Inter-Theoretic Reduction

- As is apparent to any student in science and to any practicing scientist, science seems to possess at some basic or general level a kind of *methodological unity.* Recall Duhem's likening a of scientific theory-complex to "an organism." (**Lecture XXI** .) Regardless how some philosophers may skeptically question such a notion of "unity" (e.g., Cartwright[1]) or argue that unifying activity comes in different flavors and shouldn't be thought of as explanatory (e.g., Morrison[2]), the fact remains that ***no one*** *would disagree with the (weak) claim* that *there exists at least* ***some*** *degree of interconnection at least among* ***some*** *collection of* ***theories*** {*T*} *within* ***some*** *theory-complex* (Duhem), *paradigm or disciplinary matrix* (Kuhn), *research programme* (Lakatos), *etc*. Indeed, this above mentioned weak claim concerning interconnection among theories is the primary fodder for the arguments of most scientific realists,[3] as well as of course those like Kitcher who argue that to scientifically explain *is* to unify.[4]

- Ernest Nagel (1974) was the first to sketch a detailed theory concerning characterizing a stronger form of interconnection among theories in a theor-complex, as alluded to in the above weak claim, *namely, the case of reduction: what does it mean when we say a theory T (or group of interrelated theories* {*T*}) ***reduces to*** *some theory T'* ? (I'll abbreviate this with: ***T* = =>*T'*** or **{*T*} = => *T* '** representing respectively single and the plural cases, i.e. when *a* theory reduces to another theory, or when *a group of theories* does.[5]

- Nagel argues that ***T* = =>*T'* is** (a form of) **explanation:**

  [There are] certain relations of dependence between one set of distinctive traits of a given subject matter [that] **are allegedly explained by** … **"reduced" to** assumptions concerning more inclusive relations…**They [i.e., such relations of dependence] raise**

[1] Recall Cartwright (**Lecture XXIV**): The *laws L* a that a physical theory *T* generate actually apply to very few factual cases in nature, if they apply at all. The very generality of *L* is an *explanatory* virtue, which comes at a cost of *representational* virtue. In her later writings, *The Dappled World* (1999) Cartwright expands on this notion to argue (as the world "dappled" suggests) that the theoretical picture offered by physics and other sciences like biology is very fragmented indeed. Any intuitions of 'unity' we have come methodologies of 'bottom up,' 'top down,' and 'cross-wise' reduction. As in the case of her (1980) article (**Lecture XXIV**) Cartwright deflates that such activities of reduction to the status (as in the case of component forces) of possessing heuristic or 'metaphorical' value at the expense of being truth-preserving.

[2] Recall (**Lecture XX**)

[3] Who, as you may recall from previous lectures concerning realism, would wield such a notion of interconnection as *an inference to the best explanation* to the (weak realist) claim that science (in the long run) "gets it right," i.e., that [T]truth constitutes one of science's necessary aims and virtues. (Recall, for instance, **Lecture IX.**)

[4] Recall **Lectures XVIII, XIX.**

[5] Note the **non-uniqueness** claim presupposed in both cases: *one* theory *T from a group of many* {*T* }may reduce to *T'*. Hence the {*T*} = => *T'* case can be viewed as just a stronger re-statement of the *T* = => *T'*.

> **the question of what, in fact, is the logical structure of such reductive explanations—whether they differ from other sorts of scientific explanation, what is achieved by reductions, and under what conditions are they feasible.** (**CC1998**, 906)
>
> Nagel's "reduction *is* explanation"[6] captures a rather powerful (but not wholly unquestioned[7]) intuition shared by many across the board, even skeptics concerning the epistemic status of laws like Nancy Cartwright.[8] Moreover, appealing obliquely to the "sciences as sentences" picture of the logical empiricists[9] as we'll see Nagel's "**correspondence theory of intertheoretic reduction (explanation)"** (CTIR) shares much in common with the "deductive chauvinism" Kitcher defends as constitutive of scientific explanation,[10] and which of course Hempel drew out schema for to illustrate how one can conceive of such in a generally deductive framework.[11]

- Similar in spirit to Hempel's DN versus IS schema, Nagel describes two kinds of reduction: *homogeneous* (907-910) versus *inhomogeneous* reductions (910-915) which I'll abbreviate by: ***T* = => *T'*** and ***T* ≅ ≅ > *T'*,** respectively. As in Hempel's IS, Nagel considers the inhomogeneous case as most problematic. Prior to summarizing both cases, some terminology is in order. Note the ***T*** is the **reduced theory** while ***T'*** is the **reducing theory**. Reflecting Hempel's terminology, ***T'*** functions as the *explanans* while ***T*** the *explanandum.* Be careful here! So for example, when we say "Galileo's Theory of falling bodies is reduced to Newton's theory of universal gravtation," *T* is the former (Galileo's theory) while *T'* is the latter. *T* ' is the *explanans*, since *T' does the explaining.* Conversely *T* is the explanandum, as (Nagel aims to show) *T* is *derived from T'*.[12]

