MATH 261
EXAM II
November 1, 2007

Name: KEY

Directions: For maximum credit, please show all work in most reasonable detail. An answer, *even if correct*, without any accompanying problem-solving steps, utilizing in a clear and logical manner calculus principles and formalae, as well as algebra, will receive minimal credit (5 pts out of 20 at most). I am grading you primarily for your clearly demonstrated problem solving strategies and tactics If you run out of room, please write on the back page of exam sheet and clearly indicate with a note. No books or notes. Formula sheet provided. Calculator permitted, with programming/memory mode shut off. (However, you may use the graphing utility). Please choose FIVE from the following SIX (worth 20 pts each.) If you do more, I will grade the best five. Bonus problem included. Good luck!

I. (20 pts) Evaluate the following:

a.) (10) $\int (x^3+2)^{10} x^2 dx$

$u(x) = x^3+2 \qquad \frac{du}{dx} = 3x^2 \Rightarrow x^2 dx = \frac{1}{3} du$

$$\int (x^3+2)^{10} x^2 dx = \frac{1}{3}\int u^{10} du = \frac{1}{3}\cdot\frac{1}{11} u^{11} + c$$

$$= \boxed{\frac{1}{33}(x^2+2)^{11} + c}$$

b.) (10) $\int_1^2 (x+1)(x-1)^{1/3} dx$

$u(x) = x - 1 \Rightarrow \frac{du}{dx} = 1 \Rightarrow du = dx$

$\Downarrow$

$x = u+1$

$$\int_1^2 (x+1)(x-1)^{1/3} dx = \int_{u(1)}^{u(2)} (u+2)u^{1/3} du = \int_0^1 (u^{4/3} + 2u^{1/3}) du$$

$$= \left[\frac{3}{7}u^{7/3} + 2\cdot\frac{3}{4}u^{4/3}\right]\Big|_0^1 = 3u^{4/3}\left[\frac{1}{7}u + \frac{1}{2}\right]\Big|_0^1$$

$$= 3\left(\frac{1}{7}+\frac{1}{2}\right) - 0 = \boxed{\frac{27}{14}}$$

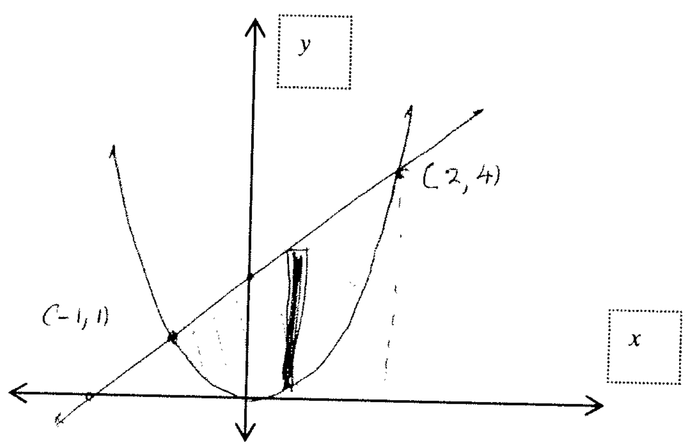
II. (20) Given the functions $y_1 = x + 2$ $y_2 = x^2$

a.) (3) Find the points of intersection by solving the equation: $y_1 = y_2$

$$x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0 \Rightarrow x_1 = 2,\ x_2 = -1$$

a.) (2) Sketch the region these two graphs define, clearly labeling the x and y coordinates of where the graphs intersect



c) (15) Find the area of the region by evaluating a dx integral

$$A = \int_{-1}^{2} [y_1(x) - y_2(x)]\,dx$$

$$= \int_{-1}^{2} [x + 2 - x^2]\,dx = \frac{1}{2}x^2 + 2x - \frac{1}{3}x^3 \Big|_{-1}^{2}$$

