Carl Hempel: "Inductive-Statistical Explanations"

- Recall (from "Two Basic Types of Scientific Explanation"):
  1. The Deductive-Nomological Model (D-N) and the Inductive-Statistical[1] (I-S) models are two fundamentally different kinds of models of scientific explanation.
  2. These models (which Hempel assumed were *exhaustive*, i.e. exhausted all the fundamental kinds of scientific explanation[2]) are *explications*; i.e. serve as

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[1] Hempel also discusses a special case of statistical explanations which are *deductive* (D-S). However, recall from today's lecture that what distinguishes D-S from I-S models is the contingent issue concerning whether or not all the premises comprising the *explanans* happen to be true (for the realist) (**CC1998**, 716) or empirically adequate/instrumentally reliable, etc. (one might add, for the anti-realist.) The issue of truth being a contingent one primarily because theories of *explanation* for Hempel and others are primarily based on **logical** and **epistemological** issues (recall **Lecture I**). While those like Hempel who argue from the logical empiricist tradition (which sharply divides contexts of discovery/justification as well viewing scientific theories and bodies of knowledge primarily in terms of *sentences characterized by a particular logical syntax*) want to remain metaphysically agnostic concerning issues of truth. Stated another way, in terms of the *structure* of a scientific explanation, notions like "truth" and similar predicates (anti-realist analogues) are *semantic*, and hence *contingent* to the *syntactic* approach to scientific theories and knowledge. **By *semantics* I am referring to the strictly *logical* sense in which predicates like 'truth' are characterized in terms of an *interpretation*, which is itself a *mapping from* the formal language Λ (conceived of as a 'domain') *to something outside* of Λ, i.e. the 'range'. For example, in the ordinary binary interpretation of 'truth' such an interpretation is characterized as a mapping $\Pi: \Lambda \rightarrow \{ \bot, T \}$** (or equivalently **{ 0 , 1}**) where the 'range' is a two-valued set denoting the 'falsum,' and 'verum' conceived as a binary "filter." *It's important however to keep in mind that the syntactic view focuses just on the formal language Λ itself,* whether regimented by FOPL or otherwise, **since Λ is conceived of purely as a *syntactic* structure**. In the case of FOPL its 'sentences' φ (or well-formed formulae (*wffs*)) are generated by primitive constants (*a,b,c…*), logical *variables* (*x,y,z,…*), *quantifiers* (∀, ∃), logical *connectives* (→,∧,∨,¬,↔) as well as by *deductive consequence* ⊢ , where ⊢ itself is *syntactically* characterized by *rules of inference*. Recall from **Lecture II**: **footnote 13, p. 6:** FOPL has 14 rules of inference: 6 **introduction rules** and 6 **elimination rules** (pertaining to →,∧,∨,¬, ∀, ∃) as well as EFSQ and DN where EFSQ means that *any* proposition can follow from a premise list containing a **contradiction** and the Double Negation rule means that for any sentence φ: ¬¬φ ⊢ φ.

On the other hand, those in the (later) *semantic* tradition (like van Fraassen) speak in terms of *models M of a formal language* Λ characterizing scientific theories and knowledge, as opposed to just *sentences in a formal language* Λ. (Recall **Lecture X**) **Models are *interpreted* sets of sentences (in the above technical sense).** That is to say, *M* is a set of sentences in Λ such that they hold *true* according to Λ's axioms. The notion of *semantic* consequence is *entailment* (⊨) which is defined as "holding true in all models *M*", i.e. for some set of sentences Σ and some other sentence φ, Σ ⊨ φ *if* all the models *M* in which all the sentences in Σ hold true, then φ holds true in all those same models *M*.

