Expanding Joseph Sneed's Analysis into Category Theory

William Kallfelz
May 16, 2006

Abstract

Joseph Sneed (1971) characterizes classical mechanics using set theory, which comprises part of his research programme of axiomatizing the logical structure of mathematical physics. "The set-theoretic predicate defined by the axiomatization characterizes the formal, mathematical structure associated with the theory. This predicate is used to make the empirical statements of the theory."(vii) However, in certain branches of contemporary physics, ranging from solid state to fluid dynamics, traditional formal schemes of reduction have been called into serious question (Batterman (2002, 2003, 2004)).

Here, I show how to extend Sneed's structuralist characterization to *category theory*. A category is a semigroup (i.e., a set with an associative product) containing enough identities (i.e., for every element in the category, there exists a left and a right hand multiplication identity.) Category theory provides a universal framework for much contemporary fields in pure and in applied mathematics, and gives important insights into the common features of structures that all mathematical systems share, in ways that set theory cannot.

Aside from extending Sneed's metatheoretic characterization into physical systems that were off-limits in his study (namely irreducibly probabilitistic theories like statistical and quantum mechanics), a categorical-theoretic modification of Sneed provides a far more robust and rigorous means to characterize intertheoretic reduction. This result holds because categorical-theoretic metatheoretical characterizations apply to physical systems characterized by mutlilinear algebras, in a manner analogous to the way set theoretic metatheoretical characterizations apply to phase-space formalisms, as in the case of classical mechanics. I have shown, however, that the former class of (multilinear algebraic) structures exhibit a regularizibility (i.e., an ability to overcome singularities) in ways that latter (phase-space based) formalisms cannot. Regular structures exhibit a natural means of reduction, since they can be algebraically expanded and contracted. Reduction occurs in the latter case, when the contracted structure is interpreted as the superseded theory, while the former is interpreted as the superseding theory.

I. Introduction: A Brief Overview of Category Theory

Category theory (CT) provides a unique insight into the general nature, or universal features of the construction process that practically all mathematical systems share, in one way or another. It simultaneously refines and subsumes set theory, in the following senses: Category theory provides a more detailed account for both the 'natural' transformations and their associated objects that arise in any mathematical system, than the language of sets and classes can capture. Set theory (ST) can be embedded into category theory, but not vice versa.[1]

Such basic universal features involved in the construction of mathematical systems, which category theory generalizes and systematizes, include, at base, the following:

Feature	**Underlying Notion**
Objects	The collection of primitive, or stipulated, entities of the mathematical system.
Product	How to 'concatenate and combine,' in a natural manner, to form new objects or entities in the mathematical system respecting the properties of what are characterized by the system's stipulated objects.
Morphsim	How to 'morph' from one object to another.
Isomorphism (structural equivalence)	How all such objects, relative to the system, are understood to be equivalent.

Table A.2

For an informal demonstration of how such general aspects are abstracted from three different mathematical systems (sets, groups, and topological spaces[2]), for instance, see Table A.1, Appendix A.1, below.

[1] A formal instantiation of a category is the category of all sets, for instance.

[2] Such systems, of course, are not conceptually disjunct: topological spaces and groups are defined, of course, in terms of sets (See Table A.1, Appendix A.1). The additional element of structure comprising the concept of group includes the notion of a binary operation (which itself can be defined set-theoretically in terms of a *mapping*) sharing the algebraic property of associativity. The structural element distinguishing a topological space is also described, set-theoretically by use of notions of 'open' sets. Moreover, groups and topological spaces can conceptually overlap as well, in the notion of a *topological group*. So in an obvious sense, set theory remains a general classification language for mathematical systems as well. However, the *expressive power* of set theory (ST) pales in comparison to that of category theory (CT). Or to put it

A *category* is defined as follows:

