the heat and pressure cannot even sustain the attractive gravitational field on the star's surface, so the star begins to implode. Sometimes, if the star is massive enough, the implosion is so catastrophic that star will Supernova, which occurs when the collapsed outer shell 'bounces' off its dense core back into outer space. For most stars, this doesn't happen, however. Most stars die a quiet death, insofar as in their collapse they're reduced to cool, glowing dense spheres of equally heavy materials. These objects are known as 'white dwarfs.' However, if a star is more massive, the intense graviation won't stop at white dwarf stage, as the internuclear forces cannot sustain the gravitational collapse, so formerly outer shell electrons get ground up into the nucleii of the former elements of the star, becoming a dense putty of neutrons, as uniform as creamy peanut butter. Since atoms are mostly space, neutron material is incredibly dense, a teaspoonfull of the stuff possesses the same mass as the earth's. But even in other cases, if the star is massive enough, the process of collapse continues past the neutron stage, and the object becomes a gravitational singularity.

Mass and Asymptotic Limits

[42] There is plenty of physical evidence that seems to positively confirm the existence of such phenomena: high X-ray bursts as predicted by Stephen Hawking's theories of what happens at the 'edge' of black holes (their event horizon). Hawking's theory contains quantum theory as well as general relativity. Inadvertently, such X-ray pulses strongly confirm quantum theory, as general relativity alone would render black holes as utterly unobservable (since light cannot escape them). Among other interesting items, Hawking's theories predict that black holes 'evaporate' over time, since by conservation of energy they radiate, whence they must lose mass (recall from Special Relativity: mass and energy are interchangeable)

- Chandrasekar was able to determine an absolute upper mass limit for White Dwarf formation. Unfortunately, there are no clear upper mass limits demarcating Neutron Stars from black holes, since competing effects from nuclear and relativity take place, however, Schwarzschild was able to determine the radius of compression (R_s) necessary for black hole formation. The limits are surmized in the table below: (M_S = mass of Sun, m_p = mass of proton, h= Planck's constant, c= speed of light, μ = ratio of nucleons to electrons, G= Universal gravity constant)

White Dwarfs	Cut-off Mass (Chandrasekar Upper Limit) $M_C = {}^1/_{32\mu^2 mp^2}(6h^3/\pi)^{1/2} \approx 1.3M_S$
Neutron Stars	$M_C < M$
Black Holes	$R < R_s = {}^{2GM}/_{c^2} \approx 3(M/\ M_S)$ km

What's intriguing about the Chandrasekar and the Schwarzschild limits is how they involve many fundamental constants of physics from different fields. The Schwarzschild estimation formula provides a convenient measure for the compresison radius (note that it depends on the mass of the object) For example, if the star is 2 solar masses, it must be compressed to a radius of 3(2Ms/Ms) km = 6 km

Appendix V Derivation of Heisenberg Uncertainty Relations[43]

The canonical commutation relation for operators p , x are:$[\ x\ ,\ p] = i^h/_{2\pi}$
(a result stemming from the replacement with the classical Poisson bracket relation, which indicate that momentum is the generator of translations)

For any linear operator A, B, the deviation from expectation is defined as:

$\Delta A = A - \langle A\rangle \qquad \Delta B = B - \langle B\rangle$

Calculating the product of their variances: $\sigma^2(\Delta A)\sigma^2(\Delta B) \equiv \langle(A - \Delta A)^2\rangle\,\langle(B - \Delta B)^2\rangle$

$\geq |\langle\Delta A\Delta B\rangle|^2$

(according to the Schwarz Inequalities)

However: $\Delta A\Delta B = {}^1/_2(\Delta A\Delta B - \Delta A\Delta B) + {}^1/_2(\Delta A\Delta B + \Delta A\Delta B)$

$\equiv {}^1/_2[\Delta A,\Delta B] + {}^1/_2\{\Delta A,\Delta B\}$

Certainly: $[\Delta A,\Delta B] = [A,B]$ (as their [real-valued] expectations $\langle A\rangle,\langle B\rangle$ always commute)

