MA261 : Assignment III KEY

Ib.) i) $y(t) = t^2\sin t + 2t\cos t - 2\sin t$

(2pts) $y'(t) = \frac{d}{dt}(t^2\sin t + 2t\cos t - 2\sin t) = 2t\cancel{\sin t} + t^2\cos t + 2\cancel{\cos t} - 2\cancel{t}\sin t - 2\cancel{\cos t}$

$= t^2\cos t + 2\sin t\,(t - t) = t^2\cos t$

Check: $\int t^2\cos t\,dt$:

u		dv
t^2	$+$	$\cos t$
$2t$	$-$	$\sin t$
2	$+$	$-\cos t$
0		$-\sin t$

$\Rightarrow t^2\sin t + 2t\cos t - 2\sin t + C$

(where $C = 0$ in this particular case)

I.b.ii) (2pts) $y = \frac{\sin\theta}{1-\cos\theta} \Rightarrow y' = \frac{d}{d\theta}\left(\frac{\sin\theta}{1-\cos\theta}\right) = \frac{(1-\cos\theta)\cos\theta - \sin\theta(\sin\theta)}{(1-\cos\theta)^2}$

$= \frac{\cos\theta - \cos^2\theta - \sin^2\theta}{(1-\cos\theta)^2} = \frac{\cos\theta - 1}{(1-\cos\theta)^2} = \frac{\cos\theta - 1}{(1-\cos\theta)^2}$

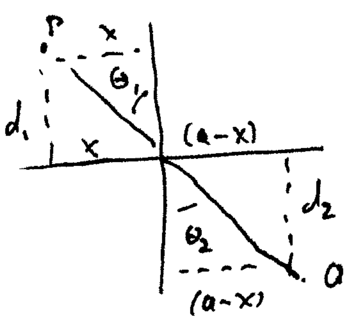
I.b.iii) (2pts) a) $y = \sin x \Rightarrow y' = \frac{d}{dx}\sin x = \cos x \Rightarrow m_t = y'(0) = \cos 0 = 1$

The number of complete cycles of $\sin x$ in $[0, 2\pi] = 1$ as well

b) $y = \sin 2x \Rightarrow y' = \frac{d}{dx}\sin 2x = 2\cos 2x \Rightarrow m_t = y'(0) = 2\cos 0 = 2$

The number of complete cycles of $\sin 2x$ in $[0, 2\pi] = 2$ as well

I.b.iv.) (4PTS) :



SNELL'S LAW: $n_1\sin\theta_1 = n_2\sin\theta_2$

where $n_1 = \frac{c}{v_1}$, $n_2 = \frac{c}{v_2} \Rightarrow n_1\sin\theta_1 = n_2\sin\theta_2$

$\frac{c}{v_1}\sin\theta_1 = \frac{c}{v_2}\sin\theta_2$

$\Rightarrow \frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$

$T(x) = T_1(x) + T_2(x)$ = total time

$T(x) = \frac{\sqrt{d_1^2 + x^2}}{v_1} + \frac{\sqrt{d_2 + (a-x)^2}}{v_2}$

$\frac{d}{dx}T(x) = \frac{1}{v_1}\frac{d}{dx}(d_1^2 + x^2)^{1/2} + \frac{1}{v_2}\frac{d}{dx}(d_2 + (a-x)^2)^{1/2}$

$= \frac{1}{v_1}\cdot\frac{1}{2}(d_1^2 + x^2)^{-1/2}(2x) + \frac{1}{v_2}\cdot\frac{1}{2}\{d_2 + (a-x)^2\}^{-1/2}(-2(a-x))$

$= \frac{x}{v_1(d_1^2 + x^2)^{1/2}} - \frac{(a-x)}{v_2[d_2 + (a-x)^2]^{1/2}}$

$= \frac{1}{v_1}\sin\theta_1 - \frac{1}{v_2}\sin\theta_2$ (from above diagram) $= 0$ (according to Snell's Law)

