MA261 **Calculus I: Notes for November 13**
Fall 2007 Kallfelz

REMAINING MATERIAL FROM § 8.4

***Recall the last two formulae for the anti-derivative of* sec*x and* csc*x*:**

$$\int \sec x dx = \ln|\sec x + \tan x| + C$$
$$\int \csc x dx = -\ln|\csc x + \cot x| + C$$

Recall also the Pythagorean Identities (all three forms):

$$\sin^2 x + \cos^2 x = 1$$
$$\tan^2 x + 1 = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

(From exercises 65, 66, § 8.4) the above formulae can be recast in different form:

$$\int \sec x dx = \ln|\sec x + \tan x| + C = \ln\left|(\sec x + \tan x)\cdot\frac{(\sec x - \tan x)}{\sec x - \tan x}\right| + C = \ln\left|\frac{\sec^2 x - \tan^2 x}{\sec x - \tan x}\right| + C$$
$$= \ln\left|\frac{(\tan^2 x + 1) - \tan^2 x}{\sec x - \tan x}\right| + C = \ln\left|\frac{1}{\sec x - \tan x}\right| + C = -\ln|\sec x - \tan x| + C$$

(Using the 'trick of 1' on the right hand side, and the second Pythagorean identity to simplify, along with the properties of logarithms.)

This can also be verified directly (by integrating on the left hand side):

$$\int \sec x dx = \int \sec x\left(\frac{\sec x - \tan x}{\sec x - \tan x}\right)dx = \int \frac{\sec^2 x - \sec x \tan x}{\sec x - \tan x}dx$$
$$u = \sec x - \tan x \Rightarrow \frac{du}{dx} = \sec x \tan x - \sec^2 x \Rightarrow -(\sec^2 x - \sec x \tan x)dx = du$$
$$\Rightarrow (\sec^2 x - \sec x \tan x)dx = -du$$
$$\therefore \int \frac{\sec^2 x - \sec x \tan x}{\sec x - \tan x}dx = -\int \frac{du}{u} = -\ln|u| + C = -\ln|\sec x - \tan x| + C$$

- Likewise in the case of the csc*x*:

$$\int \csc x dx = -\ln\left|\csc x + \cot xx\right| + C = \ln\left|\frac{1}{\csc x + \cot x}\right| + C = \ln\left|\frac{1}{\csc x + \cot x}\cdot\frac{\csc x - \cot x}{\csc x - \cot x}\right| + C$$

$$= \ln\left|\frac{\csc x - \cot x}{\csc^2 x - \cot^2 x}\right| + C = \ln\left|\frac{\csc x - \cot x}{\csc^2 x - \left(\csc^2 - 1\right)}\right| + C = \ln\left|\csc x - \cot x\right| + C$$

This can also be verified directly (by integrating on the left hand side):

$$\int \csc x dx = \int \csc x\left(\frac{\csc x - \cot x}{\csc x - \cot x}\right)dx = \int \frac{\csc^2 x - \csc x \cot x}{\csc x - \cot x}dx$$

$$u = \csc x - \cot x \Rightarrow \frac{du}{dx} = -\csc x \cot x + \csc^2 x \Rightarrow \left(\csc^2 x - \csc x \cot x\right)dx = du$$

$$\therefore \int \frac{\csc^2 x - \csc x \cot x}{\csc x - \cot x}dx = \int \frac{du}{u} = \ln|u| + C = \ln\left|\csc x - \cot x\right| + C$$

It's also a useful exercise to **differentiate** the right hand sides and see that they yield the results sec$x$ and csc$x$

$$\frac{d}{dx}\left[-\ln\left|\sec x - \tan x\right| + C\right] = -\frac{\frac{d}{dx}(\sec x - \tan x)}{(\sec x - \tan x)} = -\frac{\sec x \tan x - \sec^2 x}{\sec x - \tan x}$$