Later philosophers of science like Michael Friedman (1987) questioned whether there *is* such a significant difference between these two research traditions (i.e. the Hempel-Carnap-…*syntactic/sentential* approach versus the van Fraassen-Sneed-…*semantic/model* approach.) For, as you see in the above remarks in this footnote, certainly every model *M* can be 'translated' into some sets of sentences and vice versa! Granted, though this may be true from a *methodological* standpoint, a *logical asymmetry* certainly exists between ⊢ and ⊨, i.e. between *deductive consequence* and *semantic entailment*. Hence, the issue separating semantic from syntactic approaches in philosophy of science may hinge on a *methodological symmetry* but a *logical asymmetry,* which of course is exactly the opposite sense of Popper's notion of falsification ☺.

rational reconstructions of the phenomenon of scientific explanation. Hempel likens his models to those adopted by proof-theorists in *metamathematics*:

> "[O]ur models might be compared to the concept of mathematical proof (within a given theory) as construed in meta-mathematics…**exhibit[ing] the rationale of mathematical proofs by revealing the logical connections underlying the successive steps…provid[ing] standards for a critical appraisal of any proposed proof constructed in the mathematical system to which the model refers…afford[ing] a basis for a precise and far-reaching theory of proof, provability, decidability, and related concepts. I think the two models of explanation can fulfill the same functions, if only on a much more modest scale."** (691)
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> In other words, *explications* serve dual roles of normative as well descriptive devices.

- **Ambiguities associated with the I-S model**

Consider three scenarios discussed in *"Inductive-Statistical Explanation"* (**1d, 2a, 2b**) (708-709)

S : Property of having a streptococcal infection.
S^* : Property of having a streptococcal infection which is penicillin-resistant.
S^{**} : Property of having a streptococcal infection and suffering from a weak heart and being and octogenarian.
R : Property of recovering from streptococcal infection.
P : Property of taking penicillin.

Then three different I-S models can be constructed, *all with a high degree of inductive support* [3]:

1d) $P(R \mid S \cap P) \approx 1$
$Sj \wedge Pj$
========= $[p \approx 1]$
Rj

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[2] The band plays on: Recent work in the theory of explanation viz. philosophy of physics is Robert Batterman's (2002) book: *The Devil In The Details*, in which he argues that *another fundamental kind of scientific explanation exists*: the *asymptotic explanation* which he claims occurs frequently in applied physics (especially when two or more theories are enlisted to explain some phenomenon: like the semi-classical methods in physics which involve a hodge-podge of aspects from classical and quantum theories.) I have critically responded to his arguments in my dissertation, as well as in some of the papers I have posted on home page, available also in the Philosophy of Science archives.

[3] For the sake of uniformity, I am using the same notation of probability I introduced in **Lectures XVI, XVII** which is basically the standard contemporary notation. Also, recall (**Lecture II**) that notations like *"Pa"* in FOPL stand for *singular* statements (where *a* is a logical constant, i.e. a name, and *P* is some property). So, for instance, *Sj* means "John (*j*) has property *S*, i.e. "Jones has streptococcal infection." Last of all recall (**Lecture XVIII**) that *p* is the measure of inductive support, *which need not be the same as the probability used in one of the statistical laws of the explanans.* *p* could be some complicated function of the conditional probabilities. By " ≈ 1" this means "close to 1," i.e. with virtual certainty.

2a) $P(\sim R \mid S^* \cap P) \approx 1$
$S^*j \wedge Pj$
========= [$p \approx 1$]
$\sim Rj$

2b) $P(\sim R \mid S^{**} \cap P) \approx 1$
$S^{**}j \wedge Pj$
========= [$p \approx 1$]
$\sim Rj$

…which amounts to two 'rival arguments' (**2a), 2b)**) to **1d)** having all the all same logical *form* but deriving a contradictory conclusion ~*R* with virtual certainty as in the case of deriving *R* with virtual certainty.

- **Note 1:** To anyone not ascribing to the sentential/syntactic approach (see footnote 1 above) this concern may appear almost silly. "*Of course*," one might want to argue, "someone can draw opposing conclusions based on subtle *semantic* modifications of the predicate *S*."[4] However, to anyone ascribing to the syntax/sentential view, the issue hangs on *consistency* and *logical form*. One way to think of Hempel's worry here was that his efforts to characterize forms of I-S explanations in terms of logical form would render them robust and general enough to free them from the 'tyranny' of context-dependence. On the other hand, someone in the *semantic/model* tradition may welcome such context dependence, provided such contextual variations of meaning can be adequately characterized in terms of appropriate *interpretations* (which, as clarified in footnote 1. above, are treated as *mappings* from the language Λ to extra-logical/ extra-linguistic factors.