I.C) From Dartmouth Problem Set (10pts) 2

(2pts) 1.) $y = \sin(7\sqrt{x}) \Rightarrow y' = \frac{d}{dx}\sin(7x^{1/2}) = 7\cos(7x^{1/2}) \cdot \frac{1}{2}x^{-1/2} = \frac{7\cos(7\sqrt{x})}{\sqrt{x}}$

(2pts) 2.) $y = \cos(2\sin x)$. Method 1: $y = \cos^2(\sin x) - \sin^2(\sin x)$

$$y' = \frac{d}{dx}\left(\cos^2(\sin x) - \sin^2(\sin x)\right) = 2\cos(\sin x)\left[-\sin(\sin x)\cos x\right] - 2\sin(\sin x)\left[\cos(\sin x)\cos x\right]$$

$$= -2\cos(\sin x)\sin(\sin x)\cos x - 2\sin(\sin x)\cos(\sin x)\cos x$$

$$= -4\cos(\sin x)\sin(\sin x)\cos x$$

(2pts.) 3.) $y = 8\csc(10x^{-1})$

$$y' = \frac{d}{dx}\left(8\csc(10x^{-1})\right) = 8\frac{d}{dx}\csc(10/x) = -8\csc(10x^{-1})\cot(10x^{-1})(-10x^{-2})$$

$$= (+80x^{-2})\csc(10x^{-1})\cot(10x^{-1}) = +\frac{80\csc(10/x)\cot(10/x)}{x^2}$$

(Answer B.)

(2pts.) 4.) $y = \tan(-7x)\cot(-7x)$

Method 1: $y = \tan(-7x)\cot(-7x) = \frac{\sin(-7x)}{\cos(-7x)} \cdot \frac{\cos(-7x)}{\sin(-7x)} = \frac{-\cancel{\sin(7x)}}{\cancel{\cos(7x)}}\frac{\cancel{\cos(7x)}}{-\cancel{\sin(7x)}}$

$= 1 \Rightarrow y' = \frac{d}{dx}1 = 0$

Method 2: $\frac{dy}{dx} = \frac{d}{dx}\left(\tan(-7x)\cot(-7x)\right) = -7\sec^2(-7x)\cot(-7x) + 7\csc^2(-7x)\tan(-7x)$

$$= -7\left[\sec^2(-7x)\cot(-7x) - \csc^2(-7x)\tan(-7x)\right]$$

$$= -7\left[\frac{\cos(-7x)}{\sin(-7x)} \cdot \frac{1}{\cos^2(-7x)} - \frac{1}{\sin^2(-7x)}\frac{\sin(-7x)}{\cos(-7x)}\right]$$

$$= -7\left[\frac{1}{\sin(-7x)\cos(-7x)} - \frac{1}{\sin(-7x)\cos(-7x)}\right]$$

$$= -7\left[\frac{-1}{\sin 7x\cos 7x} + \frac{1}{\sin 7x\cos 7x}\right] = 0$$

(2pts) 5. $f(t) = 10t\cos t - 10\sin t = 10(t\cos t - \sin t)$

$f'(t) = \frac{d}{dt} 10(t\cos t - \sin t) = 10\frac{d}{dt}(t\cos t - \sin t) = 10[\cos t - t\sin t - \cos t]$

$= -10t\sin t$

Check: $\int f'(t)dt = -10\int \underset{u}{t}\,\underset{dv}{\sin t} = -10\{-t\cos t + \int \cos t\,dt\} = -10\{-t\cos t + \sin t\}$

$= 10(t\cos t - \sin t)$

(2pts.) 6. $g(t) = \sqrt{\frac{3\cos t}{t}} = \left[\frac{3\cos t}{t}\right]^{1/2}$

Method 1: $g'(t) = \frac{d}{dt}\left[\frac{3\cos t}{t}\right]^{1/2} = \frac{1}{2}\left[\frac{3\cos t}{t}\right]^{-1/2}\left\{\frac{-3t\sin t - 3\cos t}{t^2}\right\}$

$= -\frac{3}{2}\frac{t^{1/2}}{(3\cos t)^{1/2}} \cdot \frac{t\sin t + \cos t}{t^2} = -\frac{3(\sin t \cdot t + \cos t)}{2\sqrt{3}\sqrt{\cos t}\, t^{3/2}}$