$$= -\frac{(-\sec x)(\sec x - \tan x)}{\sec x - \tan x} = \sec x$$

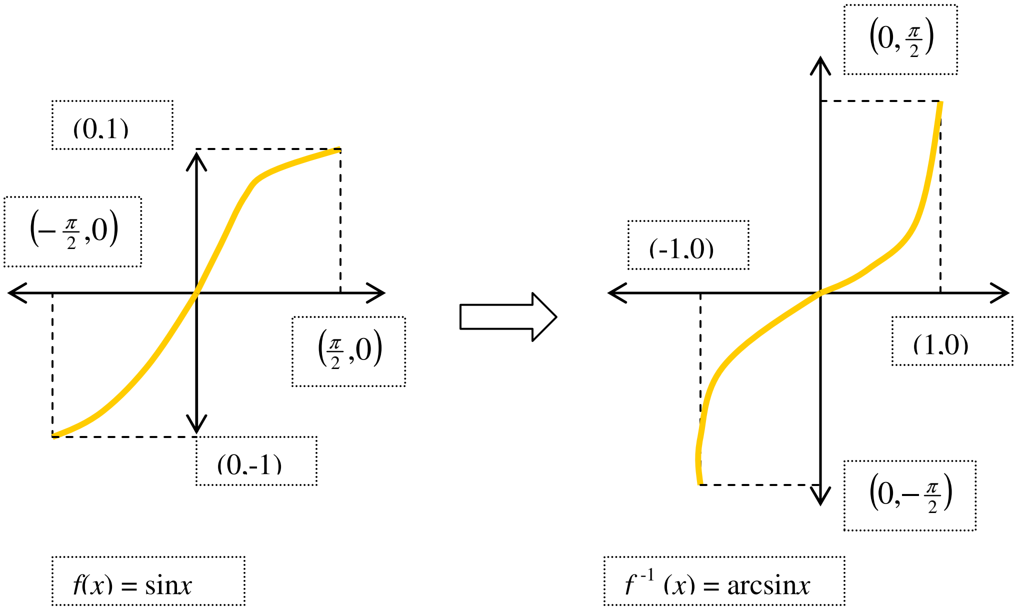
- Note1: One can drop the absolute value notation upon differentiation because the *argument $u(x)$* of $\ln(u(x))$ need not remain positive once the natural log function is discharged.

$$\frac{d}{dx}\left[\ln\left|\csc x - \cot x\right| + C\right] = \frac{\frac{d}{dx}(\csc x - \cot x)}{(\csc x - \cot x)} = \frac{-\csc x \cot x + \csc^2 x}{\csc x - \cot x} = \frac{\csc x(\csc x - \cot x)}{\csc x - \cot x} = \csc x$$

CALCULUS OF THE INVERSE TRIGONOMETRIC FUNCTIONS § 8.5

Recall page 4 (Nov. 6) concerning fixing the domains and ranges of the 6 trigonometric functions, and their inverses. We can perform this analysis graphically, reproducing the graphs on p. 460.

We start with the sine function. Shrinking its domain to $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ will leave its range $[-1,1]$ unaffected:



- From the above graphical features, we infer the following:

$Domain(\arcsin x) = [-1,1]$ $\qquad$ $Range(\arcsin x) = [-\frac{\pi}{2},\frac{\pi}{2}]$

Also, viewing the graph at the right, we obtain the following qualitative information:

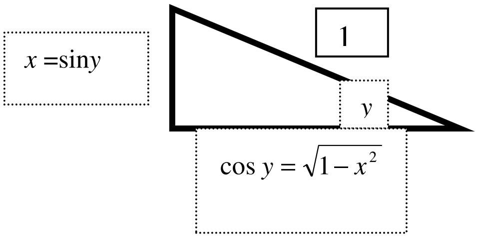
$Domain\left(\frac{d}{dx}\arcsin x\right) = (-1,1)$ (the tangent is vertical at the endpoints {-1, 1}

$\frac{d}{dx}\arcsin x > 0$ (arcsin$x$ is strictly monotone increasing)

- Having established the above facts graphically, we are ready to obtain the derivative formula for arcsin$x$ which is done via an implicit derivative procedure:

$$y = \arcsin x \Rightarrow \sin y = x \Rightarrow \frac{d}{dx}\sin y = \cos y y' = 1 \Rightarrow y' = \frac{1}{\cos y}$$

Now after completing the calculus, we'd like our expression transformed back into an explicit function with respect to x. This can be done via the following trick (using the Pythagorean Thm): sin$y$ = x relates an *angle* y to an opposite side x in terms of the right triangle below:

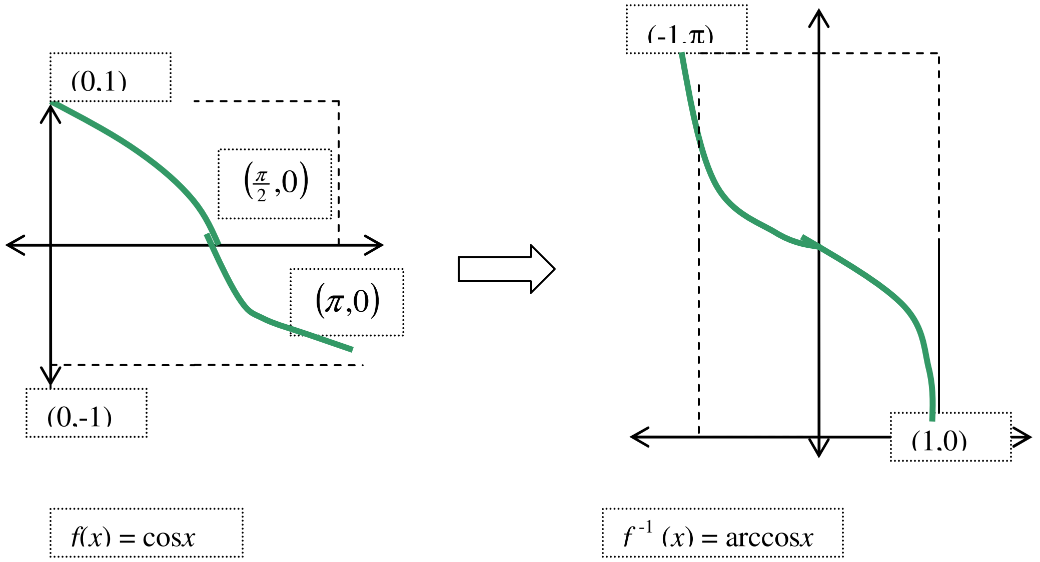


Hence, based on the above geometric derivation, the derivate formula expressed explicitly in terms of x becomes:

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \quad ,$$

or expressed in general chain rule fashion: $\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}}$

Repeating the same procedure for **arccos*x*:**



- From the above graphical features, we infer the following:

$$\mathit{Domain}(\arccos x) = [-1,1] \qquad \mathit{Range}(\arccos x) = [0,\pi]$$

Also, viewing the graph at the right, we obtain the following qualitative information:

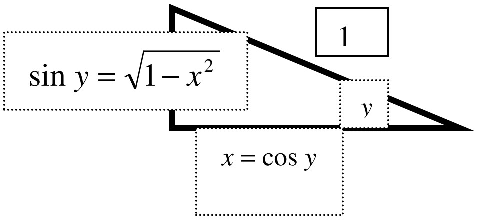
$$\mathit{Domain}\left(\frac{d}{dx}\arcsin x\right) = (-1,1)$$ (the tangent is vertical at the endpoints {-1, 1}

$$\frac{d}{dx}\arccos x > 0$$ (arccos*x* is strictly monotone decreasing)

- The derivative formula for arccos*x* which is done via an implicit derivative procedure:

  $$y = \arccos x \Rightarrow \cos y = x \Rightarrow \frac{d}{dx}\cos y = -\sin y y' = 1 \Rightarrow y' = -\frac{1}{\sin y}$$

  we'd like our expression transformed back into an explicit function with respect to *x*. This can be done via the following trick (using the Pythagorean Thm): cos*y* = *x* relates an *angle y* to an opposite side *x* in terms of the right triangle below:

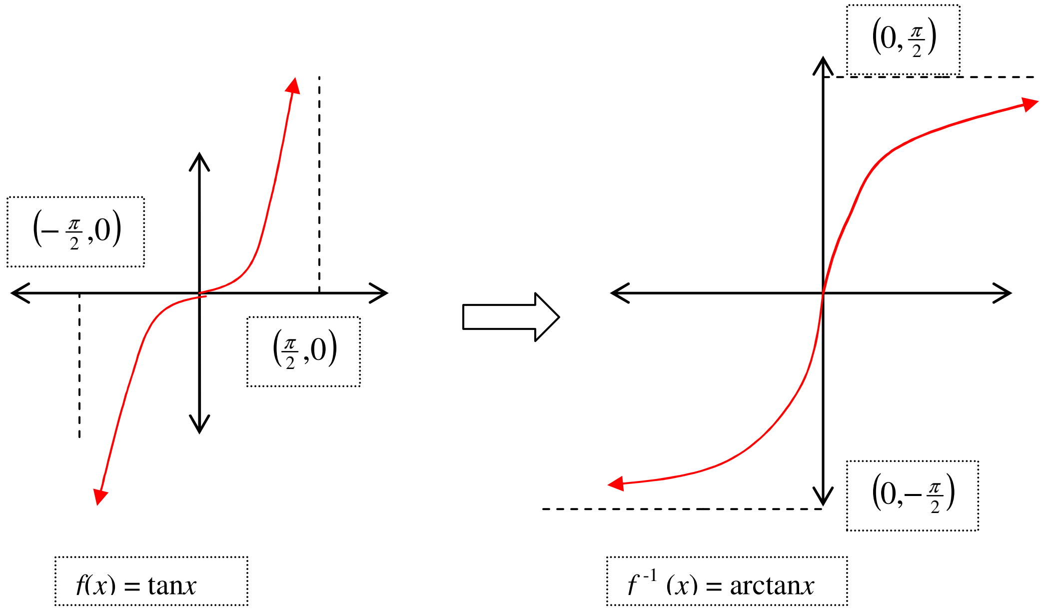


Hence, based on the above geometric derivation, the derivate formula expressed explicitly in terms of *x* becomes:

$$\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}} \qquad ,$$

or expressed in general chain rule fashion: $\frac{d}{dx}\arccos u = -\frac{u'}{\sqrt{1-u^2}}$

