

I. LOGIC (Cont.)

Recall the “challenge exercise” I suggested in the previous handout: using Rules A-E, how can we determine the total number of *valid* standard form categorical syllogisms (SFCS)? Recall the total number of possible combinations is:

$$4^4 = 256 \quad \text{cases}$$

Let’s ask a “simpler” question: What are the total number of *invalid* cases, *independent of syllogistic figure*? We’ve gained enough familiarity with Rules A-E to list explicitly their invalid cases:

Rule A: No SFCS is valid that has two negative premises.

According to this rule, cases like *EOX*, *OEX* are therefore forbidden (where *X* means *any* syllogistic form.) Why? Recall that *E* and *I* are negative categorical forms (universal and particular, respectively.) Listing these cases explicitly:

$$EOX = \{EOA, EOE, EOI, EOO\} \quad OEX = \{OEA, OEE, OEI, OEO\}$$

Therefore, in set notation (i.e., in notation using operations involving sets) the total number of *forbidden* cases according to Rule A = 8, listed explicitly as¹:

$$EOX \cup OEX = \{EOA, EOE, EOI, EOO, OEA, OEE, OEI, OEO\}$$

Rule B: No SFCS is valid that has either a negative premise but does not have a negative conclusions or vice versa.²

¹ Note: The symbol: “ \cup ”, if you’re not familiar with it, means: “union.” That is to say, the “union” of two sets is defined (informally) as follows: $A \cup B = \{\text{all elements } x \text{ such that } x \text{ is in } A \text{ or } x \text{ is in } B\}$. It’s ***extremely important*** to understand the sense of “or” used here: it’s ***inclusive***, and therefore ***not exclusive***! What does this mean? Here’s an example: “Either I am alive or I am dead” is an example of “exclusive or,” (i.e., I cannot obviously be both alive and dead.) On the other hand, “either she will name her newborn daughter ‘Sue’ or she will name her newborn daughter ‘Ann’” is an example of *inclusive ‘or.’* Why? Because there’s no logical reason why the mother to her new daughter cannot name her child: “Sue Ann.” In other words, inclusive ‘or’ involves the maximum number of cases with respect to ‘or.’ That is to say, inclusive ‘or’ occupies both the regions of *A* alone and *B* alone and where they may happen to overlap. Unless otherwise (explicitly) noted, in logic we always use “or” in this inclusive sense.

² Note that the sense of “or” used in Rule B is clearly exclusive. Why? Explain this to yourself. (Hint: Consider the introduction of “*either...or...*”

Case i) (Negative premise & no negative conclusion):

Clearly this forbids the following cases:

$$EX\{A \text{ or } I\} \text{ or } OX\{A \text{ or } I\} \text{ or } XE\{A \text{ or } I\} \text{ or } XO\{A \text{ or } I\}$$

Or:

$$\begin{aligned} & (EXA \cup EXI) \cup (OXA \cup OXI) \cup (XEA \cup XEI) \cup (XOA \cup XOI) = \\ & \{EAA, EEA, EIA, EOA, EAI, EEI, EII, EOI\} \\ & \cup \{OAA, OEA, OIA, OOA, OAI, OEI, OII, OOI\} \cup \{AEA, EEA, IEA, OEA, AEI, EEI, IEI, OEI\} \\ & \cup \{AOA, EOA, IOA, OOA, AOI, EOI, IOI, OOI\} \end{aligned}$$

Whew! But are all these cases mutually exclusive? No! Performing the (pre-)union, I highlight (in boldface) all the redundant cases:

$$= \{EAA, \mathbf{EEA}, EIA, EOA, EAI, \mathbf{EEI}, EII, \mathbf{EOI}, OAA, \mathbf{OEA}, OIA, \mathbf{OOA}, OAI, \mathbf{OEI}, OII, \mathbf{OOI}, AEA, \mathbf{EEA}, IEA, \mathbf{OEA}, AEI, \mathbf{EEI}, IEI, \mathbf{OEI}, AOA, \mathbf{EOA}, IOA, \mathbf{OOA}, AOI, \mathbf{EOI}, IOI, \mathbf{OOI}\}$$

(Note the interesting pattern!³)