- Homogeneos reduction occurs when the essential terms in *T* are also present in *T'*. For example, in the case of Galileo's theory of falling bodies T_{Gal} (where he argues that they accelerate at a uniform rate) generates the law that can be expressed in

---

[6] Recall **Lecture II**, the word "is" connotes material implication (i.e., "implies": →). **So Nagel is not implying the converse!** (That explanation *is* reduction). This is signaled in the above passage by thee phrase "other sorts of scientific explanation".

[7] Recall **Lecture XX**: Morrison doesn't buy it.

[8] Recall (p. 2, **Lecture XXIV**): "Cartwright pushes the **methodological** notion that **explanation *is* reduction.** She states this explicitly, as a matter of fact: '[W]e explain complex phenomena by **reducing them to their simpler components.** This is not the only kind of explanation we give, but it is an important and central kind.' (869)"

[9] "[S]trictly speaking, it is not phenomena which are deduced from other phenomena, but rather ***statements*** about phenomena from other statements. This is obvious if we remind ourselves that a given phenomenon can be subsumed under a variety of distinct descriptions…phenomena make no assertions or claims." (**CC1998**, 907). For a summary of the "sentence view" see footnote 1 in **Lecture XVIIIb (Lecture 18 Supplement).**

[10] Recall **Lectures XIX, XX.**

[11] Recall **Lecture XVIII**. The I-S model is of course still a 'deductive' schema insofar as being couched in premise-conclusion form (laying aside the difficulties of characterizing notions like inductive inference, i.e. notions pertaining to 'degrees of support' *p*.)

[12] Unfortunately Paul Feyerabend reverses the prime convention: rendering *T'* as the reduced theory and *T* as the reducing theory. This can obviously cause confusion when reading Feyerabend's passage Nagles cites in pp. 916-917. I'll stick with Nagel's notation. See also footnote 7 in **Commentary** (p. 1045)

mathematical form as: $F_{\text{Gal}} = mg$.[13] When we say that $T_{\text{Gal}} = = > T_{\text{Newt}}$ we mean that the law of universal gravitation generated by the Newtonian theory: $F_{Newt} = G\frac{M_E m}{r^2}$ "covers" (recalling Hempel's DN schema) the $F_{\text{Gal}} = mg$ law.[14] To the objection that F_{Gal} is actually logically incompatible with F_{Newt}, and hence the former *cannot be logically derived* from the latter,[15] since they say different things (the Galilean laws that an object's weight is *constant*, while Newton's law says it *varies* with respect to $1/r^2$) Nagel responds to this objection by appealing to the *necessarily approximating conditions* involved in $T_{\text{Gal}} = = > T_{\text{Newt}}$:

> [T]he initial hypotheses may be reasonably close approximations to the consequences entailed by the more comprehensive theory…**it is correct to say that in homogeneous reductions the reduced laws are either derivable from the explanatory premises, or are good approximations to the laws derivable from the latter….[Moreover] in actual scientific practice, the derivation of laws from theories usually involves simplifications and approximations of various kinds…** (909)

So for instance in the above example, we may write: $r = R_E + h$, where R_E is the Earth's radius (6.34×10^6 m) and h is the height of the body's center of mass measured from the Earth's surface. Certainly in scenarios Galileo was describing, bodies were close to the Earth's surface (h of the order of magnitude of 10^1- 10^2 m) i.e. $h << R_E$ hence we may say: $r = R_E + h \approx R_E$, hence: $F_{Newt} = G\frac{M_E m}{r^2} \approx G\frac{M_E}{R_E^2}m$. Hence the reduction is complete, the Earth's surface gravity constant g from Galileo is reduced to a multiplicative combination of constants involving the universal gravity constant (G), the Earth's mass, and the Earth's radius.[16] Similar approximating schemes hold in the case of reducing Galileo's law of constant period for pendular motion to the Newtonian torque law.[17] Though mathematical adjustments need to be made, no incompatibilities in logic arise.

---

[13] In present-day textbook terms, of course, where F_{Gal} is the magnitude of the object's weight force, and g is the Earth's *surface gravity* (= 9.8 $^{m}/_{sec^2}$ in the MKS system or = 32 $^{ft}/_{sec^2}$ in the British system). Galileo of course did not use such terms like 'mass' but instead used a precursor to the notion, known as 'impetus.'

