KEY - ASSIGNMENT I

MA 261 FALL 2007

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20 I. a) (5) $\lim_{x\to 2^+} \frac{2-x}{x^2-4} = \lim_{x\to 2^+} \frac{-(x-2)}{(x-2)(x+2)} = -\lim_{x\to 2^+} \frac{\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = -\lim_{x\to 2^+} \frac{1}{(x+2)} = -\frac{1}{4}$

b.) i) (5) $f(x) = 4x + 5x^2$ $\lim_{\Delta x\to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x\to 0} \frac{4(x+\Delta x) + 5(x+\Delta x)^2 - (4x+5x^2)}{\Delta x}$

$= \lim_{\Delta x\to 0} \frac{\cancel{4x} + 4\Delta x + \cancel{5x^2} + 10x\Delta x + 5\Delta^2 x - \cancel{4x} - \cancel{5x^2}}{\Delta x}$

$= \lim_{\Delta x\to 0} \frac{4\Delta x + 10x\Delta x + 5\Delta^2 x}{\Delta x} = \lim_{\Delta x\to 0} \frac{\Delta x(4 + 10x + 5\Delta x)}{\Delta x} = \lim_{\Delta x\to 0} (4 + 10x + 5\Delta x)$

$= 4 + 10x$

b.) ii) (10) $f(x) = (x+2)^{1/2}$ $\lim_{\Delta x\to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x\to 0} \frac{1}{\Delta x}\left(\sqrt{x+\Delta x+2} - \sqrt{x+2}\right)$

$= \lim_{\Delta x\to 0} \frac{1}{\Delta x}\left\{\left(\sqrt{x+\Delta x+2} - \sqrt{x+2}\right)\left(\frac{\sqrt{x+\Delta x+2} + \sqrt{x+2}}{\sqrt{x+\Delta x+2} + \sqrt{x+2}}\right)\right\}$

$= \lim_{\Delta x\to 0} \frac{1}{\Delta x}\left\{\frac{\cancel{x} + \Delta x + \cancel{2} - \cancel{x} - \cancel{2}}{\sqrt{x+\Delta x+2} + \sqrt{x+2}}\right\} = \lim_{\Delta x\to 0} \frac{1}{\cancel{\Delta x}} \frac{\cancel{\Delta x}}{(\sqrt{x+\Delta x+2} + \sqrt{x+2})}$

$= \lim_{\Delta x\to 0} \left(\frac{1}{\sqrt{x+\Delta x+2} + \sqrt{x+2}}\right) = \frac{1}{2\sqrt{x+2}} = \frac{1}{2}(x+2)^{-1/2}$

30 II. a) (5 PTS) $f(x) = \frac{|x-3|}{x-3} = \begin{cases} \frac{(x-3)}{(x-3)} & x > 3 \\ -\frac{(x-3)}{(x-3)} & x < 3 \end{cases} = \begin{cases} 1 & x > 3 \\ -1 & x < 3 \end{cases}$

Note: $\lim_{x\to 3^-} f(x) = -1$ & $\lim_{x\to 3^+} f(x) = +1$ & $f(3) = \frac{0}{0}$ DNE

A ~~re~~discontinuity at $x = 3$. It's not removable, since $\lim_{x\to 3^+} f(x) \neq \lim_{x\to 3^-} f(x)$

b.) (5 PTS) $f(x) = x^2 - 6x + 8$ $[0,3]$

Note that since $f(x)$ is a degree 2 polynomial, it's everywhere continuous.

$f(0) = 8$ $f(3) = -1$. According to I.V.T., there exists at least one

$c \in [0,3]$ such that $f(3) < f(c) < f(0)$. Consider, for example: $1 \in [0,3]$

$0 < 1 < 3$ $f(1) = 3$. Note: $f(3) < f(1) < f(0)$

c.) (5 PTS) $\lim_{x\to 1} \frac{x^3-1}{x^2+x+1} = \lim_{x\to 1} \frac{(x-1)\cancel{(x^2+x+1)}}{\cancel{(x^2+x+1)}} = \lim_{x\to 1} (x-1) = 0$

