Formula Sheet-ExamII **Math 261**

- **Algebraic formualae:**

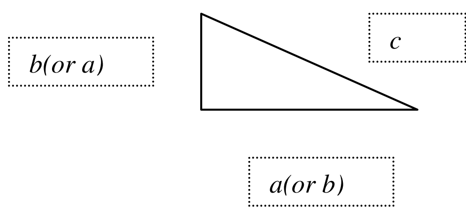
Rational exponents: $x^{\frac{p}{q}} \equiv \sqrt[q]{x^p} = \left(\sqrt[q]{x}\right)^p$

(Difference between two cubes) $x^3 \pm y^3 = (x \pm y)\left(x^2 \mp xy + y^2\right)$

Rationalizing denominator/numerator makes use of the "trick of 1": For example, for any functions *u*(*x*), *v*(*x*), *r*(*x*) :

$$\frac{\sqrt{u(x)} \pm \sqrt{v(x)}}{r(x)} = \frac{\sqrt{u(x)} \pm \sqrt{v(x)}}{r(x)} \cdot \left(\frac{\sqrt{u(x)} \mp \sqrt{v(x)}}{\sqrt{u(x)} \mp \sqrt{v(x)}}\right) = \ldots \text{(etc.)}$$

Pythagorean Formula:



$c^2 = a^2 + b^2$

- **Absolute value function: for any function *u*(*x*):**

$$|u(x)| = \begin{cases} u(x) & all\_x : u(x) \geq 0 \\ -u(x) & otherwise(all\_x : u(x) < 0 \end{cases}$$

- **Definition of Derivative**

$$f'(c) = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = \lim_{x \to c} \frac{f(x) - f(x)}{x - c}$$

- **Derivative Formulae:**

<u>Const. times a function rule & addition/subtraction rule:</u>

$$\frac{d}{dx}\left(af(x) \pm bg(x)\right) = a\frac{df}{dx} \pm b\frac{dg}{dx} = af'(x) \pm bg'(x)$$

Power Rule (**Note**: q is any rational number, i.e. $q = {}^{n}/_{m}$)

$$\frac{d}{dx}x^q = qx^{(q-1)}$$

Product Rule

$$\frac{d}{dx}(f(x)g(x)) = \left(\frac{df}{dx}\right)g(x) + f(x)\left(\frac{dg}{dx}\right) = f'(x)g(x) + f(x)g'(x)$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g(x)}{(g(x))^2}$$

Chain Rule

$$\frac{d}{dx}f(g(x)) = f'(g)g'(x) = \frac{df}{dg}\frac{dg}{dx}$$

- ***INTEGRATION (DEFINITE AND INDEFINITE)***

- **Defn.:** Given a continuous function $f(x)$, its antiderivative is a function $F(x)$ such that $\frac{d}{dx}F(x) = F'(x) = f(x)$

Note1: The antiderivative for a function $f(x)$ gets the notation: $F(x) = \int f(x)dx$

And is denoted (for reasons we'll see shortly) as an **indefinite integral.**

1.) $\int 0 \cdot dx = C$

2.) $\int [af(x) \pm bg(x)]dx = a\int f(x)dx \pm b\int g(x)dx$

3.) $\int x^q dx = \frac{x^{q+1}}{(q+1)} + C$

The **Fundamental Theorem of Calculus** states is:

$\int_a^b f(x)dx = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(c_i)\Delta x = F(b)_i - F(a)$, where: $\frac{d}{dx}F(x) = f(x)$, i.e. F is the **antiderivative** or **indefinite integral** of f.

Note 3: It's often convenient to adopt the shorthand notation: $F(x)\Big|_a^b = F(b) - F(a)$

$$\frac{d}{dx}\int_a^x f(t)dt = \frac{d}{dx}[F(x) - F(a)] = F'(x) = f(x)$$

Area between curves : (for region where $g > f$)

$$A = \int_a^b [g(x) - f(x)]dx$$

Area between curves : (for region where $g^{-1}(y)$ is to the right of $f^{1}(y)$)

$$A = \int_{y_1}^{y_2} \left[g^{-1}(y) - f^{-1}(y)\right] dy$$

Shells: (revolving around *y*-axis) $V = 2\pi \int_a^b x\left[g(x) - f(x)\right] dx$ (for region where $g > f$)

Washers: (revolving around *y*-axis) $V = \pi \int_{y_1}^{y_2} \left[(g^{-1}(y))^2 - (f^{-1}(y))^2\right] dy$ (for region where $g^{-1}(y)$ is to the right of $f^{-1}(y)$)

Recall Formulae **(A) – (L)**

$$\frac{d}{dx} \ln x = \frac{d}{dx} \int_1^x \frac{1}{t} dt = \frac{1}{x} \qquad \textbf{(A)}$$

$$\frac{d}{dx} \ln(u(x)) = \frac{d}{du} \ln u \frac{du}{dx} = \frac{1}{u(x)} u'(x) \qquad \textbf{(B)}$$

$$\int \frac{1}{x} dx = \ln|x| + C \qquad \int_a^b \frac{1}{x} dx = \ln|x| \Big|_a^b = \ln|b| - \ln|a| = \ln\left|\frac{b}{a}\right| \qquad \textbf{(C)}$$

$$\int \frac{1}{u} du = \ln|u| + C \qquad \int_{u(a)}^{u(b)} \frac{1}{u} du = \ln|u| \Big|_{u(a)}^{u(b)} = \ln|u(b)| - \ln|u(a)| = \ln\left|\frac{u(b)}{u(a)}\right| \qquad \textbf{(D)}$$

$$\frac{d}{dx} e^x = e^x \qquad \textbf{(E)} \qquad \frac{d}{dx} e^{u(x)} = e^{u(x)} \frac{d}{dx} u = e^u u' \qquad \textbf{(F)}$$

$$\int e^x dx = e^x + C \qquad \int_a^b e^x dx = e^x \Big|_a^b = e^b - e^a \qquad \textbf{(G)}$$

$$\int e^u du = e^u + C \qquad \int_{u(a)}^{u(b)} e^u du = e^u \Big|_{u(a)}^{u(b)} = e^{u(b)} - e^{u(a)} \qquad \textbf{(H)}$$

$$\frac{d}{dx} a^x = a^x \ln a = (\ln a) a^x \qquad \textbf{(I)}$$

$$\frac{d}{dx}a^{u(x)} = (\ln a)a^{u(x)}u'(x) \qquad \textbf{(J)}$$

$$\int a^x dx = \frac{1}{\ln a}a^x + C \qquad \int_c^d a^x dx = \frac{1}{\ln a}a^x \Big|_c^d = \frac{1}{\ln a}\left(a^d - a^c\right) \qquad \textbf{(K)}$$

$$\int a^u du = \frac{1}{\ln a}a^u + C \qquad \int_{u(c)}^{u(d)} a^u du = \frac{1}{\ln a}a^u \Big|_{u(c)}^{u(d)} = \frac{1}{\ln a}\left(a^{u(d)} - a^{u(c)}\right) \qquad \textbf{(L)}$$