Inter-Theoretic Reduction II: Nickles' Response to Nagel

Recall (<u>Lecture XXV</u>):

- Nagel argues that ***T* = =>*T'* is** (a form of) **explanation:** Nagel's "reduction *is* explanation"[1] captures a rather powerful (but not wholly unquestioned[2]) intuition shared by many across the board, even skeptics concerning the epistemic status of laws like Nancy Cartwright.[3] Moreover, appealing obliquely to the "sciences as sentences" picture of the logical empiricists[4] as we'll see Nagel's "**correspondence theory of intertheoretic reduction (explanation)"** (CTIR) shares much in common with the "deductive chauvinism" Kitcher defends as constitutive of scientific explanation,[5] and which of course Hempel drew out schema for to illustrate how one can conceive of such in a generally deductive framework.[6] Note the ***T*** is the **reduced theory** while ***T'*** is the **reducing theory**. Reflecting Hempel's terminology, ***T'*** functions as the *explanans* while ***T*** the *explanandum.* Be careful here! So for example, when we say "Galileo's Theory of falling bodies is reduced to Newton's theory of universal gravitation," *T* is the former (Galileo's theory) while *T'* is the latter. *T* ' is the *explanans*, since *T'* *does the explaining*. Conversely *T* is the explanandum, as (Nagel aims to show) *T* is *derived from T'*.[7]

- Similar in spirit to Hempel's DN versus IS schema, Nagel describes two kinds of reduction: *homogeneous* (907-910) versus *inhomogeneous* reductions (910-915) which I'll abbreviate by: ***T* = => *T'*** and ***T* ≅ ≅ > *T'*,** respectively. As in Hempel's IS, Nagel considers the inhomogeneous case as most problematic. **Homogeneos reduction (*T* = => *T'*)** occurs when **the essential terms in *T* are also present in**

[1] Recall <u>**Lecture II**</u>, the word "is" connotes material implication (i.e., "implies": →). **So Nagel is not implying the converse!** (That explanation *is* reduction). This is signaled in the above passage by thee phrase "other sorts of scientific explanation".
[2] Recall <u>**Lecture XX**</u>: Morrison doesn't buy it.
[3] Recall (p. 2, <u>**Lecture XXIV)**</u>: "Cartwright pushes the **methodological** notion that **explanation *is* reduction.** She states this explicitly, as a matter of fact: '[W]e explain complex phenomena by **reducing them to their simpler components.** This is not the only kind of explanation we give, but it is an important and central kind.' (869)"
[4] "[S]trictly speaking, it is not phenomena which are deduced from other phenomena, but rather ***statements*** about phenomena from other statements. This is obvious if we remind ourselves that a given phenomenon can be subsumed under a variety of distinct descriptions…phenomena make no assertions or claims." (**CC1998**, 907). For a summary of the "sentence view" see footnote 1 in <u>**Lecture XVIIIb (Lecture 18 Supplement).**</u>
[5] Recall <u>**Lectures XIX, XX**</u>**.**
[6] Recall <u>**Lecture XVIII**</u>. The I-S model is of course still a 'deductive' schema insofar as being couched in premise-conclusion form (laying aside the difficulties of characterizing notions like inductive inference, i.e. notions pertaining to 'degrees of support' *p*.)
[7] Unfortunately Paul Feyerabend reverses the prime convention: rendering *T'* as the reduced theory and *T* as the reducing theory. This can obviously cause confusion when reading Feyerabend's passage Nagles cites in pp. 916-917. I'll stick with Nagel's notation. See also footnote 7 in **Commentary** (p. 1045)

T'. **Inhomogenous reduction ($T \cong \cong > T'$)** is the opposite, and becomes **logically problematic**[8]**, as essential terms occurring in *T* may connote and denote, (possess *intensions or extensions*[9]) not found in *T'*.**

- Nagel's 'solution' to the $T \cong \cong > T'$ case is to stipulate (semantic) *bridge laws/principles:*

[S]uch bridge laws are **empirical hypotheses concerning the *extensions* of the predicates mentioned in these correspondence rules—that is, concerning the class of individual things or processes designated by these predicates. …**[or they] **assert that certain logically non-equivalent expressions describe identical entities.** (914-915)

