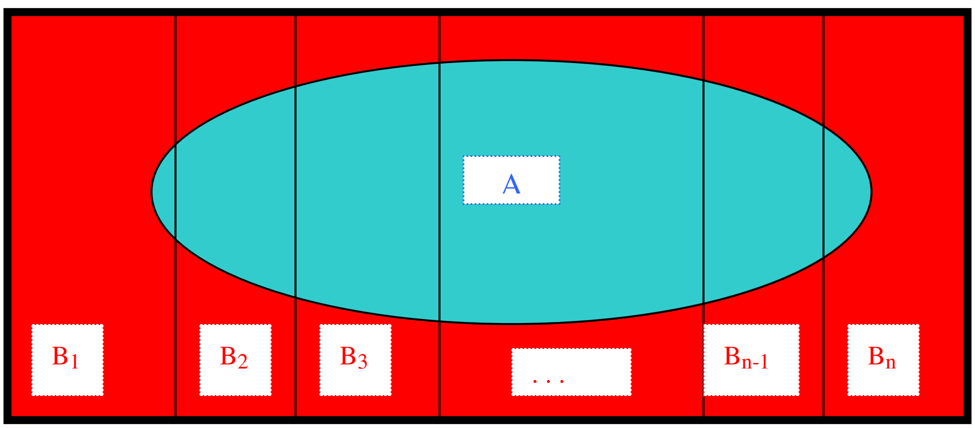
- **Recall (Lecture XV)**

Bayes' Formula:

For (disjoint) events $B_1, B_2, B_3, \ldots, B_n$, where: $B_1 \cup B_2 \cup B_3 \cup \ldots \cup B_n = S$

$$P(B_k \mid A) = \frac{P(B_k)P(A|B_k)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + \ \ldots \ + P(B_n)P(A|B_n)} \qquad \text{for } 1 \leq k \leq n$$



…which is obtainable through the simple transformations:

$$P(B_k \mid A) = \frac{P(A \cap B_k)}{P(A)} = \frac{P(A \cap B_k)}{P(A \cap B_1) + P(A \cap B_2) + \ldots + P(A \cap B_n)}$$

$$= \frac{P(A \mid B_k)P(B_k)}{P(A \mid B_1)P(B_1) + P(A \mid B_2)P(B_3) + \ldots + P(A \mid B_n)P(B_n)}$$

Wesley Salmon: "Rationality in Science or Tom Bayes Meets Tom Kuhn"

- **From Kuhn's article: "Objectivity & Value-Choice" Salmon correctly (or at least charitably) interprets Kuhn's defense of his view that theory-choice, though a *rational* process, cannot be fully logically regimented.**

***Recall Kuhn's Five Criteria*: Simplicity, Fecundity, Accuracy, Consistency (both external and internal), Scope.**

"What he [Kuhn] had intended to convey was the claim that the decision by the community of trained scientists *constitutes* the best criterion of objectivity and rationality we can have." - **CC1998, 552**

(Recall Longino) " [T]he interactions among individual members of the community of scientists produces produce a consensus for the group for the group…**the outcome of a group can be considered objective and fully rational…**Kuhn's major claim seems to be that observation and experiment, along with hypothetico-deductive reasoning, do not adequately account for the choice in scientific theories…Kuhn…has tried to locate the additional factors involved." (553)

- Could those 'additional factors' be regimented according to Bayes' Theorem, interpreted in terms of *theory confirmation* in the light of new evidence?

Consider (from the formula in page 1 above) the following association:

1. B_k is some hypothesis/theory[1] T_k for a collection of n mutually exclusive theories which exhaust all possibilities. The latter stronger condition is guaranteed via the formation of the *catch-all* hypothesis/theory T_n which states: "*none* of the previous $(n - 1)$ theories are true."[2] This can be phrased:

   $$T_n = \sim(T_1 \cup T_2 \cup \ldots \cup T_{n-1}) = (\sim T_1 \cap \sim T_2 \cap \ldots \cap \sim T_{n-1})$$

   ..by way of **DeMorgan** Rule.[3]

2. A on the **left hand side** is the *conjuction* of 'background' assumptions B with new evidence E, i.e.: $P(T_k \mid B \cap E)$.[4] This represents a **posterior probability.** Then Bayes' Thm can be rederived according posterior/prior probabilities via the following simple transformations (recall **Lecture XV**):

$$P(T \mid E \cap B) = \frac{P(T \cap E \cap B)}{P(E \cap B)} = \frac{P(E \mid B \cap T)P(B \cap T)}{P(E \mid B)P(B)} = \frac{P(E \mid B \cap T)P(T \mid B)}{P(E \mid B)}$$

…which is Salmon's formula (3) version of Bayes Thm (the index of the theory was suppressed.)