$= \frac{-3(t\sin t + \cos t)}{2\sqrt{3t^3\cos t}}$

Method 2: $g(t) = \frac{\sqrt{3\cos t}}{\sqrt{t}} = (3\cos t)^{1/2} t^{-1/2}$

$g'(t) = \frac{1}{2}(3\cos t)^{-1/2}(-3\sin t)t^{-1/2} - \frac{1}{2}t^{-3/2}(3\cos t)^{1/2}$

$= \frac{-3(\sin t \cdot t + \cos t)}{2\sqrt{t^3}\sqrt{\cos t}\sqrt{3}} = \frac{-3(t\sin t + \cos t)}{2\sqrt{3t^3\cos t}}$

(2pts. 7.) $f(t) = \cos(\tan(\sin t))$

$f'(t) = \frac{d}{dt}\cos(\tan(\sin t)) = -\sin(\tan(\sin t)) \cdot \sec^2(\sin t) \cdot \cos t$

$= -\frac{\sin(\tan(\sin t))\cos t}{\cos^2(\sin t)}$

(2 pts.) 8 $y = \cos(7x + \frac{\pi}{2})$ $y' = \frac{d}{dx}\cos(7x + \pi/2) = -\sin(7x + \pi/2)\cdot 7 = -7\sin(7x + \pi/2)$ (4.)

$P(0,0) \Rightarrow y'(0) = -7\sin(7 0 + \pi/2) = -7\sin\pi/2 = -7 = m_t$

$y - y_0 = m_t(x - x_0) \Rightarrow y - 0 = -7(x - 0) \Rightarrow y_t = -7x$

$m_n = -\frac{1}{m_t} = -\frac{1}{-7} = \frac{1}{7} \Rightarrow y_n = \frac{1}{7}x$

(2pts) 9 $y = \sin(2x^2)$

$y' = \frac{d}{dx}\sin(2x^2) = \cos(2x^2)\cdot 4x = 4x\cos(2x^2)$

$y'' = \frac{d}{dx}y' = \frac{d}{dx}[4x\cos(2x^2)] = 4\frac{d}{dx}[x\cos(2x^2)] = 4[\cos(2x^2) - x\sin(2x^2)\cdot 4x]$

$= 4[\cos(2x^2) - 4x^2\sin(2x^2)] = 4\cos(2x^2) - 16x^2\sin(2x^2)$

$y''' = \frac{d}{dx}[4\cos(2x^2) - 16x^2\sin(2x^2)] = -8x\sin(2x^2) - 32x\sin(2x^2)$
$- 16x^2\cos(2x^2)(4x)]$

$= -40x\sin(2x^2) - 64x^3\cos(2x^2) = -4x[10\sin(2x^2) + 16x^2\cos(2x^2)]$

$= -8x[5\sin(2x^2) + 8x^2\cos(2x^2)]$

2pts 10 $y = \tan(-9x)$

$y' = -9\sec^2(-9x)$

$y'' = -18\sec(-9x)\tan(-9x)(-9) = 162\sec(-9x)\tan(-9x)$

$= 162\sec(-9x)y$ Recall: $\sec^2\theta = 1 + \tan^2\theta \Rightarrow \sec\theta = \sqrt{1+\tan^2\theta}$

$\boxed{y'' = 162\sqrt{1+y^2}\,y}$

2pts, 11 $f(x) = \sec x$ $g(x) = f'(x) \Rightarrow g'(x) = f''(x) = \frac{d^2}{dx^2}(\sec x)$

$= \frac{d}{dx}\left(\frac{d}{dx}\sec x\right) = \frac{d}{dx}(\sec x\tan x) = (\sec x\tan x)\tan x + \sec x\cdot\sec^2 x$

$= \sec x \tan^2 x + \sec^3 x = \sec x [\tan^2 x + \sec^2 x] = \sec^2 [\sec^2 x - 1 + \sec^2 x]$

$= \sec x [2\sec^2 x - 1]$

(2pts) #12 : $h(t) = 19 \sin^2 t \Rightarrow h'(t) = \frac{d}{dt}(19 \sin^2 t) = 19 \frac{d}{dt} \sin^2 t$