- **Defn. I.1:** A category $\mathrm{C} = \langle \Omega, \mathrm{Mor}(\Omega), \circ \rangle$ is the ordered triple where:
  a.) Ω is the class of C's *objects.*
  b.) $\mathrm{Mor}(\Omega)$ is the set of *morphisms* defined on Ω. Graphically, this can be depicted (where $\varphi \in \mathrm{Mor}(\Omega)$, $A \in \Omega$, $B \in \Omega$): $A \xrightarrow{\varphi} B$
  c.) The elements of $\mathrm{Mor}(\Omega)$ are connected by the *product* $\circ$ which obeys the law of composition: For $A \in \Omega$, $B \in \Omega$, $C \in \Omega$: if φ is the morphism from A to B, and if ψ is a morphism from B to C, then $\psi \circ \varphi$ is a morphism from A to C, denoted graphically: $A \xrightarrow{\varphi} B \circ B \xrightarrow{\psi} C = A \xrightarrow{\psi\circ\varphi} C$. Furthermore:
    c.1) $\circ$ is *associative*: For any morphisms ϕ, φ, ψ with product defined in as in c.) above, then: $(\psi \circ \phi) \circ \varphi = \psi \circ (\phi \circ \varphi) \equiv \psi \circ \phi \circ \varphi$.
    c.2) Every morphism is equipped with a left and a right identity. That is, if ψ is any morphism from A to B, (where A and B are any two objects) then there exists the (right) *identity* morphism on A (denoted ι_A) such that: $\psi \circ \iota_A = \psi$. Furthermore, for any object C, if φ is any morphism from C to A, then there exists the (left) *identity* morphism on A (ι_A) such that: $\iota_A \circ \varphi = \varphi$. Graphically, the left (or right) identity morphisms can be depicted as *loops.*[3]

There is a virtually limitless variety of examples of mathematical structures classifiable as categories. For instance, Table A.1 in Appendix A.1 indicates that:

a) The category Set consists in an object class Ω which is the class of all sets, whose morphisms $\mathrm{Mor}(\Omega)$ is the set of all mappings on the class Ω, and $\circ$ is the usual associative law of composition between mappings.
b) The category Gr consists in an object class Ω which is the class of all groups, whose morphisms $\mathrm{Mor}(\Omega)$ is the set of all group homomorphisms.

---

another way, if category theory and set theory are conceived of as deductive systems (Lewis), it could be argued that (CT) a better combination of "strength and simplicity" (Carroll (2003, 3) than ST. Admittedly, however, this is not a point which can be easily resolved, as far as the simplicity issue goes, since the very concept of a category is usually cashed out in terms three fundamental notions (objects, morphisms, associative composition), whereas, at least in the case of 'naïve' set theory (NST), we have fundamentally the two notions: a) of membership $\in$ defined by extension, and b) the hierarchy of *types* (i.e., for any set X, $X \subseteq X$, but $X \notin X$. Or to put more generally, $Z \in W$ is a meaningful expression, though it may be false, provided, for any set, X: $Z \in \wp^{(k)}(X)$ and $W \in \wp^{(k+1)}(X)$, where k is any non-negative integer, and $\wp^{(k)}(X)$ defines the kth-level power-set operation, i.e.: $\wp^{(m)}(X) = \wp(\wp(\ldots k \text{ times} \ldots(X)\ldots)$.

[3] Since identities are defined for every object, one can, in principle identify each object with its associated (left/right) identity. Formally, for any morphism φ from A to B, with associated left/right identities ι_B, ι_A, identify: $\iota_B = \lambda$, $\iota_A = \rho$. Hence condition c2) above can be re-stated as c2$'$): "For every φ there exist λ, ρ such that: $\lambda \circ \varphi = \varphi$, and $\varphi \circ \rho = \varphi$." Algebraically, then with this apparent identification, DefnI.1 is coextensive with that of a "semigroup with enough identities," i.e. that is to say, a set S with an associative binary operation obeying c2$'$). Nevertheless, it is helpful to conventionally define a category, explicitly via its object set, since most applications involving a particular mathematical system require this.

c) The category TOP consists in an object class Ω which is the class of all topological spaces, whose morphisms MOR(Ω) is the set of all continuous mappings.

Other examples include (but obviously are not limited to): POS (the set of all partially ordered sets whose morphisms are monotone functions[4]), BOOL (the set of all Boolean algebras whose morphisms are the Boolean homomorphisms[5]), AUT (the set of all automata and their associated transition maps), etc.[6] In one instance, most especially relevant to this discussion, is Robert Geroch (1985) derives, from a purely categorical-theoretic basis, *all* relevant structures[7] used in mathematical physics.