[1] Salmon uses these notions interchangeably, see **n8, p. 580**

[2] For those with realist propensities. Anti-realists may substitute notions like 'empirically adequate,' 'instrumentally reliable,' etc.

[3] Recall **Lecture II.**

[4] Note a subtlety for those who chose paper topic #1. Hawthorne, in addition to B and E tosses in (T-independent) condition C of **how the evidence *E* was obtained** (i.e. notions like experimental design [recall Hacking].) One can assume that Salmon absorbs Hawthorne's C into B. Moreover, Salmon's worry concerning the status of prior probabilities during periods of **scientific revolutions [CC1998, 567]** is taken care of in Hawthorne's stipulation that Inductive Logic applies to cases of normal science only.

3. One achieves the full-blown Bayesian formula (Salmon's (4)) via partitioning the denominator term into the explicit alternatives (including the catch-all)[5]:

$$P(E|B) = P(E|T_1 \cap B)P(T_1|B) + P(E|T_2 \cap B)P(T_2|B) + \ldots + P(E|T_n \cap B)P(T_n|B)$$

..each one of these terms on the RHS is obtained via a similar transformation as in 2. above:

For any given T_i (where $1 \leq i \leq n$):

$$P(E \mid B) = \frac{P(E \cap B)}{P(B)} \xrightarrow{T_i} \frac{P(E \cap T_i \cap B)}{P(B)} = \frac{P(E \mid T_i \cap B)P(T_i \cap B)}{P(B)} = P(E \mid T_i \cap B)P(T_i \mid B)$$

4. Based on the transformations (1.-3.) above we have the following terminology:

- $P(T_i|B)$ are the **prior probabilities**, as one assumes the correctness/truth/reliability of T_i on the basis of background notions alone. (554)
- $P(E|T_i \cap B)$ are the **likelihoods,** which yield probability that the evidence would occur or hold given that our background assumptions B and the truth/reliability/correctness (or lack thereof) of T_i . "[L]ikelihoods …in contrast to prior probabilities, must be established independently; the value of one does not automatically determine the value of the other." (554)
- $P(E|B)$ is the **expectedness** or evidential value. "It was made plausible by assuming that the subject was completely naïve concerning the relevant physical theory." (565). **This assumption, however, is problematic.** Note that the transformations in 3. above indicate that "we really can't avoid dealing with likelihoods [and prior probabilities]." (ibid.)

Based on the remarks/transformations/qualifications made in 1.-4. above notes Salmon that:

A.) We need at least three items of information to establish posterior probability $P(T_k \mid E \cap B)$ **a' la Bayes Formula: prior probabilities, likelihoods, expectedness.**

Note1: As we'll see later when reviwing Quine/Duhem issue of theoretical undertermination, as Salmon himself remark (n. 27, 582) concerning the issue of likelihoods if our expectations aren't met, we have an ambiguity. It could very well be that T_i holds but "we may be led to reexamin our background knowledge B." (For those who wrote on topic #1) recall Hawthorne's (partial) gesture toward this issue in his more nuanced: $B\&C\&T_i$

[5] Note: I am using n as the total number of alternatives, whereas Salmon uses k. I use k for the index of each theory, whereas Salmon uses i.

B.) **(Recall Lecture XV). In the case of *empirical* interpretations of probability, Salmon holds forth on the *frequentist* (or *frequency ratio*) interpretation, as opposed to some who speak of a *propensity* interpretation (i.e. that objective propensities exist). This is mainly due to the reversal of conditionalization of probabilities (between prior and posterior in the RHS and LHS of the Bayes formula respectively). Since Salmon is charitable towards Bayesian approaches, such a reversal of conditionals renders the propensity interpretation implausible.** "In the example of the can opener factory, [though] each machine [may] ha[ve] a propensity to produce defective can openers, it does not make sense to speak of the propensity of a given defective can opener to have been produced by a new machine." (n16., 581)

C.) **Thus (similar to Hawthorne) and many in inductive logic, Salmon ascribes to a 'tempere personalist' approach[6] when it comes to the case of probabilities that cannot be assigned a direct empirical (frequentist) interpretation (whether prior[7], likelihoods, expectedness) i.e., *a measure of belief* (constrained according to axioms of probability).**

"Bayes' theorem provides a mechanical algorithm, but the judgments of individual scientists are involved in procuring the values that are to be fed into it. This is the general feature of algorithms; they are not responsible for the data they are given." (558)

- Example: Oesterd's *naturphilosophie* versus the 'Cartesian' metaphysical presuppositions of the inverse-square law.[8]

Can cognitive values can be reconciled with tempered personalism?