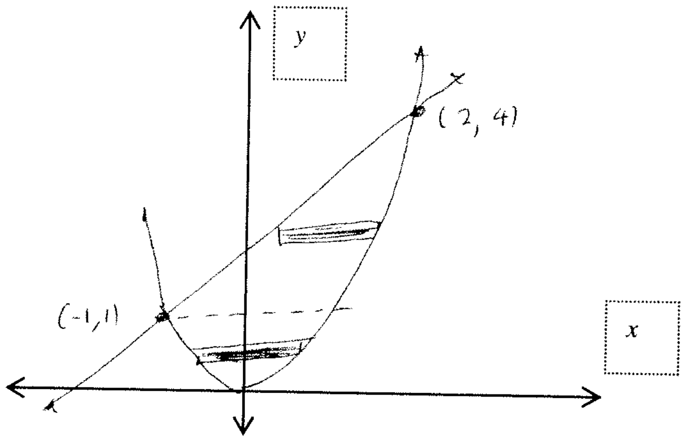
$$= x\left(\frac{1}{2}x + 2 - \frac{1}{3}x^2\right)\Big|_{-1}^{2}$$

$$= 2(5/3) - (-1)(7/6)$$

$$= 10/3 + 7/6 = \boxed{27/6}$$

$$= \boxed{9/2}$$

III (20) Given the functions $y_1 = x + 2$ $y_2 = x^2$ (same figure as in II above), find the area of the region by evaluating a dy integral. Show that your answer is the same as in II. above.



$$y_1(x) = x+2 \Rightarrow x_1(y) = y-2$$
$$y_2(x) = x^2 \Rightarrow x_2(y) = \pm\sqrt{y}$$

$$A = \int_0^1 (\sqrt{y} - (-\sqrt{y}))\,dy + \int_1^4 (\sqrt{y} - (y-2))\,dy$$

$$= \int_0^1 2y^{1/2}dy + \int_1^4 (y^{1/2} - y + 2)\,dy$$

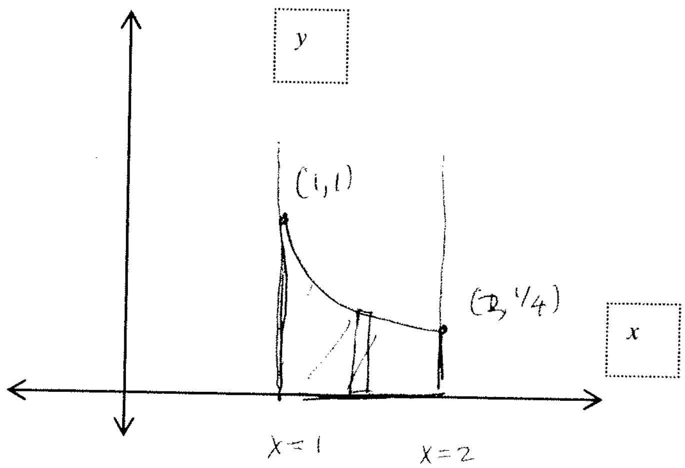
$$= 2\cdot\frac{2}{3}y^{3/2}\Big|_0^1 + \left[\frac{2}{3}y^{3/2} - \frac{1}{2}y^2 + 2y\right]\Big|_1^4$$

$$= 4/3 + y\left[\frac{2}{3}\sqrt{y} - \frac{1}{2}y + 2\right]\Big|_1^4 = 4/3 + 4\left[\frac{2}{3}\cdot 2 - 2 + 2\right] - \left[\frac{2}{3} - \frac{1}{2} + 2\right]$$

$$= 4/3 + 4[4/3] - 13/6 = 8/6 + 32/6 - 13/6 = 40/6 - 13/6 = \boxed{27/6 = 9/2}$$

IV. (20) Given the region $y = \frac{1}{x^2}$, $y = 0$, $x = 1$, $x = 2$,

a.) (5) Sketch the cross-sectional area of this region, clearly labeling the x and y coordinates of its four corners



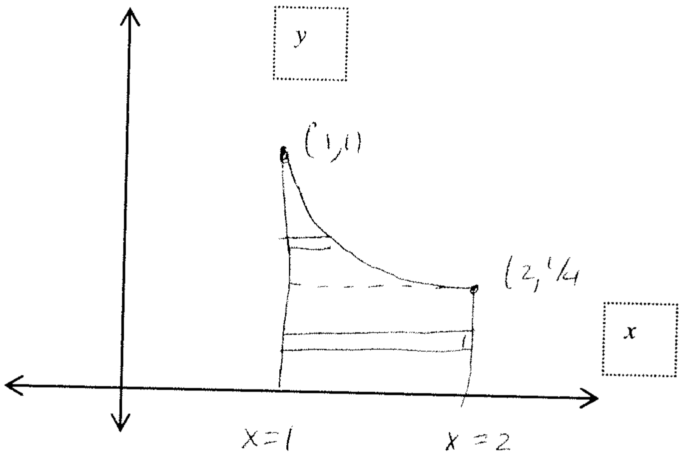
b.) (15) Find the volume of the figure generated by revolving the region around the y-axis using the Method of Shells

$$V = 2\pi \int_1^2 x\, y(x)\, dx = 2\pi \int_1^2 x \left(1/x^2 \right) dx = 2\pi \int_1^2 \frac{dx}{x}$$

$$= 2\pi \ln|x| \Big|_1^2 = 2\pi \left[\ln 2 - \ln 1 \right] = \boxed{2\pi \ln 2}$$

