# Formula Sheet-ExamI Math 261

- **Algebraic formulae:**

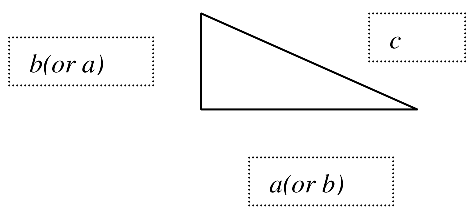
**Rational exponents:** $x^{\frac{p}{q}} \equiv \sqrt[q]{x^p} = \left(\sqrt[q]{x}\right)^p$

**(Difference between two cubes)** $x^3 \pm y^3 = (x \pm y)(x^2 \mp xy + y^2)$

**Rationalizing denominator/numerator makes use of the "trick of 1": For example, for any functions *u*(*x*), *v*(*x*), *r*(*x*) :**

$$\frac{\sqrt{u(x)} \pm \sqrt{v(x)}}{r(x)} = \frac{\sqrt{u(x)} \pm \sqrt{v(x)}}{r(x)} \cdot \left(\frac{\sqrt{u(x)} \mp \sqrt{v(x)}}{\sqrt{u(x)} \mp \sqrt{v(x)}}\right) = \ldots\text{(etc.)}$$

**Pythagorean Formula:**



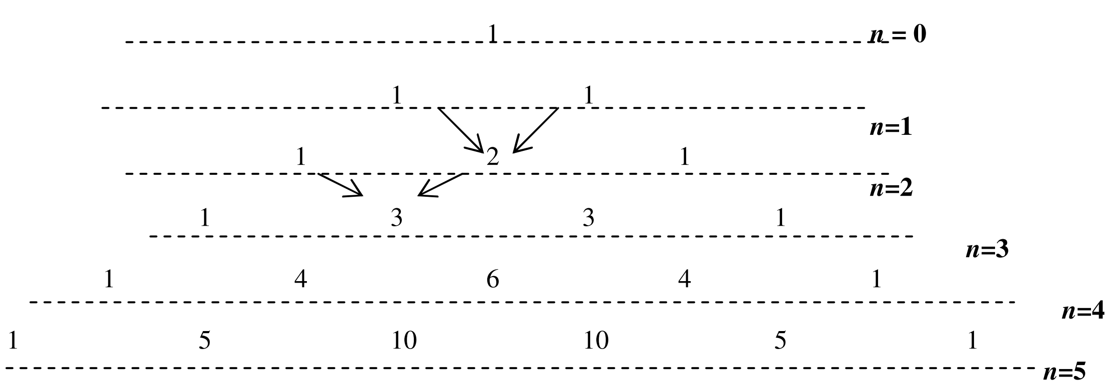

$$c^2 = a^2 + b^2$$

**Volumes:** Circular cylinder: $V = \pi r^2 h$ Sphere: $V = \frac{4}{3}\pi r^3$

**Binomial Theorem**

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k = \binom{n}{0} x^{n-0} y^0 + \binom{n}{1} x^{n-1} y^1 + \ldots + \binom{n}{n-1} x^{n-(n-1)} y^{n-1} + \binom{n}{n} x^{n-n} y^n$$

The Binomial coefficients: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ are tedious to evaluate from the formula alone. A useful shortcut is ***Pascal's Triangle***:



- **Absolute value function: for any function $u(x)$:**

$$|u(x)| = \begin{cases} u(x) & all\_x : u(x) \geq 0 \\ -u(x) & otherwise(all\_x : u(x) < 0 \end{cases}$$

- **Limits**

Note that a ***necessary condition*** for a limit to exist at any point is for its right and left hand limits to agree at the same value. That is to say, *if* $\lim_{x\to c} f(x)$ exists, *then* $\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x)$ .

Note that the above also hold in case of infinity limits! I.e., $\lim_{x\to c} f(x) = \pm\infty$ *t hen* $\lim_{x\to c^+} f(x) = \pm\infty = \lim_{x\to c^-} f(x)$ . On the other hand, *if* $\lim_{x\to c^+} f(x) \neq \lim_{x\to c^-} f(x)$, *then* $\lim_{x\to c} f(x)$ DNE (does not exist)

- **Continuity**

A function is continuous on a closed interval $[a , b]$ provided:

a.) For any $c \in (a , b)$ $f(c)$ is <u>finite</u> and $\lim_{x\to c} f(x) = f(c)$
b.) $\lim_{x\to a^+} f(x) = f(a)$ and $\lim_{x\to b^-} f(x) = f(b)$

- **Definition of Derivative**

$$f'(c) = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x} = \lim_{x\to c} \frac{f(x) - f(x)}{x - c}$$

- **Derivative Formulae:**

***<u>Const. times a function rule & addition/subtraction rule:</u>***

$$\frac{d}{dx}\left(af(x) \pm bg(x)\right) = a\frac{df}{dx} \pm b\frac{dg}{dx} = af'(x) \pm bg'(x)$$

*<u>Power Rule</u>* <u>(**Note**: $q$ is any rational number, i.e. $q = {}^{n}/_{m}$ )</u>

$$\frac{d}{dx}x^q = qx^{(q-1)}$$

*<u>Product Rule</u>*

$$\frac{d}{dx}\left(f(x)g(x)\right) = \left(\frac{df}{dx}\right)g(x) + f(x)\left(\frac{dg}{dx}\right) = f'(x)g(x) + f(x)g'(x)$$

*<u>Quotient Rule</u>*

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g(x)}{\left(g(x)\right)^2}$$

*<u>Chain Rule</u>*

$$\frac{d}{dx} f(g(x)) = f'(g)g'(x) = \frac{df}{dg}\frac{dg}{dx}$$

- A **critical point *c* is a point where either $f'(c) = 0$ or $f'(c)$ fails to exist.**
- **FDT (First derivative test)**

| ***Step*** | ***Procedure*** |
|---|---|
| 1 | **Calculate** $f(a)$ and $f(b)$ (i.e. determine the *y* values at the endpoints) |
| 2 | **Find all** critical points *c* (all points *c* where $f'(c) = 0$ or $f'(c)$ DNE) |
| 3 | **Sketch a sign chart for** $f'$ to determine which (if any) of the critical points obtained in 2. above are *local* (relative) *maxima or minima.* |
| 4a) | Find the *absolute maxima* **by maximizing over the set of the y-values consisting of *f* 's endpoints as well as *f* 's local maxima**. That is to say: find: $\max\{f(a), f(b), f(c)\}$, where *c* is a *local* maximum. |
| 4b) | Find the ***absolute minima*** **by minimizing over the set of the y-values consisting of *f* 's endpoints as well as *f* 's local minima.** That is to say: find: $\min\{f(a), f(b), f(c)\}$, where *c* is a *local* minimum. |

Suppose *c* is a **Type-1** critical point (i.e., $f'(c) = 0$ ). Then:

a) *c* is a **local maximum** if $f''(c) < 0$ (curvature is concave down at *c*)
b) *c* is a **local minimum** if $f''(c) > 0$ (curvature is concave up at *c*)
*c)* **Test fails** if $f''(c) = 0$ (other tests like the FDT need to be invoked to determine if *c* is a saddle point (special case of an inflection point, a point where the curvature changes) or otherwise.

- **Mean Value Theorem (MVT)**

If *f* is differentiable on $[a, b]$ and $f(a) = f(b)$ then there exists (at least one) point $c \in [a, b]$ such that: $f'(c) = \dfrac{f(b) - f(a)}{b - a}$