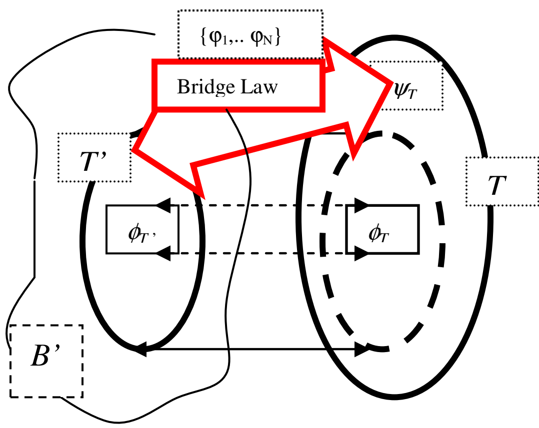

Fig. 1

[8] Recall, as discussed in class (November 30) and as shown in cases (b) and (c) (p. 4 **Lecture XXV**)why this presents a **logical problem**:

On the one hand, if essential terms occur in *T'* (the explanans) which don't occur in *T* this could be dealt with by the valid rule of inference: **If** $\{\phi_1, \phi_2,\dots, \phi_n\} \vdash \psi$, **then** $\{\phi_1, \phi_2,\dots, \phi_n, \phi_{n+1}\} \vdash \psi$, i.e. adding new terms/sentences to the explanans (the premises) preserves the truth-value of the deduction. (Recall **Lecture II** : This is a feature of the **monotonicity of standard FOPL.** Consider the following toy-demonstration: Suppose $p \rightarrow q$ is true. But $p \rightarrow q \equiv \neg p \vee q$, hence $\neg p \vee q$ is true. Then: $\neg p \vee q \vdash \neg p \vee \neg r \vee q$, i.e. **adding a new disjunct preserves the truth-value of an already true disjunction, since only one of the disjuncts must be true for the entire disjunction to be true.** However: $\neg r \vee \neg p \vee q \equiv \neg (r \wedge p) \vee q$, by DeMorgan rule. Moreover: $\neg (r \wedge p) \vee q \equiv (r \wedge p) \rightarrow q$. ***Therefore:*** $p \rightarrow q \vdash (p \wedge r) \rightarrow q$.) So one could think of the core argument $\{\phi_1, \phi_2,\dots, \phi_n\} \vdash \psi$ as corresponding with the homogenous sub-case, i.e. when **some** of the essential terms of *T'* correlate with those in *T*. (For example, both the Newtonian and the Keplerian theory possess essential terms like T, the orbital period of planets, as well as the average distance of the planets from the Sun: $\bar{r} = \frac{1}{2}(a+b)$, where *a* is the semiminor axis of the ellipse and *b* is the semimajor axis). Then the truth-value is preserved in the full reduction $\{\phi_1, \phi_2,\dots, \phi_n, \phi_{n+1}\} \vdash \psi$, in which the explanans (premises) contain essential terms in *T'* not found in *T*. (For example, the Newtonian theory contains *m*, i.e. *mass* (of planets and of course the Sun) which is not found in the Keplerian theory *T*.)

On the other hand, however, there is no valid rule of inference corresponding to the case when essential terms occur in *T* which don't occur in *T'*. On a basic level, we recognize this (yet again!) as the case of the Problem of Induction: one *cannot* deduce a sentence that contains information which isn't found in at least one of the premises! There is no rule of inference in FOPL that states: "If $\{\phi_1, \phi_2,\dots, \phi_n\} \vdash \psi$ then $\{\phi_1, \phi_2,\dots, \phi_n\} \vdash \psi \wedge \phi$." This is invalid since one cannot deduce a stronger conclusion from a previously true argument. For example, from: "*If* I oversleep, *then* I'm late for work," I *cannot* derive: "*If* I oversleep, *then* I'm late for work and I died in my sleep."