$= 19 \cdot 2\sin t \cos t = 38 \sin t \cos t = 19 \cdot 2 \sin t \cos t = 19 \sin 2t$

(2pts) #13: $f(x) = 19 \cos(\cos(6x)) \Rightarrow f'(t) = \frac{d}{dx}(19 \cos(\cos(6x))$

$= 19 \frac{d}{dx} \cos(\cos(6x)) = 19 - \sin(\cos(6x))(-\sin(6x)) \cdot 6$

$= -114 \sin(\cos(6x)) \sin(6x)$

$f'(\pi/2) = 114 \sin(\cos 3\pi) \sin 3\pi = 0$

(2pts) #14 $h(x) = \sqrt{\cos^2(5x) + \sin^2(5x)}$

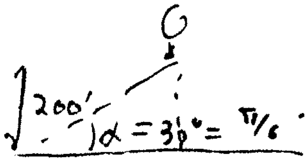
Method 1: $\cos^2 5x + \sin^2 5x = 1 \Rightarrow 1 = h(x) \Rightarrow h'(x) = 0$

Method 2: $h'(x) = \frac{d}{dx}(\cos^2 5x + \sin^2 5x)^{1/2} = \frac{1}{2}(\cos^2 5x + \sin^2 5x) \cdot$

$\{2\cos 5x(-\sin 5x)5 + 2\sin 5x \cos 5x\}$

$= \frac{-10 \cos 5 \sin 5x + 10 \cos 5x \sin 5x}{\sqrt{\cos^2 5x + \sin^2 5x}} = \frac{0}{\sqrt{\ldots}} = \underline{0}$

II.) a) (5)



$s(t) = 200 - 16t^2$

$\tan\alpha = h/x \Rightarrow x = h\cot\alpha \Rightarrow x(t) = s(t)\cot\pi/6 = (200-16t^2)\sqrt{3}$

$\Rightarrow \dot{x}(t) = \frac{d}{dt}(200t - 16t^2)\sqrt{3} = \sqrt{3}(200-32t)$

When $h = 125 = 200 - 16t^2 \Rightarrow 75 = 16t^2 \Rightarrow t = \sqrt{\frac{75}{16}} = \frac{5}{4}\sqrt{3}$ sec

$\therefore x'(t_0) = \sqrt{3}(200 - 32\cdot\frac{5}{4}\sqrt{3}) = \sqrt{3}(200-40\sqrt{3}) = 40\sqrt{3}(5-\sqrt{3})$

b) (2pts) $\int(\theta^2 + \sec^2\theta)d\theta = \frac{1}{3}\theta^3 + \tan\theta + C$

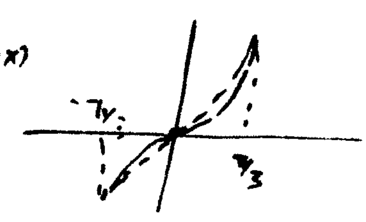
Check: $\frac{d}{d\theta}\left(\frac{1}{3}\theta^3 + \tan\theta + C\right) = \theta^2 + \sec^2\theta$

c) (3pts) $\int_0^{\pi/4}(\sec x)^3(\sec x\tan x\,dx)$

$u = \sec x \Rightarrow du = \sec x\tan x$

$= \int_{u(0)}^{u(\pi/4)} u^3 du = \int_1^{\sqrt{2}} u^3 du = \frac{1}{4}u^4\Big|_1^{\sqrt{2}} = \frac{1}{4}((2^{1/2})^4 - 1) = \frac{1}{4}(4-1) = \frac{3}{4}$

d.) (5pts) $f(x) = 2\sin x$ $\quad g(x) = \tan x$



$f(x) = g(x) \Rightarrow 2\sin x = \tan x \Rightarrow 2\sin x = \frac{\sin x}{\cos x} \Rightarrow \sin x(2 - \sec x) = 0$