"The peculiar logical phenomenon here illustrated…[i.e.] the *ambiguity…of statistical explanation…*derives from the fact that a given individual event (e.g., Jones's illness) will often be obtainable by random selection from any one of several 'reference classes' (such as [$S \cap P$, $S^* \cap P$, $S^{**} \cap P$] with respect to the kind of occurrence (e.g., *R*) instantiated by the event has very different statistical probabilities. Hence, for a proposed probabilistic explanation with true explanans which confers near certainty upon a particular event, there will often exist a rival argument in the same probabilistic form with equally true premises which confers near certainty upon the nonoccurrence of the same event…***This predicament has no analogue in the case of deductive explanation;* for if the premises of a proposed explanation are true then so is its conclusion; and its contradictory, being false, cannot be a logical consequence of a rival set of premises that are equally true.**" (709)

Another example Hempel offers along these lines is:

W : Property of being a warm and sunny day.
N : Property of being November at Stanford.
n : Nov 27th .
S : Property of being a successor to a cold and rainy day at Stanford.

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[4] A not-so-subtle modification could involve, for instance, $S^{\wedge}$ =having a streptococcal infection derived from an *extraterrestial* source (some hardy strain attached to a meteorite) which is resistant to *all* known antibiotic agents manufactured so far.

2c) $P(W \mid N) = 0.95$
Nn
========= $[p = 0.95]$
Wn

2d) $P({\sim}W \mid S) = 0.8$
Sn
========= $[p = 0.8]$
${\sim}Wn$

The contradictories **2c)** and **2d)** "concern[] I-S arguments whose premises are in fact true, no matter whether we are aware of this or not…refer[ing] to what is presumed to be known in science rather than to what, perhaps unknown to anyone, is in fact the case." (710)

- Hempel states this point more generally by denoting K_t as the "knowledge at time t by scientists" which "**is *not* meant to convey that the elements of K_t are true, and hence neither that they are definitely *known to be true*.**"[5] (710, italics added) In addition, Hempel assumes **K_t to be closed under logical implication[6] and is consistent**.[7]

- Regardless, however, of the above stipulations of logical consistency and closure under implication the problem remains that "[t]he total set K of accepted statements contains different subsets of statements which can be used as premises in arguments of the probabilistic form just considered, and which confer high probabilities to logically contradictory 'conclusions.' " (710)

One strategy (suggested by Carnap) is to specify explicitly the *total evidence* in the premises (*explanans*) in the hopes of *uniquely* singling out the appropriate argument viz. the total evidence. For example, there are different sets of total evidence in K in which S, S^*, S^{**} *uniquely* applies to. Conversely, **"[u]sing only a part of the total evidence is permissible if the balance of evidence is irrelevant to the inductive 'conclusion,' i.e., if on the partial evidence alone, the conclusion has the same confirmation, or logical probability, as on the total evidence.**" (711) This requirement was not considered by Carnap to be a theorem, rather "a necessary condition of rationality of any such application in a given 'knowledge situation,'" (711-712) In other words, this is a methodological maxim functioning in many way analogously to Kuhn's notions of values, which (according to Kuhn) are rational, but not logically regimented.

"Indeed, one would want an acceptable explanation to be based on a statistical probability statement pertaining to the narrowest reference class[8] of which, according to our total information, the particular occurrence under consideration is a member." (712)

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[5] Obviously, a relaxation of the classical epistemological definition of 'knowledge' as "justified true belief." Consider also Edmund Gettier's famous three-page paper, which provided counter-examples to "justified," "true," and "belief" interpreted as sufficiency as well as necessary conditions for "knowledge."

[6] I.e., if $\phi \in K_t$ and $\varphi \in K_t$, then $(\phi \rightarrow \varphi) \in K_t$ (where: $\in$ means "is an element of" or "belongs to"), and where (recall **Lecture II**) $(\phi \rightarrow \varphi)$ means "If ϕ, then φ."