[9] Recall **Lecture XXIII.**

As specified in the above *Fig. 1*, a bridge law relates an essential term ψ_T in T (with no corresponding semantic correlate in the domain T ') to the extension of other term(s) in *T'*. Bridge laws are 'empirical' insofar as being understood as set of sentences $\{\varphi_1,..\ \varphi_N\}$ referring to empirical protocols occurring in the 'background' *B'* . "[They] can be understood without reference to the ideas involved in the theories to which the laws have been reduced," (912) i.e. $\{\varphi_1,..\ \varphi_N\}$ are supposedly characterized in *T'*-independent manner.[10]

[10] Recall this notion of (supposedly theory-independent) background notions *B* arising in concert to a particular theory or theory-complex. If you worked on topic 1 of the first term paper, this is where you first encountered such a notion. We also visited the notion in the section on Confirmation Theory **Lectures XVI, XVII** and in the Quine-Duhem problem of underdetermination **Lecture XXI.**) **Note that it is *not* assumed that such sentences are independent of *any* theory whatsoever! (To think otherwise would be to hanker after some 'theroy-neutral, purely observational language' that early logical empiricists like Carnap sought after and later agreed, with the rest of the philosophical community past and present, that such an aspiration was futile.)** Note carefully the qualification in the quote above: "the theories to which they have been reduced." As Nagel goes on to say, in criticizing Feyerabend's 'replacement' view:

> Feyerabend has difficulties in providing a firm observational basis for objectively evaluating the empirical worth of proposed hypotheses [which]…stem from what I believe is his exaggerated view that the meaning of every term occurring in a theory or in its observation statements is wholly and uniquely determined by that theory, so that its meaning is radically changed when the theory is modified. For theories are not quite the monolithic structures he takes them to be—their component assumptions are, in general, logically independent of one another, and their terms have varying degrees of dependence on the theories into which they enter…**although both 'theoretical' and 'observational' terms may be 'theory laden,' it does not follow that there can be no term in a theory which retains its meaning when it is transplanted into some other theory.** (919)

In essence, Nagel is accusing Feyerabend of committing an 'all-or-nothing' fallacy here. Just because "the content of such [observational] terms is not exhausted by observational procedures" (Commentary, **CC1998,** 1019) it doesn't follow that meanings of terms are *entirely* theory-dependent, "in the monolithic sense". Recall for instance Ian Hacking's use Hilary Putnam's theory of meaning (**Lecture XI**) to deflate the issue of incommensurability. Recall also (**Lecture XII**) in Musgrave's realist attack on Fine, he also brushes aside what he perceives as a similar all-or-nothing fallacy committed by anti-realist like some of the idealists of the early British Empiricist tradition as well as some of seemingly solipsist notions in Wittgenstein's writings. "We are not trapped inside language in the *serious* sense that all we ever talk *about* is language. To think otherwise is to ignore the hard-won distinction between use and mention." (**CC1998**, 1217). To which Musgrave further adds (n. 2, 1223-1224):

> Wittgenstein ignored this [use/mention] distinction in his *Tractatus*, and was led to the following 'deep' thoughts…the limits of my language are the limits of my world…Wittgenstein's 'logocentric predicament' is simply old psychologistic wine poured into new linguistic bottles. The British empiricists thought that thinking consists in having a stream of 'ideas', and concluded mistakenly that all we ever think *about* are our own ideas.

To put it (yet again!) in another way, Nickles reminds us that if one takes seriously Feyerabend's claims then not only does Nagel's *reductio ad absurdum* hold (that any way of assessing the empirical worth of a theory evaporates) but in addition one *cannot even claim that incommensurable theories are logically incompatible*! (955, n. 12, 968) This is so because for two statements to be logically *incompatible* (i.e. if included in the same set, a contradiction can be derived from that set) there must be shared meaning of the terms in that sentence. Even in the most primitive case, for example, I know p and $\neg p$ are incompatible (even if I know nothing about the semantic content of p) because I have a (syntactic) notion of the meaning

- Nagel in addition distinguishes two fundamentally different kinds of situations concerning ***T* ≅ ≅ > *T'*** and the associated bridge laws which accompany such situations. One the hand, "bridge laws…may specify conditions for the occurrence of an attribute [in *T*] which **may only be necessary**…**laws involving the attribute will, in general will not be deducible from the proposed reducing theory [*T* '].**" (914) In this case the bridge laws connect the extension of the problematic term/predicate ψ_T such that it's *included* in the extension(s) of terms in the reducing theory *T'*.[11] On the other hand, the conditions may be both necessary and sufficient, in which case such bridge laws connect the extension of ψ_T in such a manner that it *coincides with* the extension of a term in the reducing theory *T* '.[12] In this case the extensions are logically *identified*. For example (illustrative of the first case) the extension of 'viscuous liquid' (a term occurring in the macro theory *T* not occurring in the reducing microtheory *T'*) is mapped *into* the extensions of 'electrostatic intermolecular (i.e. frictional) forces' in the micro-theory *T'*. An example of the latter case is the *identification* of the extension of 'light wave' (a term occurring in classical optical theory *T*) with the extension of 'electromagnetic propagation' (of a specific wavelength or frequency band) in the reducing *EM* (electromagnetic) theory:

  > Correspondence rules of the second kind thus differ from rules of the first…[because] they assert that certain logically non-equivalent expressions describe identical entities. Although both sorts of rules have a common function in reduction and both are in general empirical assumptions, failure to distinguish them is perhaps one reason for the persistence of the mistaken belief that reductive explanations establish the 'unreality' of those distinctive traits mentioned in the reduced laws. (915)

- Thomas Nickles in his (1975) article "Two Concepts of Intertheoretic Reduction" latches on to the distinctions of Nagel (both ***T* = = > *T'*** and ***T* ≅ ≅ > *T'***, as well as the sudistinctions among bridge laws mentioned above) and basically argues that they're "differences which doesn't make a difference" as well as not covering all the cases that naturally occur:

  > I do not believe that there is a general formula for reductive relationships any more than there is an interesting common structure to all theories…we must examine each case on its individual merits.[13] (964)

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of negation (¬) which is independent of the semantic content of *p*. More 'sophisticated' examples include *p*: "It is raining now," ¬*p*: "It is not raining now". I recognize these statements to be logically incompatible because of their shared meanings "now" "raining" (their semantic content) as well as the aforementioned syntactic notion ¬.

[11] Stated more formally, the bridge law *maps* the extension *into* extension(s) of term(s) belonging to *T'*. I.e, the bridge law is a *function* (in the mathematical sense).

[12] Stated more formally (as in n. 11 above) this kind of function is a *bijection* or *isomorphism* (i.e. it's both one-to-one and the extension *onto* an extension of a term belonging to *T'*.)

[13] As we'll see when examining Kitcher (Dec. 6th readings), this deflationary claim is even further extended. Kitcher thinks there's no good reason to assume there are *any* global schemes to reduce one theory to another, however there exist 'locally' systematic means (hence it's not just something arbitrary or ad-hoc.)

> [T]he fact that some paradigm examples of reduction do not fit Nagel's conditions, establishes that Nagel's conditions on reduction are not *necessary* for (all types of) intertheoretic reduction. Two additional counterexamples will establish the *insufficiency* of Nagel's conditions. (957)

- Of course, this is *not* to say that Nickles believes that Nagel's CTIR[14] is without merit. Certainly *some* (indeed a relatively significant amount) cases can be characterized as such, once they're suitably idealized.[15] In a nutshell, however, where Nagel (according to Nickles) essentially ascribed *one* form of intertheoretic reduction, Nickles ascribes (at least) *two*:

> [W]e need to recognize that at least two main kinds of reduction, which differ both in nature and in scientific function or purpose. **'Reduction$_1$' (as I shall call it) is the achievement of postulational and ontological economy and *is* obtained chiefly by derivational reduction as described by Nagel**…amount[ing] to the *explanation* of one theory by another**…'Reduction$_2$' [on the other hand]…involves a varied collection of intertheoretic relations rather than a single, distinctive logical or mathematical relation**…The great importance of reduction$_2$ lies in its **heuristic and justificatory roles in science**. (950)

To abbreviate, I'll denote Nickles' schema of 'reduction$_2$' by: ***T'≈≈> T***.

- To illustrate a case of the second 'heuristic and justificatory' kind of reduction, Nickles' offers the example of ***T' ≈≈> T*** where *T* is (classical) Newtonian dynamics (CND) and *T'* is special-relativistic dynamics (SRD).[16] As he writes "it

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[14] Recall **<u>Lecture XXV</u>**: my abbreviation of Nagel's 'correspondence theory of intertheoretic reduction'.