V. (25) $y = \dfrac{1}{x^2}$, $y = 0$, $x = 1$, $x = 2$,

(The same region as in IV) above.) Find the volume using the method of washers



$$y = 1/x^2 \Rightarrow x(y) = \frac{1}{\sqrt{y}} = y^{-1/2}$$

$$V = \pi \int_0^{1/4} [2^2 - 1^2]\,dy + \pi \int_{1/4}^{1} [(y^{-1/2})^2 - 1^2]\,dy$$

$$= \pi \int_0^{1/4} (3)\,dy + \pi \int_{1/4}^{1} (y^{-1} - 1)\,dy$$

$$= 3\pi \int_0^{1/4} dy + \pi \int_{1/4}^{1} y^{-1}\,dy + -\pi \int_{1/4}^{1} dy$$

$$= 3\pi y\Big|_0^{1/4} + \pi \ln|y|\Big|_{1/4}^{1} - \pi y\Big|_{1/4}^{1}$$

$$= 3\pi \cdot 1/4 + \pi(\ln 1 - \ln 1/4) - \pi(1 - 1/4)$$

$$= \cancel{\tfrac{3}{4}\pi} + \pi(-\ln 1/4) - \cancel{\tfrac{3}{4}\pi}$$

$$= \pi(-\ln 4^{-1}) = \pi \ln 4 = \boxed{2\pi \ln 2}$$

VI. (20) a.) (10) Given $f(x) = xe^{x^2} + \ln\sqrt[3]{2x+1}$

Find its derivative. Make sure you express your final answer in the simplest algebraic detail

$$f(x) = xe^{x^2} + \frac{1}{3}\ln(2x+1)$$

$$f'(x) = \frac{d}{dx}(xe^{x^2}) + \frac{d}{dx}\cdot\frac{1}{3}\ln(2x+1)$$

$$= (e^{x^2} + x\cdot e^{x^2}\cdot 2x) + \frac{1}{3}(2x+1)^{-1}\cdot 2$$

$$= e^{x^2}(1+2x^2) + \frac{2}{3(2x+1)}$$

b.) (10) Find: $\int_1^2 (\ln x)^8 \frac{dx}{x}$

$$u = \ln x \qquad du = \frac{dx}{x}$$

$$\int_1^2 (\ln x)^8 \frac{dx}{x} = \int_{u(1)}^{u(2)} u^8 du = \int_{\ln 1}^{\ln 2} u^8 du = \frac{1}{9}u^9 \Big|_0^{\ln 2}$$

$$= \boxed{\frac{1}{9}(\ln 2)^9}$$

BONUS (20) Use logarithmic differentiation to find the derivative of $y = \dfrac{(x^2+1)(x-2)}{(x+3)^2(x-3)}$

Express final answer in terms of x-terms alone and simplify in most reasonable extent.

$$\ln y = \ln(x^2+1) + \ln(x-2) - 2\ln(x+3) - \ln(x-3)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2x}{(x^2+1)} + \frac{1}{(x-2)} - \frac{2}{(x+3)} - \frac{1}{(x-3)}$$

$$\frac{dy}{dx} = y\left[\frac{2x}{(x^2+1)} + \frac{1}{(x-2)} - \frac{2}{(x+3)} - \frac{1}{(x-3)}\right]$$

$$= \frac{(x^2+1)(x-2)}{(x+3)^2(x-3)}\left[\frac{2x}{(x^2+1)} + \frac{1}{(x-2)} - \frac{2}{(x+3)} - \frac{1}{(x-3)}\right]$$

$$= \frac{2x(x-2)}{(x+3)^2(x-3)} + \frac{(x^2+1)}{(x+3)^2(x-3)} - \frac{2(x^2+1)(x-2)}{(x+3)^3(x-3} - \frac{(x^2+1)(x-2)}{(x+3)^2(x-3)^2} =$$