II.d) $F(x) = -\frac{k}{x^2} + \frac{k}{(x-5)^2} \quad 0 < x < 5$

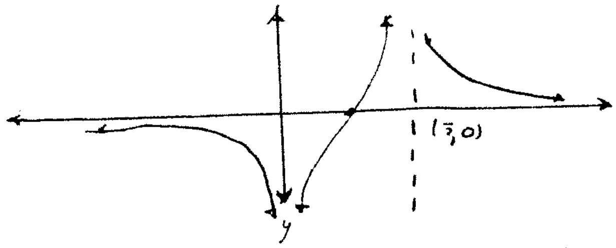
(15 PTS)

$= k\left\{\frac{-1}{x^2} + \frac{1}{(x-5)^2}\right\}$

$F=0 \Rightarrow -\frac{1}{x^2} + \frac{1}{(x-5)^2} = 0 \Rightarrow x-5 = \pm x$

$\Rightarrow x = 5/2$

$\lim_{x\to 0^+} F(x) = \lim_{x\to 0^-} F(x) = -\infty \qquad \lim_{x\to 5^-} F(x) = \lim_{x\to 5^+} F(x) = \infty \qquad \lim_{x\to\pm\infty} F(x) = 0$



25 e) (10) $|x^3-8| < \varepsilon \Rightarrow |x-2||x^2+2x+4| < \varepsilon \Leftarrow \delta\left(\underbrace{|x|^2+2|x|+4}_{\hat{\infty} \text{ when } |x-2|<\delta}\right) = k\delta$

25) III a) $f(x) = \sqrt{x-4}$

5PTS

$f'(x) = \lim_{\Delta x\to 0} \frac{\sqrt{x+\Delta x-4} - \sqrt{x-4}}{\Delta x} = \lim_{\Delta x\to 0} \frac{\sqrt{x+\Delta x-4} - \sqrt{x-4}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x-4} + \sqrt{x-4}}{\sqrt{x+\Delta x-4} + \sqrt{x-4}}$

$= \lim_{\Delta x\to 0} \frac{\cancel{x} + \cancel{\Delta x} - \cancel{4} - \cancel{x} + \cancel{4}}{\cancel{\Delta x}(\sqrt{x+\Delta x-4} + \sqrt{x-4})} = \lim_{\Delta x\to 0} \frac{1}{\sqrt{x+\Delta x-4} + \sqrt{x-4}} = \frac{1}{2\sqrt{x-4}} = \frac{1}{2}(x-4)^{-1/2}$

b.) $f(x) = 1/x \qquad c = 3$

(5PTS)

$f'(c) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c} = \lim_{x\to c} \frac{1/x - 1/c}{x-c} = \lim_{x\to c} \frac{c-x}{xc(x-c)} = -\lim_{x\to c} \frac{\cancel{(x-c)}}{xc\,\cancel{(x-c)}} = -\lim_{x\to c} \frac{1}{xc}$

$= -\frac{1}{c^2} = -1/9$

c) (5PTS) $\quad s(t) = -16t^2 + 48t$

a.) $v(t) = \frac{d}{dt}(-16t^2 + 48t) = -16\frac{d}{dt}(t^2) + 48\frac{d}{dt}(t) = -32t + 48$

$a(t) = \frac{d}{dt}v(t) = \frac{d}{dt}(-32t + 48) = -32\frac{d}{dt}(t) + \frac{d}{dt}(48) = -32$

b.) $v(t_0) = 0 \Rightarrow -32t_0 + 48 = 0 \Rightarrow t_0 = 3/2$ sec

c) $s(t_0) = -16(3/2)^2 + 48(3/2) = -36 + 72 = 36$ ft.

d.) (5PTS) $f(x) = \frac{x(x^2-1)}{x+3}$ $\qquad f'(x) = \frac{d}{dx}\left\{\frac{x(x^2-1)}{x+3}\right\} = \frac{d}{dx}\left\{\frac{x^3-x}{x+3}\right\}$ (Method 1)

$= \cancel{\frac{d}{dx}}\left\{\frac{(x+3)(3x^2-1) - (x^3-x)}{(x+3)^2}\right\} = \left\{\frac{3x^3 + 9x^2 - \cancel{x} - 3 - x^3 + \cancel{x}}{(x+3)^2}\right\}$