So we have the following set of unique cases:

$$= \{EAA, EEA, EIA, EOA, EAI, EEI, EII, EOI, OAA, OEA, OIA, OOA, OAI, OEI, OII, OOI, AEA, IEA, AEI, IEI, AOA, IOA, AOI, IOI\}$$

= 24 cases (for Case i))

Case ii) (Negative conclusion & no negative premises):

This forbids the following cases:

$$\begin{aligned} & \{A \text{ or } I\} \{A \text{ or } I\} E \text{ or } \{A \text{ or } I\} \{A \text{ or } I\} O = \\ & = \{AAE, IAE, AIE, IIE, AAO, IAO, AIO, IIO\} = 8 \text{ cases} \end{aligned}$$

...Combining both Cases, we get the following (32) forbidden cases:

³ This can be more rigorously recovered using Boolean operations like distributivity, etc.

= {EAA, EEA, EIA, EOA, EAI, EEI, EII, EOI, OAA, OEA, OIA, OOA, OAI, OEI, OII, OOI, AEA, IEA, AEI, IEI, AOA, IOA, AOI, IOI, AAE, IAE, AIE, IIE, AAO, IAO, AIO, IIO }

Rule C: No SFCS is valid unless one of the occurrences of the middle term is distributed

This forbids the following cases⁴: $IIX = \{IIA, IIE, III, IIO\}$ (= 4 cases)

Rule D: No SFCS is valid which has a distributed term in the conclusion that is not distributed in the premise in which this occurs.

This forbids the following cases:

$X\{I \text{ or } O\}A$ or $\{A \text{ or } I\}XE$ or $X\{I \text{ or } O\}E$ or $\{A \text{ or } I\}XO =$
 $\{AIA, EIA, IIA, OIA, AOA, EOA, IOA, OOA\} \cup \{AAE, AEE, AIE, AOE, IAE, IEE, IIE, IOE\}$
 $\cup \{AIE, EIE, IIE, OIE, AOE, EOE, IOE, OOE\} \cup \{AAO, AEO, AIO, AOO, IAO, IEO, IIO, IOO\}$
 $= \{AIA, EIA, IIA, OIA, AOA, EOA, IOA, OOA, AAE, AEE, AIE, AOE, IAE, IEE, IIE, IOE,$
 $AIE, EIE, IIE, OIE, AOE, EOE, IOE, OOE, AAO, AEO, AIO, AOO, IAO, IEO, IIO, IOO\}$

= {AIA, EIA, IIA, OIA, AOA, EOA, IOA, OOA, AAE, AEE, AIE, AOE, IAE, IEE, IIE, IOE,
 ,EIE ,OIE ,EOE,OOE,AAO,AEO,AIO,AOO,IAO,IEO,IIO,IOO}

=(28 cases)

Rule E: No SFCS is valid which has two universal premises and a particular conclusion

This forbids the following cases:

$AA\{I \text{ or } O\}$ or $AE\{I \text{ or } O\}$ or $EA\{I \text{ or } O\}$ or $EE\{I \text{ or } O\}$

= {AAI,AAO,AEI,AEO,EAI,EAO,EEI,EEO}

= 8 cases

...For a grand total of 80 forbidden cases. Note, however, we still need to check to see which forbidden cases from all these rules overlap, and get rid of these redundancies. Enlisting the aid of Excel:

⁴ Why aren't there any more forbidden cases? Think it over. Hint: Consider *all* the syllogistic moods.

Rule A EOA
EOE
EOI
EOO
OEA
OEE
OEI
OEO

Rule B EAA
EEA
EIA
EOA
EAI
EEI
EII
EOI
OAA
OEA
OIA
OOA
OAI
OEI
OII
OOI
AEA
IEA
AEI
IEI
AOA
IOA
AOI
IOI
AAE
AIE
IAE
IIE
AAO
AIO
IAO
IIO

RULE C IIA
IIE
III
IIO

RULE D AIA
EIA
IIA
OIA
AOA

EOA
IOA
OOA
AAE
AEE
AIE
AOE
IAE
IEE
IIE
IOE
EIE
OIE
EOE
OOE
AAO
AEO
AIO
AOO
IAO
IEO
IIO
IOO
RULE E AAI
AAO
AEI
AEO
EAI
EAO
EEI
EEO