An especially attractive feature of any category $\mathbb{C}$ is that it is 'well-balanced,' i.e., it admits a natural *duality*. "[I]n category theory the 'two for the price of one' principle holds: every concept is two concepts, and every result is two results." (Adamek, et. al (1990), 4). The *dual* is defined as follows:

- **Defn I.2.** For any category $\mathbb{C} = \langle \Omega, \text{MOR}(\Omega), \circ \rangle$ its *dual* (or *opposite*) category $\mathbb{C}^* = \langle \Omega, \text{MOR}^*(\Omega), \circ^* \rangle$ is defined such that:
  a.) For any $A \in \Omega$, $B \in \Omega$, $\varphi \in \text{MOR}(\Omega)$, such that φ is a morphism from A to B, then $\varphi^* \in \text{MOR}^*(\Omega)$ is the *dual morphism* from B to A. Graphically: $A \xleftarrow{\varphi^*} B$
  b) For any $\varphi \in \text{MOR}(\Omega)$, $\psi \in \text{MOR}(\Omega)$, with product defined in DefnI.1c), its dual product is defined by: $\psi \circ^* \varphi = \varphi \circ \psi$. Thus we may write: $(\psi \circ \varphi)^* = \varphi^* \circ^* \psi^*$.
  Graphically, if $A \xrightarrow{\varphi} B \circ B \xrightarrow{\psi} C = A \xrightarrow{\psi \circ \varphi} C$, then:
  $A \xleftarrow{\varphi^*} B \circ^* B \xleftarrow{\psi^*} C = A \xleftarrow{\varphi^* \circ^* \psi^* = (\psi \circ \varphi)^*} C$

[4] A partially ordered set is a set S with a partial ordering relation denoted by symbol: $\angle$ (i.e. a relation which is **reflexive**: for all $x \in S$, $x \angle x$, **antisymmetric**: for all $x \in S$, $y \in S$, $x \angle y$ & $y \angle x$ implies $y = x$, **transitive**: for all $x \in S$, $y \in S$, $z \in S$: $x \angle y$ & $y \angle z$ implies $x \angle z$). A **monotone function** is a mapping f on S respecting the partial ordering relation, i.e. if $x \angle y$ then $f(x) \angle f(y)$.

[5] I.e, the structure-preserving maps f respecting the Boolean "sum" ($\vee$) and "product" ($\wedge$). In other words, any mapping on Boolean elements x,y whereby: $f(x \vee y) = f(x) \vee f(y)$, $f(x \wedge y) = f(x) \wedge f(y)$.

[6] For more details, see Herrlich & Strecker (1979), 14-15 , as well as Adamek (1990), pp 16-17.

[7] Algebraic: pp. 1-133, (chapters 1-24) and topological: pp. 134-329 (chapters 25-56).

So, in a graphical sense, the dual $\mathtt{C}^*$ is simply $\mathtt{C}$'s 'mirror image,' or 'parity inverse,' since the action of the dual reverses the direction of all the arrows (i.e. the directed edges in its graphical representation.)

Similar to naïve set theory (NST) Category theory also preserves its form and structure on any level or category 'type.' That is to say, any two (or more) categories $\mathtt{C}$, $\mathtt{D}$ can be part of the set of structured objects of a *meta-category* **X** whose morphisms (functors) respect the categorical structure of its arguments $\mathtt{C}$, $\mathtt{D}$. That is to say:

- **Defn I.3.** Given two categories $\mathtt{C} = \langle \Omega, \mathrm{Mor}(\Omega), \circ \rangle$, $\mathtt{D} = \langle \Omega', \mathrm{Mor}(\Omega'), \bullet \rangle$, a *categorical functor* **Φ** is a morphism in the *meta-category* **X** from objects $\mathtt{C}$ to $\mathtt{D}$ assigning each $\mathtt{C}$-object (in Ω) a $\mathtt{D}$-object (in Ω') and each $\mathtt{C}$-morphism (in $\mathrm{Mor}(\Omega)$) a $\mathtt{D}$-morphism (in $\mathrm{Mor}(\Omega')$) such that:
  a.) **Φ** preserves the 'product' (compositional) structure of the two categories, i.e., for any $\varphi \in \mathrm{Mor}(\Omega)$, $\psi \in \mathrm{Mor}(\Omega)$: $\mathbf{\Phi}(\varphi \circ \psi) = \mathbf{\Phi}(\varphi) \bullet \mathbf{\Phi}(\psi) \equiv \varphi' \bullet \psi'$ (where φ', ψ' are the **Φ-images** in $\mathtt{D}$ of the functors φ, ψ in $\mathtt{C}$.
  b.) **Φ** preserves identity structure across all categories. That is to say, for any $A \in \Omega$, $\iota_A \in \mathrm{Mor}(\Omega)$, $\mathbf{\Phi}(\iota_A) = \iota_{\mathbf{\Phi}(A)} = \iota_{A'}$ where A' is the $\mathtt{D}$-object (in Ω') assigned by **Φ**. (I.e., $A' = \mathbf{\Phi}(A)$)