[7] I.e. if $K_t \vdash \sigma$, i.e. if σ is a deductive consequence of K_t then $\neg(K_t \vdash \neg\sigma)$, i.e. $\neg\sigma$ (σ's contradictory, or logical negation of σ) can't be a logical consequence of K_t.

[8] The subset of K to which some predicate in the statistical law (comprising part of the explanans) refers to. For example, the reference class of $S^* = \{$all strepoccocal bacteria $\mid$ that are penicillin-resistant.$\}$

For example, in the case of radioactive decay, the conditional probability $P(G|F_1)$ (where G is the outcome: 2.4mg < $m(t)$ < 2.6mg, where $m(t)$ the residual mass of a sample of radon Rn after t = 7.64 days and where F_1 represents the scenario m_0 = 10mg in which m_0 is the initial mass) would remain unaltered if its conditions were adjoined: $P(G|F_1 \cap F_2 \cap F_3 \cap \ldots)$ where $\{F_2, F_3, \ldots\}$ represented ambient conditions like humidity, temperature, etc. This is because the theory of radioactive decay involves the weak force (the mechanism of beta-decay: when a neutron decays into a proton, antiproton, electron, and neutrino) which doesn't depend on such ambient factors. Hence $P(G|F_1 \cap F_2 \cap F_3 \cap \ldots) = P(G|F_1)$ or the probabilities are the same for the narrower class $F_1 \cap F_2 \cap F_3 \cap \ldots$ as for the original reference class F_1 . (713)

However, consider the statistically improbable but not impossible reference class F^* such that probability $P(G^*|F^*)$ (where G^* is the outcome: $m(t)$ = 2.7mg, where $m(t)$ the residual mass of a sample of radon Rn after t = 7.60 days and where F represents the scenario m_0 = 10mg in which m_0 is the initial mass). Then in particular $P(G| F_1 \cap F^*)$ is exceedingly lower, since within a rather short time (1 hour) the amount of radioactive Rn will have to plunge from 2.7mg to fall in the (2.4mg, 2.6mg) range.[9] In other words, $P(G| F_1 \cap F^*) << P(G| F_1)$ **"[T]he additional information here may not be disregarded…an explanation of the observed outcome will be acceptable only if it takes account of the probability in the narrower reference class** [$F_1 \cap F^*$ = {*Rn* samples | Initially at 10mg and after 7.6 days at 2.7 mg and after 7.64 days, between 2.4 and 2.6 mg}]" (713)

- This is an example of what Hempel refers to as **the requirement of maximum specificity** (which, he argues, as in the above case concerning decay of Rn, disambiguates what at the outset appears to evince the following ambiguity[10]:

$P(G\| F_1) = r_1 \approx 1$	$P({\sim}G\| F_1 \cap F^*) = r_2 \approx 1$
$F_1 b$	$F_1 b \& F^* b$
========= $[p \approx 1]$	========= $[p \approx 1]$
Gb	${\sim}Gb$

"The requirement of maximal specificity…is here tentatively put forward as characterizing the extent to which the requirement of total evidence properly applies to inductive-statistical explanations…*the concept of statistical explanation for particular events is essentially relative to a given knowledge situation as represesnted by a class K of accepted statements*…[w]e will refer to this characteristic as the *epistemic relativity of statistical explanation*." (714-715)

> "[T]he epistemic relativity that the requirement of maximal specificity implies for I-S explanations…has no analogue for D-N explanations…[i]t…concerns what may be called the concept of a *potential* statistical explanation. **For it stipulates that no matter how much evidential support there may be for the explanans, a proposed I-S explanation is acceptable if its potential explanatory force with respect to the…explanandum is vitiated by statistical laws which are included in *K* but not in the explanans, and which permit the production of rival statistical arguments."** (716)

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[9] The simple law of radioactive decay is a decaying exponential: $m(t) = m_0 e^{-kt}$. Of course, in the case of quantum theory, the decaying exponential curve represents a trend of maximum likelihood. Hence the statistically improbable (but not impossible) scenario of sudden mass loss in the range of 0.1mg to 0.3mg in one hour.

[10] where b is some specific element in the reference class, i.e. in instance of such a sample of Rn, with initial mass of 10mg)