[15] This is one of Nickles' main bones of contention against Feyerabend and other of Nagel's critics:

> I think it is important to distinguish those logical incompatibilities [between *T* ' and *T*] which are historical accidents…from those based on irreconcilable conceptual differences. Except where alternative formulations of essentially the same theory are being investigated, and perhaps even there**, it would be good fortune indeed if, without any adjustment of details, one theory should be logically derivable from another theory arrived at by a different line of research.** (955)

> Nagel's critics tend to forget that **philosophical models are deliberately idealized somewhat and not intended to give a completely accurate representation of intellectual history**….I would [even] go so far as to say that Nagel's derviational reduction is a useful concept even if not one single historical example perfectly exemplifies the pattern he describes…**Philosophical models are tools that must be applied with care and with attention to the special features of the individual case.** (956)

[16] Nickles writes 'SR' for 'special relativity' but in his example, this abbreviation isn't specific enough. For one speaks of the *kinematics* of SR (i.e. the study of general behavior of space, time, spatiotemporal trajectories, i.e. *descriptions of possible classes of motion of a system*) versus the *dynamics* of SR (the study of the *causes* of actual dynamical behavior of some system.) Recall the distinction of kinematics versus dynamics as discussed in class (November 30) as well as in footnote 8, p. 3 **<u>Lecture XVII.</u>** In any dynamical theory, usually non-spatiotemporal (i.e. unextended in space and time) quantities or 'physical dimensions' are included, like momentum, force, energy, etc. Kinematic theories usually (but not always) contain purely geometric (in the strict spatiotemporal sense) quantities. (Exceptions include more advance

is more natural to say that the *more* general theory [SRD] reduces to the *less* general [CND] in the limit at low velocities." (951) This is so because the relativistic formula for momentum[17] is: $\vec{p} = m\vec{v}$ where m is the (special) relativistic mass of an object, defined according to formula: $m = \gamma(v)m_0$, where: $\gamma(v) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is the so-called 'gamma factor' which can be thought of describing how much classical quantities (like distance, time, and in this case mass) 'deform' as their speeds approach the speed of light c. While m_0 is the so-called 'rest mass' (i.e. the mass of the particle in a reference frame in which the particle is locally at rest.) [18] Now in the limit $v << c$, then $\gamma(v) \rightarrow 1$[19] hence the SRD formula converges to the CND formula, since the relativistic mass becomes

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kinematic analyses which introduce 'generalized coordinates' in a so-called phase space, not to be interpreted as actual physical space.)

[17] Recall **Lecture XXIV** : the arrow superscript refers to a three-dimensional vector in ordinary 3D geometry. For the record, however, SRD is conducted in 4D Minkowski geoemetry, where the notion of '4-vector' is applied. (The convention is to denote components of 4-vectors by Greek letter subscripts or superscripts, referring to 'covariant' and 'contravariant' indices respectively, while in the ordinary 3-vecotr case to denote such components with Latin letter indices). For purposes of the demonstration, Nickles (justifiably) ignores such geometric subtleties here.

[18] Observe a few facts here concerning the above formula for relativistic momentum: a.) The 'deformation' (or discrepancy between rest mass and relativistic mass) only becomes significant for speeds appreciably close to c. For example, even if the particle were traveling at 0.5c (i.e. at half the speed of light or at $1.5 \times 10^8\ {}^{m}/_{sec}$) its gamma factor would be (only) $\gamma(0.5c) = (1 - .5^2)^{-1/2} = 1.154$, i.e. the mass discrepancy would have only increased by a factor of ~15%). b.) Any object with nonzero rest mass would require infinite energy to reach the speed of light. This is so because special relativistic kinetic energy (KE = $^1/_2mv^2$) is of course directly proportional to (special relativistic) mass. However, in the limit $v \rightarrow c$, $\gamma \rightarrow \infty$, since inserting c into the formula for the gamma factor produces a zero in the denominator, making the quantity therefore infinite!) Of course nothing like this happens *at all* in CND: i.e. masses don't change when accelerated close to c (much less become infinite) and, moreover, and most fundamentally, in CND (as well as in classical kinematics of course) *fundamental quantities like space, time, mass do* <u>*not*</u> *depend on the motion of a particle at all.* In SR, of course, they do. This constitutes a central rationale for Feyerabend's 'replacement theory' as well as others towing the 'incommensurability' line: they're basically saying (among other things) that the meanings of quantities like space, time, mass depend on the theory in which they're characterized (whether Newtonian, special, or general relativity, or quantum theory, etc.) *and*, moreover, *they mean completely different things* (depending on the theory). For Feyerabend (and others) this is true *even as a matter of logical syntax*. For in the class of CND, for example, mass is a 1-place predicate: $m(a)$ mean (in logical terms) 'the mass of a.' Whereas in SRD, mass becomes a *two-place predicate*: we must say in this case $m(a,v)$, i.e.: 'the mass of a with speed v.'