Examples of functors include the 'forgetful functor' **For**: $\mathtt{C} \rightarrow \mathrm{Set}$ (where Set is the category of all sets) which has the effect of 'stripping off' any extra structure in a mathematical system $\mathtt{C}$ down to its 'bare-bones' set-structure only. That is to say, for any $\mathtt{C}$-object $A \in \Omega$, $\mathbf{For}(A) = S_A$ (where S_A is A's underlying set), and for any $\psi \in \mathrm{Mor}(\Omega)$: $\mathbf{For}(\psi) = f$ is just the mapping (or functional) property of ψ. Moreover, within the category Set the (covariant) power-set functor is defined $\wp$: Set→Set, such that, for sets U, V and mapping f: $P\left(U \xrightarrow{f} V\right) = P(U) \xrightarrow{P(f)} P(V)$ where $P(U)$, $P(V)$ are the power-sets of U and V, and $P(f(X))$ is the image of set X under f, i.e. $f(X)$. Robert Geroch (1985), for example, builds up the toolchest of the most important mathematical structures applied in physics, via a combination of (partially forgetful[8]) and (free construction functors.) Part of this toolchest, for example, is suggested in the diagram below. The boxed items represent the categories, the solid arrows are the (partially) forgetful functors, and the dashed arrows represent the free construction functors.[9]

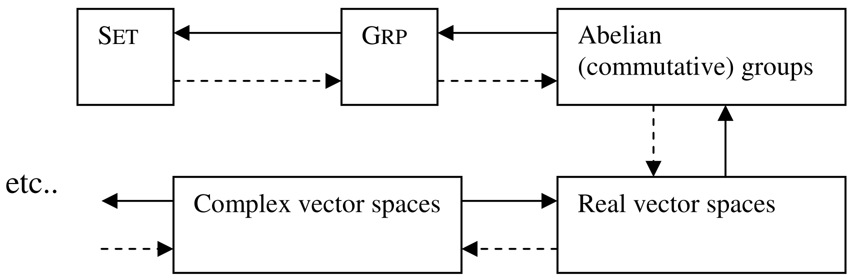

Figure I.1

II. Joseph Sneed's "Set Theoretic Predicate" Characterization of Theories in Mathematical Physics

Joseph Sneed (1971) is exemplary of the *structuralist* philosophy of science, a view that flourished in the 1960s and 1970s in Anglo-American philosophy of science.[10] Structuralism can be roughly understood as a rather formal sub-discipline within the *semantic* view of scientific theories. The latter was initially promoted by vanFraasen and Suppes, which conceived of scientific theories primarily as *collections of models*, instead of axiomatically characterized sets of (context-independent) propositions, as understood in the traditionally 'syntactic' view (Carnap, et. al).[11] In other words:

[8] 'Partially forgetful' in the sense that the action of such functors does not collapse the structure entirely back to its set-base, just to the 'nearmost' (simpler) structure.
[9] For more detail, see pages 132, 248 (Geroch (1985)).
[10] Though German philosophers and mathematicians like Erhart Scheibe and Guenther Ludwig, among others, continue this research tradition. (Schneider (2003), 9)
[11] The concept of 'model', however, is notoriously broad and open-textured. On the one hand, writers such as Suppes and vanFraasen appealed to the strictly logical definitions thereof, i.e., as subcollections of statements instantiating the truth-conditions of the premises or general axioms in any deductive system (i.e., a 'consistent world' in modal-logical terms). This prompted writers like Michael Friedman to question, for instance, whether or not the difference between the syntactic and the semantic view, so strictly conceived, really *makes a difference*, since models are characterized by their axioms. On the other hand, other writers (Nersessian (2002), Giere (1988)), for example, broaden the concept of model to include more