Repeating the procedure for **arctan:**



- From the above graphical features, we infer the following:

$$Domain(\arctan x) = (-\infty,\infty) \qquad Range(\arctan x) = \left(-\tfrac{\pi}{2},\tfrac{\pi}{2}\right)$$

Also, viewing the graph at the right, we obtain the following qualitative information:

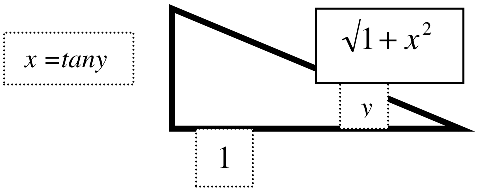
$$Domain\left(\frac{d}{dx}\arctan x\right) = (-\infty,0)\cup(0,\infty) \quad \text{(the tangent is vertical at the origin)}$$

$$\frac{d}{dx}\arctan x > 0 \qquad \text{(}\arctan x\text{ is strictly monotone increasing)}$$

- Having established the above facts graphically, we are ready to obtain the derivative formula for arctan*x* which is done via an implicit derivative procedure:

$$y = \arctan x \Rightarrow \tan y = x \Rightarrow \frac{d}{dx}\tan y = 1 \Rightarrow \sec^2 y y' = 1 \Rightarrow y' = \frac{1}{\sec^2 y}$$

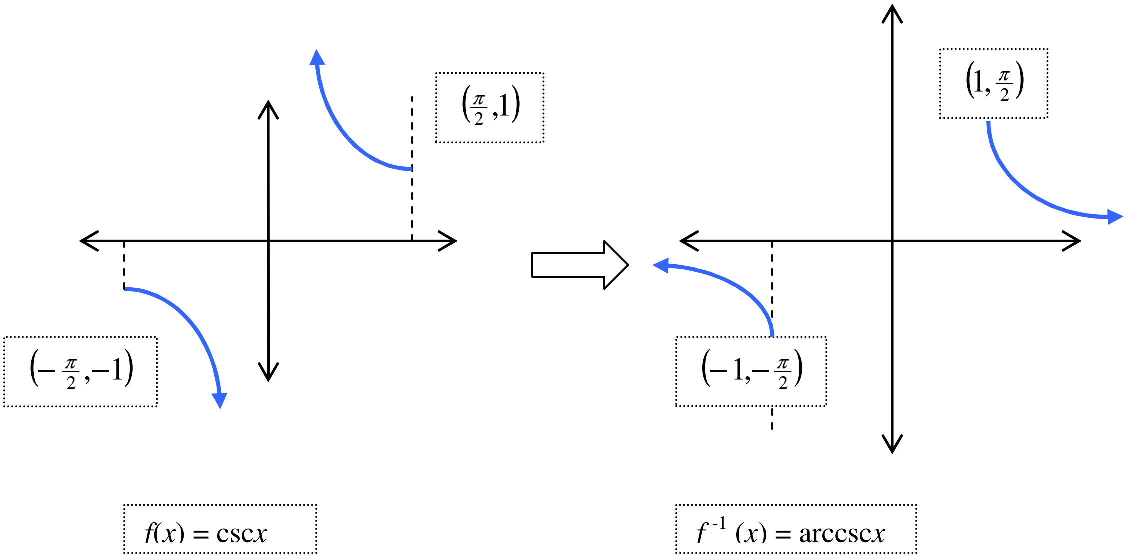
Now after completing the calculus, we'd like our expression transformed back into an explicit function with respect to *x*. This can be done via the following trick (using the Pythagorean Thm): sin$y$ = x relates an *angle* y to an opposite side x in terms of the right triangle below:



Recall that for the tangent, the inscribed triangle is larger (its base has length 1). Hence, based on the figure above: $\cos y = \dfrac{1}{\sqrt{1+x^2}} \Rightarrow \sec y = \sqrt{1+x^2}$

Hence: $\dfrac{d}{dx}\arctan x = \dfrac{1}{\sec^2 y} = \dfrac{1}{1+x^2}$, and in chain rule form: $\dfrac{d}{dx}\arctan u = \dfrac{u'}{1+u^2}$

Consider now the arccsc function:



- From the above graphical features, we infer the following:

$$Domain(\text{arc}\csc x) = (-\infty,-1] \cup [1,\infty) \qquad Range(\text{arc}\csc x) = [-1,0) \cup (0,1]$$

Also, viewing the graph at the right, we obtain the following qualitative information:

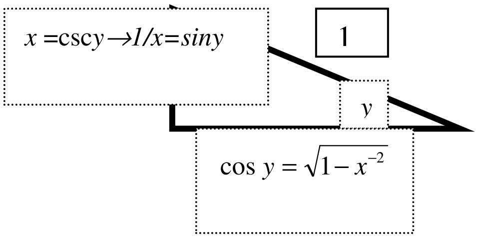
$$Domain\left(\frac{d}{dx}arc\csc x\right)=(-\infty,-1)\cup(1,\infty)$$ (the tangent is vertical at the endpoints {-1, 1}

$$\frac{d}{dx}arc\csc x<0$$ (arccsc*x* is strictly monotone decreasing)