$= \frac{2x^3 + 9x^2 - 3}{(x+3)^2}$

Method 2.

$$\begin{array}{r|l} & x^2 - 3x + 8 \\ x+3 & x^3 - x \\ & x^3 + 3x^2 \\ & -3x^2 - x \\ & -3x^2 - 9x \\ & 8x \\ & 8x + 24 \end{array}$$

$\Rightarrow \frac{x^3-x}{x+3} = x^2 - 3x + 8 - \frac{24}{(x+3)}$

$$\therefore\ f'(x) = \frac{d}{dx}\left\{ x^2 - 3x + 8 - \frac{24}{(x+3)} \right\} = 2x - 3 + \frac{24}{(x+3)^2} = \frac{(2x-3)(x+3)^2 + 24}{(x+3)^2}$$

$$= \frac{(2x-3)(x^2+6x+9)+24}{(x+3)^2} = \frac{2x^3 + 12x^2 + \cancel{18x} - 3x^2 - \cancel{18x} - 27 + 24}{(x+3)^2}$$

$$= \frac{2x^3 + 9x^2 - 3}{(x+3)^2}$$

e.) (5PTS) $f(x) = x + \frac{32}{x^2} = x + 32x^{-2}$

$f'(x) = \frac{d}{dx}(x + 32x^{-2}) = 1 - 64x^{-3}$ $\quad f''(x) = \frac{d}{dx}(1 - 64x^{-3}) = 192x^{-4}$

25 IV. a) $f(x) = -3(2-9x)^{1/4}$

5PTS $\quad f'(x) = -3\frac{d}{dx}(2-9x)^{1/4} = -\frac{3}{4}(2-9x)^{-3/4}(-9) = \frac{27}{4}(2-9x)^{-3/4}$

b) (5PTS) $f(t) = \frac{\sqrt{t^2+1}}{t} = \sqrt{1+t^{-2}} \Rightarrow f'(t) = \frac{d}{dt}(1+t^{-2})^{+1/2}$

$= \frac{1}{2}(1+t^{-2})^{-1/2}(-2t^{-3}) = -t^{-3}(1+t^{-2})^{-1/2}$

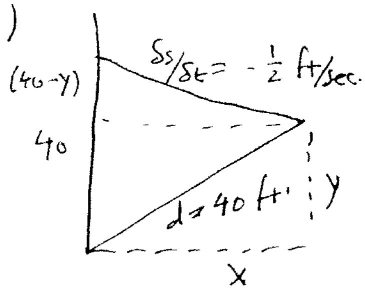
$f''(t) = \frac{d}{dt}\left(-t^{-3}(1+t^{-2})^{-1/2}\right) = +3t^{-4}(1+t^{-2})^{-1/2} - t^{-3}(-\tfrac{1}{2})(1+t^{-2})^{-3/2}(-2t^{-3})$

$= t^{-4}(1+t^{-2})^{-1/2}\left\{3 + t^{-2}(1+t^{-2})^{-1}\right\} = \frac{1}{t^4\sqrt{1+1/t^2}}\left\{3 + \frac{1}{t^2(1+1/t^2)}\right\}$

$= \frac{1}{t^3\sqrt{t^2+1}}\left\{3 + \frac{1}{t\sqrt{t^2+1}}\right\} = \frac{1}{t^3\sqrt{t^2+1}}\left\{\frac{3t\sqrt{t^2+1}+1}{t\sqrt{t^2+1}}\right\}$

$= \frac{3t\sqrt{t^2+1}+1}{t^4(t^2+1)} = 8$

c) (10PTS)



Find $\frac{\delta x}{\delta t}$ and $\delta y/\delta t$ when $\frac{\delta s}{\delta t} = -\frac{1}{2}$ & $y = 20$

$s^2 = (40-y)^2 + x^2 \Rightarrow x^2 = s^2 - (40-y)^2$

$d^2 = x^2 + y^2$

$\frac{d}{dt}(d^2) = \frac{d}{dt}(x^2+y^2) \Rightarrow 0 = 2x\frac{\delta x}{\delta t} + 2y\frac{\delta y}{\delta t}$

$\cancel{d\,\delta d}$ $\quad \Rightarrow x\frac{\delta x}{\delta t} = -y\frac{\delta y}{\delta t}$