[19] Heuristically, if you assume $v << c$, then v is negligible (i.e more or less 0). Hence $\gamma(0) = (1 - 0^2)^{-1/2} = 1$. More rigorously, one can demonstrate convergence to 1 by *expanding* $\gamma(v)$ according to its *Taylor Series Representation*: $\gamma(v) = [1 - (^v/_c)^2]^{-1/2} = 1 - {}^1/_2(^v/_c)^2 + O((^v/_c)^{2n})$, where $O((^v/_c)^{2n})$ are higher-order even powered terms of $(^v/_c)$ that are all 'big-oh', i.e. $\lim_{v/c\rightarrow 0} O(v^{2n}) = 0$. In the case where $v << c$, call $\varepsilon = (^v/_c)$. Then certainly $\varepsilon \approx 1$, and in this case $\gamma(v) = [1 - (^v/_c)^2]^{-1/2} \approx 1 - {}^1/_2\varepsilon^2 \approx 1$. As we'll see in the Batterman article (Dec. 6th reading), all this fussing with the niceties of series expansions becomes philosophically interesting!

the rest-mass in this case, i.e. the (constant) mass of the particle measured locally in its frame of reference. [20]

- The above case (as well as others Nickles mentions) *T' ≈≈> T* "is not only intuitively natural; it is the way physicists and mathematicians, in contrast to most philosophers, usually talk." (951) Philosophers, on the other hand, usually think of reduction "chiefly…involv[ing] an increase in the over-all efficiency of a conceptual scheme." (ibid.) His critique of Nagel is based on his overall complaint that:

> With few exceptions philosophers have attempted to force all scientific reductions into the same mold, the mold varying somewhat from philosopher to philosopher. This uniformity of treatment is somewhat surirising in view of the fact that reduction takes place at different levels, each with its special problems, and that reduction fulfills a variety of scientific and philosophical functions. (n.2, 967)

As a matter of fact this theme of diversity and particularity is greatly elaborated on as we'll see in the Dec. 6th readings of Kitcher & Batterman.

- Nickles borrows his terms from a previous publication (referred to in n. 3, 967) of "domain combining" to refer to Nagels' sense of derivational reduction (both in the *T = = > T'* and in the *T ≅ ≅ > T '* cases). This is so because one theory (the reduced) *is explained* via the other (the reducing), in the form of a logical derivation or logical consequence. So for instance, hearkening back to the example discussed in pp. 2-3, **Lecture XXV**, the domain of Galilean theory of terrestial mechanics is combined with Newton's, insofar as "[t]he reduction of one *science* to another…[is] a domain-combining reduction$_1$." (953) More specifically, as illustrated in the notes of the previous lecture, a *law* for falling object in Galileo's theory is logically subsumed by Newton's *law* of universal gravitation. By contrast, the *T' ≈≈> T* case refers to a *domain preserving* reduction. In other words, *T* is certainly not *explained by T'* (in the above example, Newtonian momentum certainly doesn't 'explain' Relativistic momentum!), though one could (in a loose sense) say that such cases "might be said to explain why the predecessor theory worked as well as it did…" (n. 4, 967). Nevertheless, the hallmark of such cases is heuristic and justificatory since:

> The development of new theoretical ideas is **heuristically guided** by the requirement that these ideas yield certain results **as a special case**…The **justification** derives from the fact that the reduction shows the successor theory to account adequately for the structured domain of phenomena inherited from its successful predecessor, i.e. it is 'domain-preserving.' (953) [in the sense of "domain-preserving only when the predecessor was successful." (n.5, 967)]

[20] Obviously, as discussed in n.18 above, the particle rest mass *is* the CND mass in a trivial sense, since in this theory, mass stays constant, whether the particle moves or not, with respect to whatever frame of reference.