- Having established the above facts graphically, we are ready to obtain the derivative formula for arccsc*x* which is done via an implicit derivative procedure:

$$y=arc\csc x\Rightarrow \csc y=x\Rightarrow \frac{d}{dx}\csc y=-\csc y\cot y'=1\Rightarrow y'=\frac{-1}{\csc y\cot y}$$

Now after completing the calculus, we'd like our expression transformed back into an explicit function with respect to *x*. This can be done via the following trick (using the Pythagorean Thm): sin*y* = *x* relates an *angle y* to an opposite side *x* in terms of the right triangle below:



So based on the above picture:

$$\cot y=\frac{\cos y}{\sin y}=\frac{\sqrt{1-x^{-2}}}{x^{-1}}=x\sqrt{1-x^{-2}}=\sqrt{x^2-1}$$

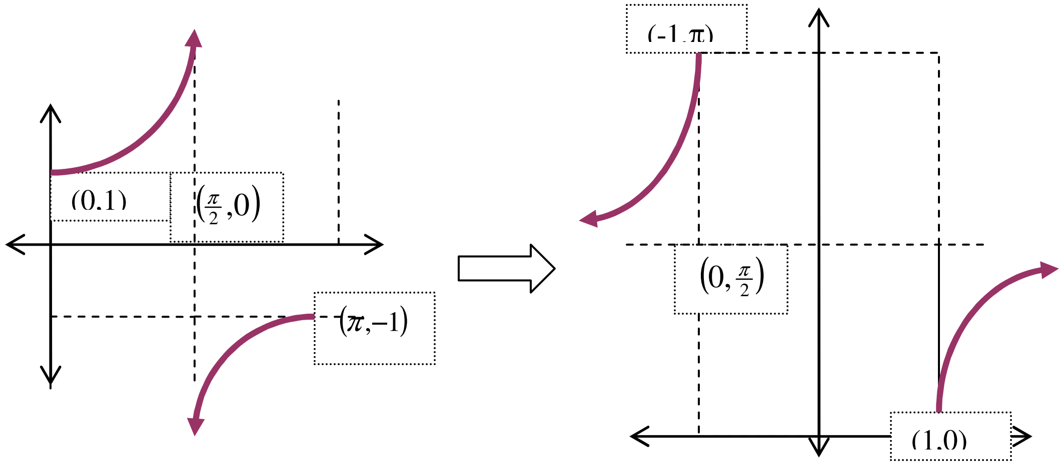
Hence, based on the above geometric derivation, the derivate formula expressed explicitly in terms of *x* becomes:

$$\frac{d}{dx}arc\csc x=\frac{-1}{\csc y\cot y}=\frac{-1}{|x|\sqrt{x^2-1}}\qquad ,$$

or expressed in general chain rule fashion: $\dfrac{d}{dx} arc\csc u = -\dfrac{u'}{|u|\sqrt{1-u^2}}$

- Note2: The absolute value placed around *x* is deliberate to ensure that the derivative is always *negative* (as evidenced in the above graph). In other words, the numerator is negative (-1) so the denominator should always be positive, to ensure a negative fraction

Repeating the same procedure for **arcsec*x*:**



f(*x*) = sec*x*

$f^{-1}(x)$ = arcsec*x*

- From the above graphical features, we infer the following:

$$Domain(arc\sec x) = (-\infty,-1]\cup[1,\infty) \qquad Range(arc\sec x) = [0,\tfrac{\pi}{2})\cup(\tfrac{\pi}{2},\pi]$$

Also, viewing the graph at the right, we obtain the following qualitative information:

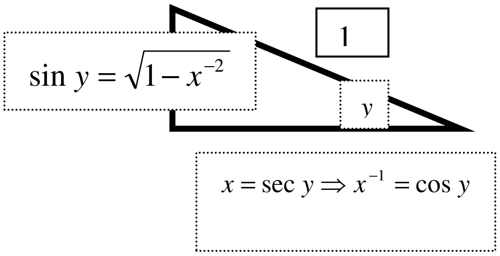
$$Domain\left(\frac{d}{dx} arc\sec x\right) = (-\infty,-1)\cup(1,\infty)$$ (the tangent is vertical at the endpoints {-1, 1}

$$\frac{d}{dx} arc\sec x > 0$$ (arcsec*x* is strictly monotone increasing)

- The derivative formula for arcsec*x* which is done via an implicit derivative procedure:

$$y = arc\sec x \Rightarrow \sec y = x \Rightarrow \frac{d}{dx}\sec y = \sec y \tan y y' = 1 \Rightarrow y' = \frac{1}{\sec y \tan y}$$

we'd like our expression transformed back into an explicit function with respect to *x*. This can be done via the following trick (using the Pythagorean Thm): cos*y* = *x* relates an *angle* *y* to an opposite side *x* in terms of the right triangle below:



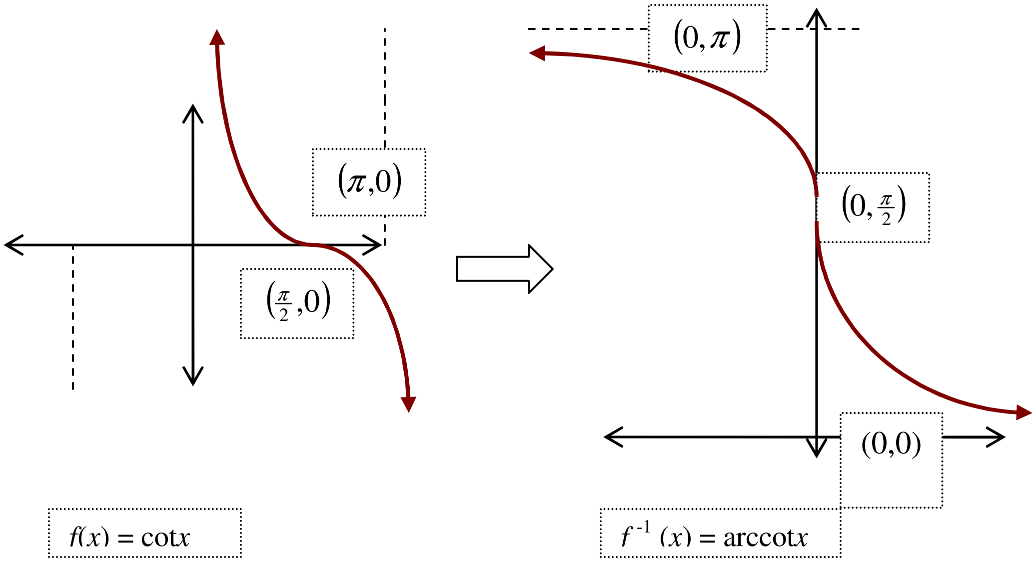
Hence, based on the above depiction, $\tan y = \frac{\sin y}{\cos y} = \frac{\sqrt{1-x^{-2}}}{x^{-1}} = \sqrt{x^2 - 1}$

So: $\frac{d}{dx} arc\sec x = \frac{1}{\sec y \tan y} = \frac{1}{|x|\sqrt{x^2 - 1}}$

or expressed in general chain rule fashion: $\frac{d}{dx} arc\sec u = \frac{u'}{|u|\sqrt{u^2 - 1}}$

- Note3: As in the case of arcccsc, the absolute value term was inserted to guarantee that the derivative is always *positive*

Repeating the procedure for **arccot**:



- From the above graphical features, we infer the following:

$$Domain(arc\cot x) = (-\infty, \infty) \qquad Range(arc\cot x) = (0, \pi)$$

Also, viewing the graph at the right, we obtain the following qualitative information:

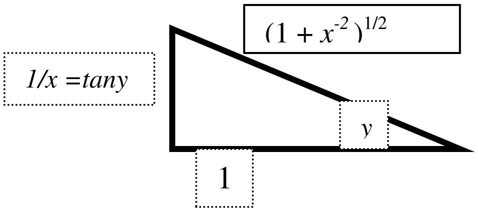
$$Domain\left(\frac{d}{dx}arc\cot x\right) = (-\infty, 0) \cup (0, \infty) \quad \text{(the tangent line is vertical at the origin)}$$

$$\frac{d}{dx}arc\cot x < 0 \qquad \text{(arccot}x\text{ is strictly monotone decreasing)}$$

- Having established the above facts graphically, we are ready to obtain the derivative formula for arctan$x$ which is done via an implicit derivative procedure:

$$y = arc\cot x \Rightarrow \cot y = x \Rightarrow \frac{d}{dx}\cot y = 1 \Rightarrow -\csc^2 yy' = 1 \Rightarrow y' = \frac{-1}{\csc^2 y}$$

opposite side x in terms of the right triangle below:



So from the above figure: $\csc y = \dfrac{1}{\sin y} = \dfrac{\sqrt{1+x^{-2}}}{x^{-1}} = x\sqrt{1+x^{-2}} = \sqrt{x^2+1}$

So: $\dfrac{d}{dx} arc\cot x = \dfrac{-1}{\csc^2 y} = \dfrac{-1}{x^2+1}$, and in chain rule form: $\dfrac{d}{dx} arc\cot u = \dfrac{-u'}{u^2+1}$

- Summarizing:

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \Rightarrow \frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}}$$
$$\frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1-x^2}} \Rightarrow \frac{d}{dx}\arccos u = \frac{-u'}{\sqrt{1-u^2}}$$
$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2} \Rightarrow \frac{d}{dx}\arctan u = \frac{u'}{1+u^2}$$
$$\frac{d}{dx} arc\csc x = \frac{-1}{|x|\sqrt{x^2-1}} \Rightarrow \frac{d}{dx} arc\csc u = \frac{-u'}{|u|\sqrt{u^2-1}}$$
$$\frac{d}{dx} arc\sec x = \frac{1}{|x|\sqrt{x^2-1}} \Rightarrow \frac{d}{dx} arc\sec u = \frac{u'}{|u|\sqrt{u^2-1}}$$
$$\frac{d}{dx} arc\cot x = \frac{-1}{1+x^2} \Rightarrow \frac{d}{dx} arc\cot u = \frac{-u'}{1+u^2}$$

The above formulae were presented in the order in which they were derived from the procedures above (using the graphs and implicit differentiation). As you notice, (and as the text presents in Thm 8.7) these formulae look redundant, in other words, we get the same expressions (except for a minus sign). To see this more clearly, note (as in Thm 8.7):

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \Rightarrow \frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}}$$
$$\frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1-x^2}} \Rightarrow \frac{d}{dx}\arccos u = \frac{-u'}{\sqrt{1-u^2}}$$
$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2} \Rightarrow \frac{d}{dx}\arctan u = \frac{u'}{1+u^2}$$
$$\frac{d}{dx}arc\cot x = \frac{-1}{1+x^2} \Rightarrow \frac{d}{dx}arc\cot u = \frac{-u'}{1+u^2}$$
$$\frac{d}{dx}arc\sec x = \frac{1}{|x|\sqrt{x^2-1}} \Rightarrow \frac{d}{dx}arc\sec u = \frac{u'}{|u|\sqrt{u^2-1}}$$
$$\frac{d}{dx}arc\csc x = \frac{-1}{|x|\sqrt{x^2-1}} \Rightarrow \frac{d}{dx}arc\csc u = \frac{-u'}{|u|\sqrt{u^2-1}}$$

We can exploit this ambiguity in the following way, concerning anti-derivative formulae:
Note that only *three* are required:

$$\int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C = -\arccos u + C$$
$$\int \frac{du}{1+u^2} = \arctan u + C = -arc\cot u + C$$
$$\int \frac{du}{u\sqrt{u^2-1}} = arc\sec u + C = -arc\csc u + C$$

By convention, the **positive** answers are selected.

EXAMPLES

- Exercise 24 (§8.5)

  $\sec(\arcsin(x-1))$ Let $y = \arcsin(x-1) \Rightarrow \sin y = (x-1)$

  Hence according to Pythagorean identity (#1):

  $\sin^2 y + \cos^2 y = 1 \Rightarrow \cos y = \sqrt{1-(x-1)^2} = \sqrt{2x-x^2}$

  So $\sec y = \frac{1}{\cos y} = (2x-x^2)^{-\frac{1}{2}}$

- Exercise 55 (§8.5)

$$f(x)=\frac{1}{2}\left(\frac{1}{2}\ln\frac{x+1}{x-1}+\arctan x\right)=\frac{1}{4}\ln(x+1)-\frac{1}{4}\ln(x-1)+\frac{1}{2}\arctan x$$

$$f'(x)=\frac{d}{dx}\left(\frac{1}{4}\ln(x+1)-\frac{1}{4}\ln(x-1)+\frac{1}{2}\arctan x\right)$$

$$=\tfrac{1}{4}\cdot\frac{1}{x+1}-\tfrac{1}{4}\cdot\frac{1}{x-1}+\tfrac{1}{2}\cdot\frac{1}{1+x^2}=\tfrac{1}{4}\left(\frac{1}{x+1}-\frac{1}{x-1}+\frac{2}{x^2+1}\right)=\tfrac{1}{4}\left(\frac{-2}{x^2-1}+\frac{2}{x^2+1}\right)$$

$$=\tfrac{1}{2}\left(\frac{-1}{x^2-1}+\frac{1}{x^2+1}\right)=\tfrac{1}{2}\left(\frac{-2}{(x^2-1)(x^2+1)}\right)=\frac{-1}{(x^4-1)}=\frac{1}{1-x^4}$$

- Exercise 61 (§8.5)

$$f(x)=arc\sec x-x\Rightarrow f'(x)=\frac{d}{dx}(arc\sec x-x)=\frac{1}{|x|\sqrt{x^2-1}}-1$$

$$\therefore f'(x)=\begin{cases}\dfrac{1}{x\sqrt{x^2-1}}-1=\dfrac{1-x\sqrt{x^2-1}}{x\sqrt{x^2-1}} & (If\_x\geq 0)\\ -\dfrac{1}{x\sqrt{x^2-1}}-1=\dfrac{-\left(1+x\sqrt{x^2-1}\right)}{x\sqrt{x^2-1}} & (If\_x<0)\end{cases}$$