$\Rightarrow x_1 = 0 \quad x_2 = \frac{\pi}{3} \quad x_3 = -\frac{\pi}{3}$

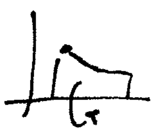
$\Rightarrow A = \int_{-\pi/3}^{0}(\tan x - 2\sin x)dx + \int_0^{\pi/3}(2\sin x - \tan x)dx = 2\int_0^{\pi/3}(2\sin x - \tan x)dx$

$$= 2\int_0^{\pi/3}(2\sin x\,dx) - 2\int_0^{\pi/3}\tan x\,dx = -4\cos x\Big|_0^{\pi/3} - 2\ln|\cos x|\Big|_0^{\pi/3}$$

$$= -2\left\{2\left[\tfrac{1}{2}-1\right] + \ln\left|\tfrac{1}{2}\right| - \ln 1\right\}$$

$$= -2\{-2 - \ln 2 + 0\} = \underline{4 + 2\ln 2}$$

e.) (5pts) $y_1 = \csc x \quad y_2 = 0 \quad x_1 = \pi/6 \quad x_2 = 5/6\,\pi$



$$V = \pi\int_{\pi/6}^{5/6\pi}(\csc x)^2\,dx = \pi\int_{\pi/6}^{5/6\pi}\csc^2 x\,dx = -\pi\cot x\Big|_{\pi/6}^{5/6\pi}$$

$$= -\pi\left\{\cot \tfrac{5}{6}\pi - \cot \tfrac{\pi}{6}\right\} = -\pi\left\{-\sqrt{3} - \sqrt{3}\right\} = 2\pi\sqrt{3}$$

f.) $\int \sin x \cos x\,dx$ __Method 1:__ $u = \sin x \quad du = \cos x\,dx \Rightarrow \int u\,du = \frac{1}{2}u^2 + C_1$

5pts $\quad = \frac{1}{2}\sin^2 x + C_1$

__Method 2:__ $u = \cos x \quad du = -\sin x\,du \Rightarrow -\int u\,du = -\frac{1}{2}\cos^2 x + C_2$

* Though the results __appear__ to be different, they're the same. Recall:

$\sin^2 x + \cos x^2 = 1 \Rightarrow \sin^2 x = 1 - \cos^2 x$ Hence C_2 absorbs the $\frac{1}{2}$ from the Pythagorean identity so that: $C_2 = C_1 + \frac{1}{2} \quad \therefore \quad \frac{1}{2}\sin^2 + C_1 = C_1 + \frac{1}{2} - \frac{1}{2}\cos^2 x$

$= C_1 + \frac{1}{2}(1 - \cos^2 x)$

III a) (3pts.) $f(x) = x\arctan 2x - \frac{1}{4}\ln(1+4x^2)$

$$f'(x) = \frac{d}{dx}\left(x\arctan 2x - \frac{1}{4}\ln(1+4x^2)\right)$$

$$= \arctan 2x + \cancel{x}\, 2\cdot\left(\frac{1}{1+(2x)^2}\right) - \frac{1}{4}\cdot\frac{8x}{1+4x^2}$$

$$= \arctan 2x + \frac{2}{1+4x^2} - \frac{2}{1+4x^2} = \arctan 2x$$

<u>Check:</u>

$$\int \underset{u}{\arctan 2x}\,\underset{dv}{dx} = \underset{v}{x}\,\underset{u}{\arctan 2x} - \int x\left(\frac{2}{1+(2x)^2}\right)dx$$

$$= x\arctan 2x - \int \frac{2x}{1+(4x^2)}\,dx$$

$u = 1+4x^2 \quad du = 8x\,dx$

$\therefore\ 2x\,dx = \frac{1}{4}du$

$$= x\arctan 2x - \frac{1}{4}\int\frac{du}{u}$$

$$= x\arctan 2x - \frac{1}{4}\ln u + C$$

$$= \underline{x\arctan 2x - \frac{1}{4}\ln(1+4x^2) + C} \quad ✓$$

60) (b.) (5pts) $f(x) = \operatorname{arccot} 2x$

$$f'(x) = \frac{d}{dx}\operatorname{arccot} 2x = \frac{2}{1+(2x)^2} = \frac{2}{1+4x^2}$$

$$f''(x) = \frac{d}{dx}\left(\frac{2}{1+4x^2}\right) = 2\frac{d}{dx}(1+4x^2)^{-1} = -2(1+4x^2)^{-2}(8x)$$

$$= \frac{-16x}{(1+4x^2)^2}$$

$$f''(x) = 0 \Rightarrow \frac{-16x}{(1+4x^2)^2} = 0 \Rightarrow x = 0 \quad \text{(inflection pt.)}$$