It’s important to note that actual cases of reduction can actually involve both of the kinds as described above, as discussed in detail in §IV (960-962) characterizable by the following schema:

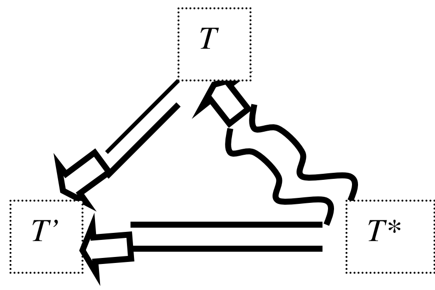

Fig. 2

…where the solid lines refer to domain-combining reduction and the wavy lines refer to domain-preserving. *T** is a ‘corrected’ version of the predecessor theory: a notion he borrows from Ken Schaffer which loosely speaking can be thought of as redacted version of the predecessor theory which “’provid[es] more accurate experimentally verifiable predictions…indicat[ing] why [*T*] was incorrect (crucial variable ignored).’ (144)” (961) Conversely one can think of *T** as getting rid of the need for bridge laws, i.e. if $T \cong\cong > T'$ then $T^* == > T'$.[21] The important point regardless is to recognize that the justificatory and heuristic sense of reduction for Nickles indicates that “[n]ot all reduction is explanation!” (953)

- As mentioned above, Nickles finds favor in Nagel’s schema, as a suitably idealized philosophical model. Nickles also shows how the two cases (homogenous versus inhomogeneous) Nagel was concerned about (in particular the second) can be subsumed under one schema by considering $T \cong\cong > T'$ as essentially a $T \,\&\, H == > T'$, where *H* are the set of bridge laws “belonging to the meta-theory.”[22] (958) Moreover, Nickles also admits that the $T' \approx\approx > T$ case *can* be viewed as “derivational” (in a “broader sense”) as “[m]athematical derivation…includes not only logical deduction but limit processes and approximations of many kinds.” (956) Nevertheless, however:

  [I]n the standard analysis of explanation, to which Nagel subscribes, a deductive explanation is a deductive argument in the strict sense. So it is pretty clear that **Nagel’s**

[21] For the sake of consistency I use the *T* , *T*’ notation as I have been doing throughout to designate the respective predecessor and successor theories, as opposed to the “1,2” subscript notation that Nickles borrows from Schaffer here.

[22] Nagel may recoil from such an association, since it’s unclear that ‘background’ empirical rules/protocols are necessarily metathoretical (with respect to a given theory). The latter notion is stronger. Nickles qualifies this distinction however by stating that “[t]he theory/metatheory distinction here is context-dependent and is not intended to demarcate essentially different modes of scientific inquiry…*H* will [or can] consist of empirically supported empirical statements.” (n. 22, 969)

original analysis makes intertheoretic reduction derivational in the narrow sense, not the broad sense. (957)

Also, as mentioned above, the fact that *T* ‘≈≈> *T* cases can and do exist reveals that Nagels derivational reduction can’t be a *necessary condition* for intertheoretic reduction.[23] Nickels also draws some counterexamples from Lawrence Sklar’s paper to indicate that Nagels domain combining strictly derivational reduction cannot be a *sufficient condition* for intertheoretic reduction either.[24] One counterexample for instance consists of statistical mechanics and kinetic theory.[25] Statistical mechanics incorporates laws of Newtonian mechanics to describe the microstates of particles, along with statistical assumptions.[26] But then according to Nagels’ schema, it would follow that classical Newtonian mechanics reduces to statistical mechanics!

> This result is certainly counterintuitive, since [Newtomian] mechanics always has been regarded as the more basic or universal theory, partly for the very reason that it is basic to kinetic theory [and statistical mechanics] and to other theories...[Nagel’s] correlatory statements are in one sense too weak and in another sense too strong. (957)

- In the case of Nagel’s second kind of bridge law (the isomorphism alluded to in page 4 in these notes), i.e. the *identification* (of extensions) Nagel makes a special point here to illustrate a *non-uniqueness* counterclaim. In the case of quantum theory viz. classical Newtonian dynamics, though it would seem “far more natural” to correlate the quantum-theoretic notion of position and momentum with their classical counterparts, “it proves far more illuminating [instead] to correlate classical values with quantum expectation values.[27]” (960) The main point being (yet again!):

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[23] Recall **<u>Lecture II</u>**: ***q*** is a necessary condition for *p* whenever *p* cannot hold if *q* doesn’t. In other words, if *p*, then ***q***.

[24] Recall **<u>Lecture II</u>**: ***p*** is a sufficient condition for *q* whenever *p* holds then so can *q*. In other words, if ***p***, then *q*.

