MA 360 **ASSIGNMENT I**

Due: February 8, 2006 (during class (preferred due date.) <u>Absolute</u> due date: no later than 5:30 pm Monday, Feb. 20th (you can leave it in my faculty mailbox in faculty lounge)

DIRECTIONS

1. In accordance with syllabus policy (page 2, section on Academic Integrity) in order to receive credit for this assignment you must adhere to the instructions set forth in this assignment, as explained in these directions. Moreover, you must attach this page to the assignment, as a cover sheet, signed and dated. If you emailed the assignment to me, you must still hand in this signed and dated document, in order to receive credit.

2. You may consult with the instructor, the notes you took in class, and any text or URL. You may also consult with your fellow classmates, if you choose, ***but*** the solutions to the problems must be written up by your own hand individually. (In other words, 'consulting' here doesn't mean copying what your fellow classmate does without understanding what you're copying.) You may not, however, consult with any other faculty member at Capitol College or elsewhere. If you consulted any references, you should cite. Please adopt the following format when citing:

(Example)

"According to Eqn 1 [*Jones* (1992), 11] …"

…where the complete reference of *Jones* is found in your list of references, stapled at the end of the assignment. The page number follows the date (in parentheses). Though standards in technical writing vary, I would prefer you list a citation in the following format:

Example for a book:

Jones, Robert. *Elementary Linear Algebra*. (Boston: Harcourt Press, 1992)

Example for an article:

Jones, Robert. "Boundary Value Equations: An Overview". *American Mathematical Monthly*, vol. 3 n2 (1998), 1227-1135.

Example for an electronic source, (website, pdf or other):

Jones, Robert.(2003 – use date it was last updated if it is a webpage) "Some Properties of Orthogonal Functions" URL = <http://www.jrworld.org/~jroberts/... >

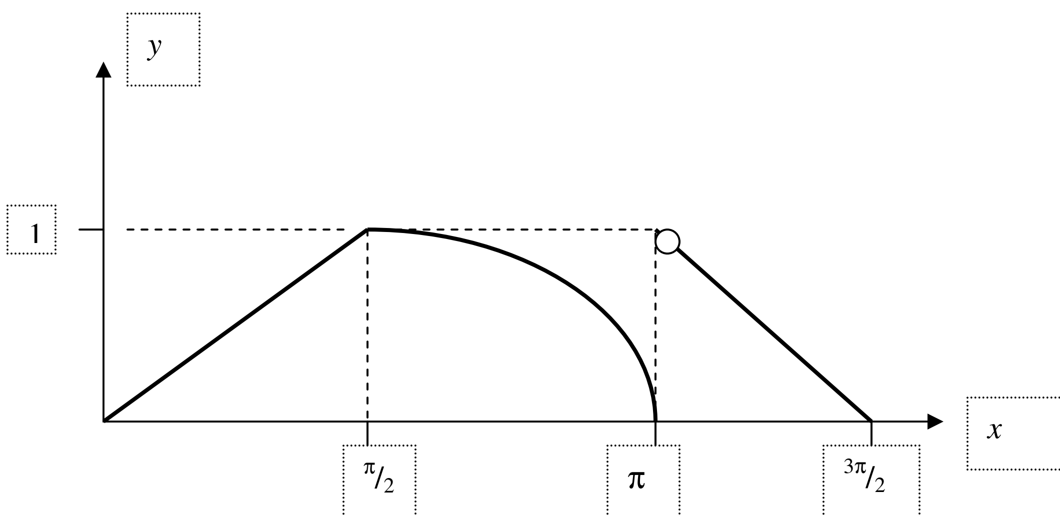
Please use discretion when citing. If you stumble across a formula in some source, which I have asked you to explicitly derive, you obviously cannot cite it. This is taken care of by the fact that you must show all your work in maximal reasonable detail. An answer with no work shown gets a 0.

I have read, understood, and have complied with the instructions set forth in this assignment

(Signature)   (date)

Directions: Please complete the following problems listed below. Start each problem on a fresh sheet of paper, in legible and neat form. Illegible work will not be graded, and marked with a 0 grade.