Therefore: $\sigma^2(\Delta A)\sigma^2(\Delta B) \geq |\langle\Delta A\Delta B\rangle|^2 \quad \geq |\langle{}^1/_2[A,B]\rangle|^2$

which is hence a general inequality relating the variance of the dispersions with any two (Hermitian) observables A, B . Inserting the case: $A = p$, $B = x$ and their canonical commutation relations, after taking the quare root of both sides one recovers the Heisenberg Uncertainty Relations

Appendix VI: Bell's Inequalities[44]

There are many ways to demonstrate how QM violates Bell's Inequalities. The easiest way is to resort to Bell's original gedankenexperiment (which he proposed in answer to the EPR papers[45]) Unlike

[43] See JJ Sakurai, sections 1.4-1.6 for further details

[44] See P Gibbins (1981) ch 4

[45] A series of papers between Bohr and Eisntein, Podowski, and Rosen, which at a certain point seem to become chicken-and-egg arguments: Bohr insisted QM was a complete theory, and locality was to be called into question. Einstein held the opposite belief. They exchanged paper after paper filled with all sorts of thought experiments proposed to shoot down the others' position…it's hard to say when the

the models proposed in the EPR papers (see footnote), Bell's experiments can be easily carried out. The simplest experiment involves the correlation of spin between two electrons (I, II.) in a *single*t state (when their combined spins are either +1, 0, -1.)

THE EXPERIMENT

Okay, let's consider making N spin measurements on the two electrons separated at an enormous distance to get a good population of data. Consider two (not necessarily perpendicular) directions (a,a') regarding the measurement of electron I's spin, and the two directions (b,b') for the measurement of electron II's spin:

Electron I   Electron II

Define the Correlations: $C(a,b)= {}^{1}/_{N} \sum a_n b_n$

$C(a',b)= {}^{1}/_{N} \sum a'_n b_n$

$C(a,b')= {}^{1}/_{N} \sum a_n b'_n$

$C(a',b')= {}^{1}/_{N} \sum a'_n b'_n$

…which measure the amount of correlation between the spins in the respective directions $a,b;\ a',b';$ etc. of the electrons I., II.

For example, suppose after N trials , I and II are perfectly **correlated**; i.e. whenever I is always spin +1 in direction a likewise is II spin +1 always in direction a'. , then: $C(a,b) =1$

If on the other hand, I and II are perfectly **anticorrelated**, (*II.) points -1* down in a' direction whenever I points in the +1 direction) then $C(a,b)=-1$

If I. and II. are wholly **uncorrelated,** $C(a,b)=0$

Consider the quantity: $g_n = a_n b_n + a_n b'_n + a'_n b_n - a'_n b'_n = a_n(b_n + b'_n) + a'_n(b_n - b'_n)$

Since b_n *or* $b'_n = +1$ *or* -1, *then:* $b_n \pm b'_n = \pm 2$

And since a_n *or* $a'_n = +1$ *or* -1, *then:* $|\, g_n| = 2$

Hence: ${}^{1}/_{N} \sum g_n = {}^{1}/_{N} \sum \{a_n b_n + a_n b'_n + a'_n b_n - a'_n b'_n\} \equiv C(a,b) + C(a,b') + C(a',b) - C(a',b')$

Therefore: $|{}^{1}/_{N} \sum g_n| = |\, C(a,b) + C(a,b') + C(a',b) - C(a',b')| \leq 2$ ***Bell's Inequality***

The above inequality relating the correlations of spins was constructed using ***elementary statistics*** Therefore we can say that *IF* classical statistics (and in turn classical logic, probability) is correct, the above inequality *MUST NOT BE VIOLATED.*

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thought experiments begin to look more like Rube Goldberg contraptions than viable physical models to test.

Okay…, on to the experiment: Suppose $a \parallel b$ and that a' 45° to the right of a and b' lies 45° to the left of a. According to the selection rules of quantum mechanics[46] ***which have been experimentally confirmed:***

$C(a,b)=-\cos\theta_{ab} = -1$ $\quad C(a,b') = C(a',b) = -\sqrt{2}/2$ $\quad C(a',b')=0$

Therefore: $|C(a,b) + C(a,b') + C(a',b) - C(a',b')| = |-1 - \sqrt{2}| = 1 + \sqrt{2}$

…which is obviously > 2! Contradiction

APPENDIX V: Entanglement of a Doublet State:

As a toy example, consider the following model of a doublet state, described by a pair of two-dimensional quantum systems A, B with states spanned by bases: $\{|-\rangle_A, |+\rangle_A\}, \{|-\rangle_B, |+\rangle_B\}$. Suppose that the initial composite system is in the (separable) state:

$$|\Psi_{AB}(0)\rangle = {}^1\!/_2\{|-\rangle_A + |+\rangle_A\} \otimes \{|-\rangle_B - |+\rangle_B\}\ ) \qquad \text{(II.3.1)}$$

$$= {}^1\!/_2\{|-\rangle_A|-\rangle_B - |-\rangle_A|+\rangle_B + |+\rangle_A|-\rangle_B - |+\rangle_A|+\rangle_B\}$$

$$\equiv {}^1\!/_2\{|--\rangle - |-+\rangle + |+-\rangle - |++\rangle\}$$ [47]

Consider the system Hamiltonian: $H = \{|--\rangle\langle--| + |-+\rangle\langle+-| + |+-\rangle\langle-+| - |++\rangle\langle++|\}$. Then the time-evolution operator $U(t, t_0 = 0) = \exp(-i2\pi Ht/h)$, after time $t = {}^h\!/_4$ becomes:

$$U({}^h\!/_4,0) = -i\{|--\rangle\langle--| + |-+\rangle\langle+-| + |+-\rangle\langle-+| - |++\rangle\langle++|\}. \qquad \text{(II.3.2)}$$

So the initially separable $|\Psi_{AB}\rangle$ now evolves into the state:

$$|\Psi_{AB}({}^h\!/_4)\rangle = U({}^h\!/_4,0)|\Psi_{AB}(0)\rangle = -{}^i\!/_2\{|--\rangle - |-+\rangle + |+-\rangle + |++\rangle\} \qquad \text{(II.3.3)}$$

It is simple to show that H is non-factorizable, and that (II.3.3) is likewise non-factorizable, and hence represents an entangled doublet state. For details, see below:

H is factorizable *iff* there exist $H_1 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, H_2 = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ such that:

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46 We summarize these formulae without deriving them

47 The last line in (II.3.1) adopts the shorthand representation for denoting the ordering of base elements in the composite system.

$$H = H_1 \otimes H_2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \otimes \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} ap & aq & bp & bq \\ ar & as & br & bs \\ cp & cq & dp & dq \\ cr & cs & dr & ds \end{pmatrix}$$

Now $H = \{ |--\rangle\langle--| + |-+\rangle\langle+-| + |+-\rangle\langle-+| - |++\rangle\langle++| \}$ can take on the following matrix representation:

$$H = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

If H were separable, then: $ap = -1$, $as = dp = ds = 1$
$c = b = 0 = q = r$
$ap = -as \Rightarrow p = -s$ (since $a \neq 0$)
$\Rightarrow dp = -dp \Rightarrow d = 0$ or $p = 0$. Contradiction.

$|\Psi_{AB}({}^{h}/_{4})\rangle = U({}^{h}/_{4},0)|\Psi_{AB}(0)\rangle = -{}^{i}/_{2}\{|--\rangle - |-+\rangle + |+-\rangle + |++\rangle\}$ can take on the following representation (as the same basis as above in the case of H):

$$\left|\Psi_{AB}\left({}^{h}\!/_{4}\right)\right\rangle = -{}^{i}\!/_{2}\begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$$

$|\Psi_{AB}({}^{h}/_{4})\rangle$ is factorizable *iff* there exist ψ_1, ψ_2 such that:

$$\left|\Psi_{AB}\left({}^{h}\!/_{4}\right)\right\rangle = \psi_1 \otimes \psi_2 = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \otimes \begin{pmatrix} \chi \\ \delta \end{pmatrix} = \begin{pmatrix} \alpha\chi \\ \alpha\delta \\ \beta\chi \\ \beta\delta \end{pmatrix}$$

$\Rightarrow \alpha\chi = \alpha\delta = 1 \Rightarrow \chi = \delta$ (since $\alpha \neq 0$)
$\Rightarrow \beta\chi = -\beta\delta = 1 \Rightarrow \beta = 0$ or $\delta = 0$. Contradiction.