MATH 261
Post-EXAM I
October 3, 2007

Name: KEY

Directions: For maximum credit, please show all work in most reasonable detail. If you run out of room, you may pick up extra paper (supplied by instructor/proctor) and attach to this exam. No books or notes. Formula sheet provided. Calculator permitted, with programming/memory/graphing mode shut off.

I.) **10) Calculate** $\lim_{x\to 1}\frac{\sqrt{2x+1}-\sqrt{3}}{x-1}$ **by rationalizing the numerator**

$$\lim_{x\to 1}\frac{\sqrt{2x+1}-\sqrt{3}}{x-1}\cdot\frac{\sqrt{2x+1}+\sqrt{3}}{\sqrt{2x+1}+\sqrt{3}}=\lim_{x\to 1}\frac{2x+1-3}{(x-1)(\sqrt{2x+1}+\sqrt{3})}$$

$$=\lim_{x\to 1}\frac{2x-2}{(x-1)(\sqrt{2x+1}+\sqrt{3})}=2\lim_{x\to 1}\frac{(x\cancel{-1})}{(x\cancel{-1})(\sqrt{2x+1}+\sqrt{3})}$$

$$=2\lim_{x\to 1}\frac{1}{\sqrt{2x+1}+\sqrt{3}}=2\cdot\frac{1}{\sqrt{3}+\sqrt{3}}=\frac{2}{2\sqrt{3}}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

II.) **(10) For the function:** $f(x) = x\sqrt{x^2-1}$

a.) (4 points) Find $f'(x)$ and algebraically simplify

Method 1 Use Product Rule

$$\frac{d}{dx}\left(x(x^2-1)^{1/2}\right) = (x^2-1)^{1/2} + x\cdot\tfrac{1}{2}(x^2-1)^{-1/2}(2x) = (x^2-1)^{1/2} + x^2(x^2-1)^{-1/2}$$

$$= (x^2-1)^{-1/2}\left[(x^2-1)+x^2\right] = (x^2-1)^{-1/2}\left[2x^2-1\right] = \frac{2x^2-1}{\sqrt{x^2-1}}$$

Method 2 : $\frac{d}{dx}\left(x\sqrt{x^2-1}\right) = \frac{d}{dx}\sqrt{x^4-x^2} = \frac{1}{2}(x^4-x^2)^{-1/2}(4x^3-2x)$

$$= \frac{(4x^3-2x)}{2\sqrt{x^4-x^2}} = \frac{2x^3-x}{\sqrt{x^4-x^2}} = \frac{2x^3-x}{x\sqrt{x^2-1}} = \frac{\cancel{x}(2x^2-1)}{\cancel{x}\sqrt{x^2-1}} = \frac{2x^2-1}{\sqrt{x^2-1}}$$

b.) (6 points) Find $f''(x)$ and algebraically simplify

$$\frac{d}{dx}\frac{2x^2-1}{\sqrt{x^2-1}} = \frac{\sqrt{x^2-1}\,(4x) - (2x^2-1)\frac{1}{2}(x^2-1)^{-1/2}(2x)}{(x^2-1)}$$

$$= \frac{(x^2-1)^{-1/2}\left[4x(x^2-1) - x(2x^2-1)\right]}{(x^2-1)}$$

$$= \frac{x\left[4x^2-4-2x^2+1\right]}{(x^2-1)^{3/2}} = \frac{x(2x^2-3)}{(x^2-1)^{3/2}}$$

III.) (10) Given $f(x) = \frac{1}{3}x^3 + x^2 + 2x - 10$

a.) (5) Find its critical points. (All points where $f'(x) = 0$)

$$f'(x) = \frac{1}{3} \cdot 3x^2 + 2x + 2 = x^2 + 2x + 2$$

$$b^2 - 4ac = 4 - 8 < 0 \Rightarrow \text{NO C.P.s}$$

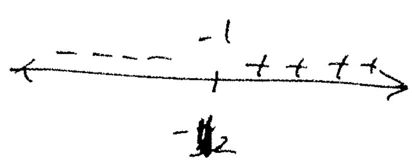
b.) (3) Find its inflection points. (All points where $f''(x) = 0$)

$$f''(x) = \frac{d}{dx}(x^2 + 2x + 2) = 2x + 2$$

$$f''(x_i) = 2x_i + 2 = 0 \Rightarrow x_{IP} = \cancel{-\tfrac{1}{2}} \; -1$$

c.) (2) Use your answer in a.) and the Second Derivative Test to find out which of those critical points are maxima or minima or neither.

N/A



← Shape

IV. (10) The volume of a cone is: $V = \frac{1}{3}\pi r^2 h$. Given $h = 4r$ and ${}^{dr}/_{dt} = 2$ ft/sec, find the rate of change of the volume of the cone, i.e. find $\frac{dV}{dt}$

$$h = 4r \Rightarrow \quad V = \frac{1}{3}\pi r^2(4r) = \frac{4}{3}\pi . r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \dot{r}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 (2) = 8\pi r^2$$

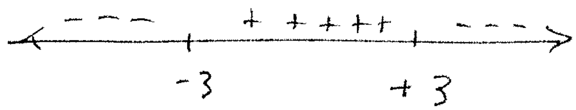
or the rate of change of $V \propto r^2$
with respect to t

(Proportional to r^2)

V.) (10) Calculate the derivative and second derivative of $f(x) = |9 - x^2|$ For maximum credit, specify the domain precisely

$$f(x) = |9 - x^2| = \begin{cases} 9 - x^2 & 9 - x^2 \geq 0 \Rightarrow (3-x)(3+x) \geq 0 \\ x^2 - 9 & 9 - x^2 < 0 \end{cases}$$

For $(3-x)(3+x) \geq 0 \Rightarrow$ True for all: $-3 \leq x \leq 3$



$$\therefore \quad f(x) = \begin{cases} 9 - x^2 & -3 \leq x \leq 3 \\ x^2 - 9 & x < -3 \text{ or } x > 3 \end{cases}$$

$$f'(x) = \begin{cases} -2x & -3 \leq x \leq 3 \\ 2x & x < -3 \text{ or } x > 3 \end{cases}$$

$$f''(x) = \begin{cases} -2 & -3 \leq x \leq 3 \\ 2 & x < -3 \text{ or } x > 3 \end{cases}$$