[14] Where G is the universal gravitational coupling constant (= 6.67×10^{-11} Nm^2/kg^2 in the MKS system) and M_{E} is the mass of the Earth (=5.9×10^{24} kg) and r is the distance measured between the body's (of mass m) center of mass and the Earth's center of mass. The Earth's center of mass is located (more or less) at the Earth's geometric center. (Note that geometric centers coincide with centers of mass only in extremely simple cases, when the object exhibits all sorts of symmetries, like sphere and cylinders do, for instance, *and* when the object is uniform, i.e. has a constant density.)

[15] Recall **Lecture II.** Two statements φ ψ are logically incompatible when combined in a set will generate a contradiction from that set, i.e. $\{\varphi, \psi\} \vdash \perp$. Colloquially phrased, when φ ψ *contradict each other. Assuming* φ *to be true in that case,* then we certainly cannot say: $\varphi \vdash \psi$, i.e. we cannot (in a truth preserving way) derive one from another.

[16] I.e. $g = GM_{\text{E}}(R_{\text{E}}^{-2})$. As a bonus of course, the formula is more general insofar as it *predicts* the surface gravity constants for some other planet(s) (once one knows the mass and radius of such other planet(s)).

[17] This is mathematically more complicated. One starts with the torque equation $\tau = I\alpha$ and assumes: a) the mass of the rod is negligible, b) the mass of the bob is located at the end of the rod, c) for small initial angles of oscillation the torque equation becomes a homogeneous linear second order differential equation

- The inhomogenous case is logically problematic, as essential terms occurring in *T* may connote and denote, i.e. possess *intensions or extensions*[18]different or disjunct from those found in *T'*. For instance, consider cases like Kepler's third law reducing to Newton's law of universal gravitation. Even after the mathematical approximations are made effecting such a reduction, Newton's formula involves the masses of the planet and the sun, while Kepler's formula doesn't. 'Mass' does not occur anywhere in Kepler's theory. Consider another example, in the reverse scenario: in the case of geometric optics reducing to Maxwell's EM theory, the former contains terms like 'light waves/light rays' while the latter doesn't. (911) Or in the most contentious or problematic case, *if* one stipulates that cognitive psychology is reducible in principle to neuroscience, the brute fact remains that *intentional mental states* are essential terms occurring in the former, but not in the latter.

  (a) Now in the first case (Kepler ≅ ≅> Newton) is not so problematic, insofar as it's an accepted rule of inference in deductive logic that adding sentences to the premise set is truth-preserving. This is what makes standard FOPL monotonic.[19]

  (b) In the second case (Optics ≅ ≅> EM) one could argue that though the *intension* 'light wave/light ray' is different from the 'intension' of the predicate 'EM wave', *they share the same extension under suitable restrictions* (namely we call light the EM bandwidth from 140-170 nm).

  (c) The third case however is the harder problem, since no easy case can be made for arguing that under suitable restrictions, the *extensions* of intentional mental state are included in the extensions of neural states.

- In all three cases above (a), (b), (c) Nagel argues that ***T* ≅ ≅ > *T'*** can only come about via the use of (semantic) *bridge laws/principles:*

  [S]uch bridge laws [in cases (a) and (b)]are **empirical hypotheses concerning the *extensions* of the predicates mentioned in these correspondence rules—that is, concerning the class of individual things or processes designated by these predicates. …**Correspondence rules of the second kind **[**attempted for instance in (c)]**..assert that certain logically non-equivalent expressions describe identical entities.** (914-915)

- Nagel defends his CTIR against Feyerabend's criticisms (the latter based on notions of incommensurability and theory-dependence and meaning holism[20]). Aside from arguing (*reductio ad absurdum*) that taking Feyerabend's ideas

---

with constant coefficients. The second coefficient is a formula expressible in terms of the angular frequency of the oscillations, hence the period of oscillations is derived as a constant.

[18] Recall **Lecture XXIII.**

[19] I.e., if $\{\phi_1, \phi_2,\ldots, \phi_n\} \vdash \psi$ then $\{\phi_1, \phi_2,\ldots, \phi_n, \phi_{n+1}\} \vdash \psi$. Laying the objection of question-begging aside, one could argue that the Newtonian theory's extra sentence(s) involve ϕ_{n+1} still produce the truth of the schema.

[20] We explored some of these issues in **Lectures XXI, XXII.**

seriously would entail that *we would have no basis for assessing the empirical worth of a theory at all* (916) Nagel also charges that Feyerabend conflates the issue of differences in intension with extension. (I.e. 'Venus' and 'evening star/morning star' all have the same extensions, though differ in intension, i.e. connotation.) (919-920)