- **Alphabetizing this list reveals many redundancies:**

AAE
AAE
AAI
AAO
AAO
AAO
AEA
AEE
AEI
AEI
AEO
AEO
AIA
AIE

AIE
AIO
AIO
AOA
AOA
AOE
AOI
AOO
EAA
EAI
EAI
EAO
EEA
EEI
EEI
EEO
EIA
EIA
EIE
EII
EOA
EOA
EOA
EOE
EOE
EOI
EOI
EOO
IAE
IAE
IAO
IAO
IEA
IEE
IEI
IEO
IIA
IIA
IIE
IIE
IIE
III
IIO
IIO
IIO
IOA
IOA
IOE
IOI
IOO

OAA
OAI
OEA
OEA
OEE
OEI
OEI
OEO
OIA
OIA
OIE
OII
OOA
OOA
OOE
OOI

Getting rid of the redundant cases, we're left with 53 forbidden cases:

# of cases	forbidden case
1	AAE
2	AAI
3	AAO
4	AEA
5	AEE
6	AEI
7	AEO
8	AIA
9	AIE
10	AIO
11	AOA
12	AOE
13	AOI
14	AOO
15	EAA
16	EAI
17	EAO
18	EEA
19	EEI
20	EEO
21	EIA
22	EIE
23	EII
24	EOA
25	EOE
26	EOI
27	EOO
28	IEA
29	IAO
30	IEA

31IEE
 32IEI
 33IEO
 34IIA
 35IIE
 36III
 37IIO
 38IOA
 39IOE
 40IOI
 41IOO
 42OAA
 43OAI
 44OEA
 45OEE
 46OEI
 47OEO
 48OIA
 49OIE
 50OII
 51OOA
 52OOE
 53OOI

- Hence, *independent of figure*, there are 53 invalid cases = $4 \times 53 = 212$ total invalid cases (we multiply by 4 when to including all four different kinds of figure.) Hence, *there can be no more than* $256 - 212 = 44$ valid syllogisms.
- **Note:** The analysis above simply established an *upper bound* to the number of totally possible valid SFCS (standard form categorical syllogisms.) In actuality, there are fewer than that number, since we're not counting SFCS that can be invalid in one figure but valid in another. (For example: AII figure 2 is invalid, but AII figure 4 is valid.) What this analysis shows is if we were to arbitrarily make up an SFCS, chances are pretty good it would be invalid. (Conversely, the *maximum* probability it could be valid is: $\frac{44}{256} = 17\%$.)

Recall (remark in Sept 9 discussion section) that grammar (i.e., the structure of language) isn't the same as logic (the structure of thought.) One way to see this is to consider all the grammatically equivalent ways of saying the same logical concept:

Logical concept	Logical Usage	Grammatical form
Conclusion	3 rd proposition in a standard form syllogism	A sentence beginning with: "Thus," "Therefore," "So," "Hence," etc.
Premise(s)	Major (first) or minor	A sentence beginning with:

	(second) proposition in a standard form syllogism	“For,” “Since,” “Because,” etc.
Universal quantifier (logical symbol: \forall)	What distributes the subject in an <i>A</i> -form categorical proposition (for example.)	A term in a sentence qualified by phrases like: “For all,” “for any,” “for each and every,” etc. Note: (recall lecture, September 19th, sometimes the universal quantification is implicit. For example: ‘Tigers are carnivorous.’
Existential quantifier (logical symbol: \exists)	What modifies the subject (or predicate) in an <i>I</i> -form categorical proposition (for example.)	A term in a sentence qualified by phrases like: “For some,” “there is a...,” “some,” etc.
Unique existence (logical symbol: $\exists!$)	What modifies a term belonging to a unit class, i.e., a class with only one member	A term referred in a sentence denoted by: a.) proper name, or b.) referred to with a definite description, c.) referred to with a demonstrative article [‘that,’ ‘this’, etc.], d.) referred to by a singular personal pronoun [‘s/he,’ ‘I,’ etc.]
Implication (logical symbol, in propositional form: $P \rightarrow Q$, where: <i>P</i> is the premise, and <i>Q</i> is the conclusion	The form of an <i>A</i> categorical proposition: “All <i>S</i> are <i>P</i> ” means: “For all <i>x</i> , if <i>x</i> is <i>S</i> , then <i>x</i> is <i>P</i> ”	Any sentence phrased like: “If <i>P</i> then <i>Q</i> ,” “ <i>P</i> implies <i>Q</i> ,” “ <i>Q</i> if <i>P</i> ,” “ <i>Q</i> is a necessary condition for <i>P</i> ,” “ <i>P</i> is a sufficient condition for <i>Q</i> ,” “ <i>P</i> [hence/so/thus/therefore] <i>Q</i> ” “[For/since/because] <i>P</i> , [hence/so/thus/therefore] <i>Q</i> .”
Conjunction (logical symbols: $\&$, \wedge)	The form of an <i>I</i> categorical proposition. “Some <i>S</i> are <i>P</i> ” means: “There are <i>x</i> such that <i>x</i> is <i>S</i> & <i>x</i> is <i>P</i> .”	Any sentence of the form: “ <i>P</i> and <i>Q</i> ,” “ <i>P</i> but also <i>Q</i> ,” etc
Disjunction (logical symbol: \vee)	Any proposition with connective “or” in the <i>inclusive</i> sense (see footnote 1 of this handout.)	A sentence of the form: “ <i>P</i> or <i>Q</i> ” where it’s understood that <i>P</i> & <i>Q</i> can also occur

- Sample Exercise Answers

I. (8) “Some women are tennis players, **so some women are athletes**, since all tennis players are athletes.”

Conclusion (highlight in boldface, because of indicator):

S: Women P: Athletes **I-Form:** “Some S are P”
Major Premise: **(A-form)** M: tennis players “All P are M”
Minor Premise: **(I-form)** M: tennis players “Some S are M”

An **AI₂** syllogism.

1. **Rule A:** No negative premises, OK
2. **Rule B:** No negative conclusion, nor premises, OK
3. **Rule C: Violated, since M isn’t distributed either in major nor minor premise.**
4. **Rule D:** No term distributed in conclusion, OK by default.
5. **Rule E:** Doesn’t have two universal premise, OK by default.

II. (d) “Nothing is both safe and exciting, so, since playing checkers is safe, it is not exciting.”

Identify conclusion and premise indicators:

“*since* playing checkers is safe.” **A premise**
“*so*...it is not exciting.”

Conclusion. (“**It**” refers to “**playing checkers**”)

S: “Checker games” **P:** “Exciting things”

No S are P (E-Form)

Minor premise: All S are M **M:** “Safe things” (A-Form)
Major Premise: No M are P (E-Form)

An **EAE₁** SFCS

- Rule A:** Just one negative premise, OK.
Rule B: Has a negative premise (Major) and a negative conclusion, OK.
Rule C: OK, M is distributed in minor premise.
Rule D: OK, P is distributed in major premise and in conclusion.
Rule E: Doesn’t have two universal premises, OK by default.

- III. To determine validity of SFGS, we can refer to the list generated at the beginning of the handout, in the (strong) cases wherein they're *always* invalid *independent* of figure. (For example, without further ado, AAO-3 is one of those cases.) *However*, this list does not include the *weaker* 'figure-dependent' cases (Recall remark on page 8.) In other words, there are cases (like mentioned in page 8, whose validity depends on figure, such as the valid AII-4 form, but the invalid AII-2 form.)

For instance OIO-4 is an OIO form, which isn't on list. So it may be valid or invalid. Let's find out:

OIO-4 Some P are not M
 Some M are S
 Therefore, some S are not P

Rule A: Just one negative premise (particular negative form-major), OK.

Rule B: Has a (particular) negative premise and a (particular) negative conclusion, OK.

Rule C: OK, M is distributed in the major premise.

Rule D: Violated. P is distributed in conclusion but not in major premise.

Rule E: Doesn't have two universal premises, OK by default.

METAPHYSICS EXERCISE:

Analyze the last remark in lecture (September 19) that questions concerning the existence of God are similar to those concerning the existence of werewolves. Consider different *ontological* properties regarding God versus werewolves. I.e., how can one distinguish between descriptions of the properties of the class of things containing God and those properties of the class of things containing werewolves? Do any of those properties predicate existence? [Is 'existence' a predicate? Etc.]