$\frac{d}{dt}s^2 = 2s\frac{\delta s}{\delta t} = -2(40-y)\frac{\delta y}{\delta t} + 2x\frac{\delta x}{\delta t} = 2\left(\frac{x}{y}\right)(40-y)\frac{\delta x}{\delta t} + 2x\frac{\delta x}{\delta t}$

$$2s\frac{\delta s}{\delta t} = \frac{2x}{y}(40-y)\frac{\delta x}{\delta t} + 2x\frac{\delta x}{\delta t} \Rightarrow s\frac{\delta s}{\delta t} = \left[\frac{x}{y}(40-y)+x\right]\frac{\delta x}{\delta t} = x[40y^{-1}-1+1]\,\delta x/\delta t$$

$$s\frac{\delta s}{\delta t} = 40\left(\frac{x}{y}\right)\frac{\delta x}{\delta t} \Rightarrow \frac{\delta x}{\delta t} = \frac{s\,\delta s/\delta t}{40(x/y)} = \frac{ys\,\delta s/\delta t}{40x} = \frac{ys\,\delta s/\delta t}{40\sqrt{40^2-y^2}}$$

$$= \frac{y[(40-y)^2+x^2]^{1/2}\,\delta s/\delta t}{40\sqrt{40^2-y^2}} = \frac{y[(40-y)^2+40^2-y^2]^{1/2}\,\delta s/\delta t}{40\sqrt{40^2-y^2}}$$

$$= \frac{y[3200-80y+y^2-y^2]^{1/2}}{40\sqrt{40^2-y^2}}\frac{\delta s}{\delta t} = \frac{y[3200-80y]}{40[1600-y^2]^{1/2}}\frac{\delta s}{\delta t} = \frac{40y[80-\cancel{2}y]\,\delta s/\delta t}{40\sqrt{1600-y^2}}$$

$$= \frac{2y[40-y]\,\delta s/\delta t}{\sqrt{1600-y^2}} = \frac{2\cdot 20\cdot 20\cdot(-1/2)}{\sqrt{1600-400}} = -\frac{400}{\sqrt{1200}} = \frac{-40}{\sqrt{12}} = -\frac{20}{\sqrt{3}}$$

$$= -\frac{20\sqrt{3}}{3}$$

e) (5PTS) $f(x) = \dfrac{x+1}{x^2+x+1}$ $\quad f(x)=0 \Rightarrow x+1=0 \Rightarrow x=-1$ (No Vertical asymptotes since denominator is an irreducible)

f: $- - - -$ on $(-\infty,-1)$, $+ + + + +$ on $(-1,\infty)$; marks at -1, 0

$$f'(x) = \frac{(x^2+x+1)-(x+1)(2x+1)}{(x^2+x+1)^2} = \frac{(x^2+x+1)-(2x^2+3x+1)}{(x^2+x+1)^2}$$

$$= -\frac{x^2-2x}{(x^2+x+1)^2} = \frac{-x(x+2)}{(x^2+x+1)^2} \qquad x_2=0,\ x_3=-2 \text{ critical pts (Type I)}$$

f': $+ + +$ on $(-\infty,-2)$, $- - - -$ on $(-2,0)$, $+ + + +$ on $(0,\infty)$

$$f''(x) = \frac{d}{dx}\left\{\frac{-x^2-2x}{(x^2+x+1)^2}\right\} = -\frac{d}{dx}\left\{\frac{x^2+2x}{(x^2+x+1)^2}\right\}$$

$$= -\frac{[(x^2+x+1)^2(2x+2)-(x^2+2x)\,2(x^2+x+1)(2x+1)]}{(x^2+x+1)^4}$$

$$= -2\frac{[(x^2+x+1)^2(x+1)-(x^2+2x)(x^2+x+1)(2x+1)]}{(x^2+x+1)^4}$$

$$= \frac{-2[(x^2+x+1)(x+1)-(x^2+2x)(2x+1)]}{(x^2+x+1)^3}$$

$$= \frac{-2[(x^3+\cancel{2x}x^2+x+x^2+x+1-2x^3-5x^2-2x]}{(x^2+x+1)^3}$$