Key Assingment 5a
PHIL280
Grader: W Kallfelz

a) The modified Turing Thesis (MTT) states that the existence of a Turing Machine **TM** using only a particular *fixed* amount of tape is a necessary condition for answering *all* questions from a set of questions S_Q that some algorithm A_Q answers. Consider the following counterexample: Let:

$$S_Q = \{q_n | A_Q(q_{k+1}) = f(A_Q(q_k)), \text{ where } f \text{ is computable, } \& \ (1 \leq k \leq n)\}$$

In other words, S_Q is a set of n questions, defined recursively, such that there always exists some effective procedure (modeled by a computable function f) generating the answer to the next (k+1)th question based on the answer given by the algorithm for the kth question. Since f is computable, according to Turing's Thesis, it is implementable by a TM. However, now consider, for a given n, fixing the length of the tape to l_n (For example, suppose our set S_Q consisted of 3 questions, and suppose we fix the length of the tape at: $l_3 = 10$.) Now consider asking the question: "Will the TM use a tape length exceeding l_n for *any* given S_Q consisting of n questions?" A moment's reflection indicates this question is impossible to answer, as it depends on the nature of the first question asked q_1 and the particular (computable) function f. So, though in principle the TM can model the procedure A_Q answering all the n questions in S_Q, there exists no way of knowing in general if the length of the tape required by the TM will exceed some given fixed amount or not (all we can say is that the TM will implement A_Q using a finite tape.)

b.) The converse, however, is true. (I.e., that the existence of an algorithm for answering a set of questions is a necessary condition for the existence of a TM with a finite tape for answering that set of questions.) The reason why the converse statement is true has to do with the fact that a TM is the simplest way to model (implement) an effective procedure, i.e., an algorithm.

2(a) This is just a restatement of Russel's Paradox. To precisify, a bibliography (consisting of n entries) is a just *sequence* $<b_1, b_2, \ldots, b_n>$ (i.e. an ordered list of terms b_j : $1 \leq j \leq n$).) A set can always be formed from a sequence, by neglecting the order of its terms. (For example, the set $S = \{b_1, b_2\}$ is formed from the two (different) sequences: $<b_1, b_2>$, $<b_2, b_1>$.) So for any given bibliography $B_n = <b_1, b_2, \ldots, b_n>$ not including itself as a member, (i.e. $B_n \neq b_j$: for all j: $1 \leq j \leq n$), then we can always form the set: $S_n = \{b_1, b_2, \ldots, b_n\}$ not listing itself as a member.)

(b) (Answer 1) There can be no World Bibliography of All Bibliographies. Consider the following counterexample: Suppose there exists a finite set of *all* bibliographies in the world (say, n of them.) Now consider the set of all possible bibliographies one can form from this set: This is the power-set, of the set of all subsets [subbibliographies] consisting of 2^n members. Call this power-set the 'world bibliography of *all* bibliographies' W. But W does not contain itself as a member. But W itself is a bibliography. So W doesn't contain all bibliographies.

(Answer 2) However, if one conceives of W as simply the (finite) union of all of the world's n bibliographies, then, since $W \subseteq W$ it's theoretically possible.