"The most reasonable way to look at simplicity [as well as Kuhn's other criteria] is to regard it as a highly relevant characteristic, but one whose applicability varies from one scientific context to another…scientists must use their judgment…[t]he kind of judgment to which I refer is not spooky …[but] bas[ed] on training and experience. " (563)

- **Problem with Catchall Hypothesis/denominator term in general**

Note that "[t]o try to evaluate the likelihood of the catchall involves, it seems to me, an attempt to guess at the future history of science. That is something we [obviously] cannot do with any reliability." (569)

[6] A notion coined by philosopher and physicist Abner Shimony. "The frightening thing about pure unadulterated personalism is that nothing prevents prior probabilities (and other probabilities as well) from being determined by all sorts of idiosyncratic and objectively irrelevant considerations." (559-560) **Salmon reveals as well his a certain suspicion (as opposed to let's say Longino's and Anderson's qualified appreciation) for contextual values here.** "Much more troubling…is the fact that any given scientist may be inadvertently influenced by ideological or metaphysical prejudices." (560)

[7] Though in the case of prior, he admits that such can represent "our best estimates of frequencies with which certain kinds hypotheses succeed…however, [if] one wants to construe them as personal probabilities, there is no harm in it, as long as we attribute to the subject who has them the aim of bringing to bear all his or her experience that is relevant to the success or failure of the hypothesis being considered. The personalist and the frequentist need not be in any serious disagreement…" (564)

[8] Recall Nersessian's story here (analogous models)

"Solution" Set up ratio between two posterior probabilities with cancels the denominator term in Bayes' formula:

$$P(T_i \mid B \cap E) / P(T_j \mid B \cap E) = P(T_i \mid B)P(E \mid T_i \cap B) / P(T_j \mid B)P(E \mid T_j \cap B)$$

"Dynamic Rationality." (578)

Answers (MT Extra Credit)

1. Given the posted midterm scores, let V1={took version 1}, V2= {took version 2}, and V3 = {took version 3}. Obviously V1, V2, V3 are mutually exclusive.
   Let B ={score| 80% ≤ score < 89%}. A.) (1 pt) Find $P(V2 \mid B)$ directly

$$P(V2 \mid B) = \frac{n(V2 \cap B)}{n(B)} = \frac{1}{10}$$

   b.) (3 pts) Use the Bayes formula to verify your answer.

$$P(V2 \mid B) = \frac{P(B \cap V2)}{P(B \cap V1) + P(B \cap V2) + P(B \cap V3)}$$

$$= \frac{P(B \mid V2)P(V2)}{P(B \mid V1)P(V1) + P(B \mid V2)P(V2) + P(B \mid V3)P(V3)}$$

$$= \frac{\frac{1}{8} \cdot \frac{1}{3}}{\frac{3}{8} \cdot \frac{1}{3} + \frac{1}{8} \cdot \frac{1}{3} + \frac{6}{8} \cdot \frac{1}{3}} = \frac{1}{8\left(\frac{3}{8} + \frac{1}{8} + \frac{6}{8}\right)} = \frac{1}{3+1+6} = \frac{1}{10}$$

2. (4) Suppose Box I contains 3 red and 4 blue marbles and Box II 2 red and 7 blue marbles. Suppose someone tosses a coin. If it's heads, a marble is chosen from Box I. If tails, a marble is chosen from Box II. Suppose whoever tosses the coin doesn't reveal whether it turned up heads or tails (assume it's a fair coin), but s/he does reveal that a red marble was chosen. Find the probability (using Bayes' formula) that Box II was chosen.

$$P(II \mid R) = \frac{P(R \mid II)P(II)}{P(R \mid I)P(I) + P(R \mid II)P(II)} = \frac{\frac{2}{9} \cdot \frac{1}{2}}{\frac{3}{7} \cdot \frac{1}{2} + \frac{2}{9} \cdot \frac{1}{2}} = \frac{2}{9\left(\frac{3}{7} + \frac{2}{9}\right)} = \frac{2}{\frac{27}{7} + 2}$$

$$= \frac{14}{27+14} = \frac{14}{41}$$

3. (2) A box contains 6 red and 5 white marbles. Two are drawn successively from the box without replacement, and it is noted that the second one is red. What is the probability that the first one is also red? (You don't need to use Bayes' formula for this problem)

   Let: $W1$ = {white ball on first draw} $R1$ = {red ball on first draw}

   $R2$ = {red ball on second draw}

   Then: $P(R1 \cap R2) = P(R2 \cap R1) = P(R2 \mid R1)P(R1) = \frac{5}{10} \cdot \frac{6}{11} = \frac{30}{110} = \frac{3}{11}$