III.e) (2pts)

$$\int_0^{1/\sqrt{2}} \frac{\arccos x}{\sqrt{1-x^2}}\,dx \qquad u = \arccos x \qquad du = -\frac{dx}{\sqrt{1-x^2}}$$

$$\Rightarrow -\int_{u(0)}^{u(1/\sqrt{2})} u\,du = -\int_{\pi/2}^{\pi/4} u\,du = -\frac{1}{2}u^2\Big|_{\pi/2}^{\pi/4} = \frac{1}{2}u^2\Big|_{\pi/4}^{\pi/2} = \frac{1}{2}\left(\frac{\pi^2}{4}-\frac{\pi^2}{16}\right)$$

$$= \frac{1}{2}\left(\frac{3}{16}\pi^2\right) = \frac{3}{32}\pi^2$$

III.f) (3pts)

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\frac{d}{dx}\frac{1}{a}\arctan(u/a) = \frac{1}{a}\frac{d}{dx}\arctan\left(\frac{u(x)}{a}\right) = \frac{1}{a}\cdot\frac{1/a}{1+u^2/a^2}$$

$$= \frac{1}{a^2}\cdot\frac{1}{1+u^2/a^2} = \frac{1}{a^2+u^2} \checkmark$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a^2}\operatorname{arcsec}\frac{|u|}{a} + c$$

$$\frac{d}{dx}\frac{1}{a^2}\operatorname{arcsec}\frac{|u|}{a} = \frac{1}{a}\frac{d}{dx}\operatorname{arcsec}\frac{u}{a} = \frac{1}{a}\cdot\frac{\frac{1}{a}}{u/a\sqrt{u^2/a^2-1}}$$

$$= \frac{1}{a^2}\frac{1}{\frac{u}{a}\sqrt{\frac{u^2}{a^2}-1}} = \frac{1}{u\cdot\cancel{a}\sqrt{u^2/a^2-1}} = \frac{1}{u\sqrt{u^2-a^2}}$$

IV. a) $y = \ln(\cosh x)$ $\quad y' = \frac{d}{dx}\ln(\cosh x) = \frac{1}{\cosh x}\cdot \sinh x = \frac{\sinh x}{\cosh x} = \tanh x$

(3pts)

b) $y = x^{\cosh x} \Rightarrow \ln y = \cosh x \ln x \Rightarrow \frac{1}{y}\frac{dy}{dx} = \sinh x \cdot \ln x + \cosh x \cdot \frac{1}{x}$

(4pts)

$$\Rightarrow y' = y\left(\ln x \sinh x + \frac{1}{x}\cosh x\right)$$

$$= x^{\cosh x}\left(\ln x \sinh x + \frac{1}{x}\cosh x\right)$$

$$= x^{(\cosh x - 1)}\left[x \ln x \sinh x + \cosh x\right]$$

$$= \frac{y}{x}\left[x \ln x \sinh x + \cosh x\right]$$

c) (5pts) $\displaystyle\int \frac{dx}{(x+1)\sqrt{2x^2+4x+8}} = \frac{1}{\sqrt{2}}\int \frac{dx}{(x+1)\sqrt{x^2+2x+4}}$

$$= \frac{1}{\sqrt{2}}\int \frac{dx}{(x+1)\sqrt{x^2+2x+1-1+4}}$$

$$= \frac{1}{\sqrt{2}}\int \frac{dx}{(x+1)\sqrt{(x+1)^2+(\sqrt{3})^2}}$$

$u = x+1 \Rightarrow du = dx$

$$= \frac{1}{\sqrt{2}}\int \frac{du}{u\sqrt{u^2+(\sqrt{3})^2}} = \frac{-1}{\sqrt{2}}\cdot\frac{1}{\sqrt{3}}\ln\left(\frac{\sqrt{3}+\sqrt{3+u^2}}{|u|}\right) + C$$