To find relative extrema, set $f'(x) = 0$

- Note: There are two critical points (of Type II) 0, -1, 1 (which make the denominator = 0)

Case 1: $(x > 0)$: $\dfrac{1-x\sqrt{x^2-1}}{x\sqrt{x^2-1}}=0\Rightarrow 1-x\sqrt{x^2-1}=0$ (for all $x\neq 1$)

$$\Rightarrow -x\sqrt{x^2-1}=-1\Rightarrow x^2\left(x^2-1\right)=1\Rightarrow x^4-x^2-1=0$$

Let $u = x^2$, then: $x^4-x^2-1=0\Rightarrow u^2-u-1=0\Rightarrow u_{1,2}=\dfrac{1\pm\sqrt{1+4}}{2}$

$$\Rightarrow \left(x^2\right)_1=\tfrac{1}{2}\left(1+\sqrt{5}\right),\left(x^2\right)_2=\tfrac{1}{2}\left(1-\sqrt{5}\right)$$

Note that the second case is forbidden, in terms of x, since we'd be taking the square root of a negative number. Hence:

$$x_{1,2}=\pm\sqrt{\frac{1+\sqrt{5}}{2}}=\pm\tfrac{1}{2}\left(\sqrt{2+2\sqrt{5}}\right)\approx\pm 1.272$$

Case 2: ($x < 0$): $\dfrac{-1-x\sqrt{x^2-1}}{x\sqrt{x^2-1}} = 0 \Rightarrow 1 + x\sqrt{x^2-1} = 0$ (for all $x \neq 1$)

$$\Rightarrow x\sqrt{x^2-1} = -1 \Rightarrow x^2\left(x^2-1\right) = 1 \Rightarrow x^4 - x^2 - 1 = 0$$

(we get the same equation as in **Case 1,** hence the one result which is negative)

To find which is the local max or min, use the Second Derivative Test:

$$\therefore f''(x) = \begin{cases} \dfrac{d}{dx}\dfrac{1}{x\sqrt{x^2-1}} - 1 = \dfrac{d}{dx}[\left(x^4-x^2\right)^{-1/2} - 1] & (If\_x \geq 0) \\ \dfrac{d}{dx} - \dfrac{1}{x\sqrt{x^2-1}} - 1 = -\dfrac{d}{dx}\left[1 + \left(x^4-x^2\right)^{-1/2}\right] & (If\_x < 0) \end{cases}$$

Simplifying:

$$\therefore f''(x) = \begin{cases} \dfrac{d}{dx}[\left(x^4-x^2\right)^{-1/2} - 1] = -\tfrac{1}{2}\left(x^4-x^2\right)^{-3/2}\left(4x^3-2x\right) = \dfrac{x-2x^3}{\left(x^4-x^2\right)^{3/2}} & (If\_x \geq 0) \\ -\dfrac{d}{dx}\left[1 + \left(x^4-x^2\right)^{-1/2}\right] = \dfrac{2x^3-x}{\left(x^4-x^2\right)^{3/2}} & (If\_x < 0) \end{cases}$$

Hence: $f''(1.272) < 0$ **(local max)**

Other point is local min

- <u>Exercise 68 (§8.5)</u>

If the function: $f(x) = \arcsin\left(\dfrac{x-2}{2}\right) - 2\arcsin\dfrac{\sqrt{x}}{2}$ is constant on $0 \leq x \leq 4$, then its derivate must = 0 on that interval:

Calculating the derivative:

$$f'(x)=\frac{d}{dx}\{\arcsin\left(\frac{x-2}{2}\right)-2\arcsin\frac{\sqrt{x}}{2}\}=\frac{d}{dx}\arcsin\left(\tfrac{1}{2}x-1\right)-2\frac{d}{dx}\arcsin\tfrac{1}{2}\sqrt{x}$$

$$=\frac{\frac{d}{dx}\left(\tfrac{1}{2}x-1\right)}{\sqrt{1-\left(\tfrac{1}{2}x-1\right)^2}}-2\frac{\frac{d}{dx}\tfrac{1}{2}\sqrt{x}}{\sqrt{1-\tfrac{1}{4}x}}=\frac{\tfrac{1}{2}}{\sqrt{x-\tfrac{1}{4}x^2}}-\frac{2\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}x^{-1/2}}{\sqrt{1-\tfrac{1}{4}x}}=\frac{\tfrac{1}{2}}{\sqrt{x-\tfrac{1}{4}x^2}}-\frac{\tfrac{1}{2}}{\sqrt{x}\sqrt{1-\tfrac{1}{4}x}}$$

$$=\frac{\tfrac{1}{2}}{\sqrt{x-\tfrac{1}{4}x^2}}-\frac{\tfrac{1}{2}}{\sqrt{x-\tfrac{1}{4}x^2}}=0$$