[25] The theories which seek to, from a microphysical account involving the individual positions and momenta of molecules or atoms (suitably idealized), i.e. their *microstates,* derive *macrostates* described by thermodynamic quantities like pressure, temperature, density, entropy, etc. of the gas such constituent particles compose. A famous result, derivable from the kinetic theory of ideal gases, includes
$<E> = {}^{3}/_{2}NkT$, where <*E*> is the average (kinetic) energy of the ensemble of atoms (assumed point like and electrically neutral) and N is the number of particles (also can be written as nN_0 where n is the mole ratio and N_0 is Avogadro’s number) and k is the Bolzmann constant, and T the temperature of the ideal gas.

[26] The *Equipartition Theorem* (EP) being one such assumption, not free from controversy. The EP states that the energy microstates for a given energy macrostate for a gas in thermal equilibrium (maximum entropy) are uniformly distributed, i.e. each microstate contains a ‘degree of freedom’ (a statistically independent measure of energy) of ${}^{1}/_{2}kT$.

[27] A rather contrived statistical notion: An expectation value is a statistical expectation defined over an ensemble, i.e. the set of all statistical likelihoods. An analogy is the “statistician’s concept of ‘average man,’ in which “I am more like a particular 18th century man than like the statistician’s ‘average man.’” (n.22, 969)

[I]n…contexts in which justification and heuristics are the main concerns, we deliberately forswear attempting the ontological and conceptual identification which is so essential to other forms of reduction. (960)

- Nickles elaborates more on his notion of *T* '≈≈> *T* by presenting a scheme of reduction $O_n \ldots O_1 T'$ ≈≈> *T* where $O_1, \ldots, O_n$ represent a list of *n* intertheoretic operations "which are reductive under certain conditions.[28]"(963) In addition, the operations $O_n \ldots O_1$ on *T'* need not cover all of the aspects of *T* ' let alone *T*…" [*T'*] and [*T*] will stand for theory-parts. Nevertheless partial reduction of this sort remains an indispensable conceptual tool in theoretical research." (963) He then consider a toy example involving the following two 'theory' parts:

  (A): $w = ax + 2y + g$ (B): $z = bx + ey + d$

  where *x, y* are (independent) physical variables while *w,z* are (dependent) physical variables and *a*, *b*, *d*, *e*, *g* are constants. Suppose (B) is the 'new' theory while (A) is the 'old' theory, in which $g = 0$. Futhermore, suppose that $a = b$, and that $0 < d << 1$.[29] "Is it surprising to say (B) reduces$_2$ to (A) in the limit as $d \rightarrow 0$? …[W]hy is it not equally legitimate to say that (A) reduces$_2$ to (B) in the limit as $g \rightarrow d$?" (964) His answer is that "letting numerical constants change value is mathematically illegitimate." (ibid) Nickles uses a *reductio ad absurdum* here to argue why he thinks so: One could, *mutates mutandis* end up asking what characterizes (A) in the "2 $\rightarrow$ 0" limit. Not only does this appear mathematically absurd, it would reduce (A) to an equation *qualitiatively different* from (A) and (B), i.e. to an equation involving *one* variable alone.[30]

  However, physicists *do* talk in such terms! (They allow constants like the speed of light $c \rightarrow \infty$ or Plank's constant $h \rightarrow 0$ when looking for heuristic justificatory reductions of SRD to CND, or quantum theory to classical theory, respectively, for example)[31]. What are we to make of this? We should think of cases like this that "*h* and *c* act as variables in a noncommital metalanguage." (966) Or (as Nickles paraphrases) "Strauss points out that when constants are taken to limits, they are being regarded as variables in the metatheory." (n. 37, 970) The fact that this occurs on a case-by-case basis in physics indicates that "there is [no] general formula for reductive relationships any more than there is an interesting common structure to all theories:…we must examine each case on its individual merits." (964)

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[28] Nickles presents two, but of course his point can be generalized to finitely many (*n* in this case). Note also that $O_n \ldots O_1 T'$ is shorthand for nesting, and should be read in parenthetical order from right to left ("first do O_1, then do $O_2, \ldots$") i.e.: $O_n(\ldots (O_1 (T'))\ldots)$

[29] All of these moves can be considered cases of the operations *O*i as discussed above.

[30] Robert Batterman (Dec. 6[th] readings) looks at this issue more closely.

[31] In fact this limit process is so commonplace that mathematical phycisists usually refer to such 'constants' as 'parameters' which could be thought of as a type of variable.