I. (20) Find the LT for the function:



II. Find (20) : a.) (8) $L\left\{\int_0^t \omega^2 \sin 3\omega d\omega\right\}$

b.) (12) Repeat **a**.) using the substitution: $\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$ Show that you get the same result.

III.) Given (20): $F(s) = \frac{8s-9}{(s+4)(s^2+8s+12)} = L[f(t)]$

a.) **(2)** Find $L\left[f\left(-\frac{t}{3}\right)\right]$ using Thm 3

b.) **(18)** Now find $L\left[f\left(-\frac{t}{3}\right)\right]$ by explicitly getting $f(t)=L^{-1}[F(s)]$ first. (Hint: complete the square in the denominator term) Show that you get the same result.

IV.) (25) Given: $f(t)=e^{2t}u(t-3)$

a.) (5) Find $L\{f(t)\}$ using $L\{u(t-a)\} = \frac{e^{-as}}{s}$ and Thm7

b.) (5) Find $L\{f(t)\}$ using Thm8. Show that you get the same result.

c.) (5) Find: $L^{-1}\left[\dfrac{2+e^{-\pi s}}{s^2+4}\right]$

d.) **(5)** Given: $F(s)=\frac{1}{(s-4)(s+4)^2}$. Use the Thm12 to find $L^{-1}[F(s)]$

e.) **(5)** Repeat (d) *without* using Thm 12

V (15) a.)(8) Problem 2c. Exercise 5 (Sheng, p53)

b.) (7) Problem 17. Exercise 6 (Sheng, p69) **Typo in Sheng: the irreducible term in the denominator (i.e., the (s^2 – 4s +5) term) is raised to the *first* power, not the second power.**

Bonus (20 pts total): (The Gamma Function)

You've seen the Gamma function (Handout 1c): $\Gamma(p+1)=\int_0^\infty x^p e^{-x}dx$ defined for any *real* number $p > -1$. For parts a.) and b.) now consider the following more general definition:

$\Gamma(z)=\int_0^\infty t^{(z-1)}e^{-t}dt$, where z is any *complex* number (i.e. $z = x+iy = re^{i\phi}$, where:

$r=\sqrt{x^2+y^2}, \phi = \arctan\left(\frac{y}{x}\right)$.)

a.) (3 pts) Show that: $\Gamma(z+1)=z\Gamma(z)$ (Hint: use integration by parts)

b.) (12 pts) Show that: $\int_0^{\frac{\pi}{2}} \cos^{(2x-1)}\theta \sin^{(2y-1)}\theta d\theta = \dfrac{\Gamma(x)\Gamma(y)}{2\Gamma(x+y)}$ (for *any* x, y real or complex.)

(Hint1: rewrite $\Gamma(x),\Gamma(y)$ in terms of the rationalizing substitutions[1]: $u^2=t, v^2=t$ for $\Gamma(x),\Gamma(y)$, respectively. Then combine the expressions together to form the product: $\Gamma(x)\Gamma(y)$. Hint2: From the integral you've formed from $\Gamma(x)\Gamma(y)$ now use the Polar Coordinate substitution[2]: $u = r\cos\theta, v = r\sin\theta$. The integral now becomes separable in terms of r, θ; i.e. expressible as a product of an integral solely involving r and another integral solely involving θ.

[1] Recall the move in the first several steps of the proof of Property2 (Handout 1c, page 2)

[2] See the subsequent steps, for example, in the proof of Property 2 (towards bottom of page2, Handout 1c)

Hint3: Look closely at the first integral (the one expressed in terms of *r*). Show that it expresses: $\Gamma(x+y)$.)

c.) (5 pts total) i.) (3 pts) Find: $L\left\{5e^{-4t}\left(\sqrt[3]{t}\right)\right\}$ (your final answer should include the expression: $\Gamma\left(\frac{1}{3}\right)$, but don't try to simplify further (i.e. don't try to derive $\Gamma\left(\frac{1}{3}\right)$.)

ii.) (2 pts) Find: $L^{-1}\left[\frac{1}{\sqrt{s+4}}\right]$ (Here, your final answer should include the term: $\sqrt{\pi}$)