$$= -\frac{1}{\sqrt{6}}\ln\left(\frac{\sqrt{3}+\sqrt{x^2+2x+4}}{|x+1|}\right) + C$$

d) (3pts) $\int \frac{-2x}{\sqrt{x^2-4}}\,dx$

$u = x^2-4 \quad du = 2x\,dx \Rightarrow \int \frac{-du}{u^{1/2}} = -\int u^{-1/2}du = -2u^{1/2}+c$

$= -2\sqrt{x^2-4} + c$

e) (5pts) a) $\int x\sqrt{4+x}\,dx$ (3pts)

$dv = \sqrt{4+x}\,dx \Rightarrow v = \int (4+x)^{1/2}\,dx = \int u^{1/2}du = \frac{2}{3}(4+x)^{3/2}$

$u = 4+x$

$u = x \quad du = dx$

$= \frac{2x}{3}(4+x)^{3/2} - \frac{2}{3}\int (4+x)^{3/2}dx = \frac{2}{3}x(4+x)^{3/2} - \frac{2}{3}\cdot\frac{2}{5}(x+4)^{5/2} + c$

$= \frac{2}{3}x(4+x)^{3/2} - \frac{4}{15}(4+x)^{5/2} + c = \frac{2}{3}(4+x)^{\frac{3}{2}}\left[x - \frac{2}{5}(4+x)\right] + c$

$= \frac{2}{3}(4+x)^{3/2}\left[\frac{3}{5}x - \frac{8}{5}\right] = \frac{8}{15}(4+x)^{3/2}(3x-8) + c$

(2pts) b) $\int x\sqrt{4+x}\,dx \qquad u = 4+x \qquad x = u-4$

$du = dx$

$\parallel$

$\int (u-4)u^{1/2}du = \int (u^{3/2} - 4u^{1/2})du = \frac{2}{5}u^{5/2} - \frac{8}{3}u^{\frac{3}{2}} + c$

$= 2u^{3/2}\left[\frac{1}{5}u - \frac{4}{3}\right] + c$

$= 2(4+x)^{3/2}\left[\frac{1}{5}(4+x) - \frac{4}{3}\right] + c$

$= 2(4+x)^{3/2}\left[\frac{4}{5} + \frac{x}{5} - \frac{4}{3}\right] + c$

$= \frac{8}{15}(4+x)^{3/2}\left[3x-8\right] + c$

f) (5pts) $\int \sec^n x\, dx = \int \underbrace{\sec^{n-2}x}_{u} \underbrace{\sec^2 x\, dx}_{dv}$ (12)

$u = \sec^{n-2}x$

$du = (n-2)\sec^{n-3}x\,(\sec x \tan x)dx$

$= (n-2)\sec^{n-2}x \tan x$

$dv = \sec^2 x dx$

$v = \tan x$

$= \cancel{(n-2)\sec}\ \underset{u}{\sec^{n-2}x}\,\underset{v}{\tan x} - \int \tan x\,(n-2)\sec^{n-2}x \tan x\, dx$

$= \sec^{n-2}x \tan x - (n-2)\int \sec^{n-2}x \tan^2 x\, dx$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$= \sec^{n-2}x \tan x - (n-2)\int \sec^{n-2}(\sec^2 x - 1)\, dx$

$= \sec^{n-2}x \tan x - (n-2)\int (\sec^n x - \sec^{n-2}x) dx$

$\int \sec^n x\, dx = \sec^{n-2}x \tan x - (n-2)\int \sec^n x\, dx + (n-2)\int \sec^{n-2}x\, dx$

$\therefore (n-1)\int \sec^n x\, dx = \sec^{n-2}x \tan x + (n-2)\int \sec^{n-2}x\, dx$

$\int \sec^n x\, dx = \frac{1}{(n-1)}\sec^{n-2}x \tan x + \frac{(n-2)}{(n-1)}\int \sec^{n-2}x\, dx$