> [T]he gist of this view…may be summarily stated thus: At the core of a physical theory there is always a coherent piece of mathematics that is intended to throw light on the processes and states of affairs in this theory's chosen field of study. *Any coherent piece of mathematics can be articulated…as the conception of a relational system or structure*….Physical theories [however] do not fall like ripe fruits from a 'place beyond heaven' (Plato, *Phaedrus,* 247c) but are gropingly fashioned by physicists on earth…The structuralist explication pairs a structure specified by axioms with a family of physical complexes conceived as particular instances of it. Live physics is murkier. Still, *structuralism offers a picture of physical theories in an idealized state of maturity that throws much light on their actual development, their mutual relations, and their semantic links with their referents in the real world.* (Torretti (1999), 412-414, italics added)

One could always take issue with the presumed ability (or even, for that matter, the *relevance*) of the structuralist program's methods in clarifying the structure of a mature physical theory as suggested in the last sentence of the passage above. Nevertheless, the above passage does provide an indication of the spirit of the research program, and its central motivations. Structuralists, moreover, seek to apply their formal schemes to the following major issues and questions motivating their research (Schmidt (2003), 1):

(i) To apply a formalization of a metatheory of a scientific theory (or theories), which may require one differing in kind from the formalization schemes of the object languages (i.e. the physical theory or theories under study.)
(ii) To provide a framework for the rational reconstruction of particular theories.
(iii) The central and primitive notion is Bourbaki's 'species of structure[12]' is developed and refined to fit the following significant are of interest concerning the study of physical theories: a.) their actual mathematical structure, b.) their empirical claims, c.) the function of their theoretical terms, d.) the role played by approximation, e.) intertheoretic relations.

---

empirical notions such as tacit, cognitive structures, which may preconfigure our basic modes of abstracting.

[12] "Suppes took his cue from Bourbaki, whose reconstruction of mathematics as the science of abstract structures achieved the peak of its influence and began to spill into undergraduate textbooks in the 1950s." (Torretti (1999), 413)

For example, concerning item the question of the status of a physical theory's theoretical and empirical terms ((iii.a), (iii.b) above), reveals an apparent circularity. Concerning the nature of the theoretical terms of a particular physical theory Θ, many of Θ's laws acquire their conceptual content from the very concepts used in the formulation of the laws in the first place. In other words, "typically every physical theory Θ requires some new concepts which cannot be defined without using Θ." This characterizes Sneed's (and others') central notion of a Θ-theoretical concept as one which cannot be defined without using Θ. (Schmidt (2003), 2)

So, for example, in the case of classical Newtonian mechanics, force, acceleration, and mass are Θ-theoretical concepts, since Newton's Second Law (NSL): $\vec{F}_{resul\tan t} = \sum_i \vec{F}_i = m\vec{a}$ simply gives a *definition* of force. Acceleration is Θ-theoretical, since it is assumed that $\vec{a}$ for the center-of-mass of the system is measured against a background inertial frame of reference (IFR), i.e., one whose origin is not accelerating.[13] Inevitably, this introduces links *L* with other theories, such as that of classical spacetime. In addition, mass is Θ-theoretical, since it's defined inertially by the magnitude of the ratios of the above-mentioned Θ-theoretical terms[14]: $m = |\vec{F}| / |\vec{a}|$ (Sneed (1971), 32) .

[13] Suppose the background IFR's origin were accelerating at rate *A*. In the body's local IFR, $a' = 0 = a - A$, hence the body would be governed by law: $ma = mA$, which, according to the General Equivalence Principle, is indistinguishable from the effects of a local gravitational field.

[14] However, one could rewrite the NSL-gravity equation (for the sake of simplicity, I write it in scalar form here): $F = ma = Gm_g\sum_i m_{gi}/r_i^2$ in Ramsey-form: " $\exists I\ \exists G \exists m_g\ \exists m_{gi}$ (where *I* is an inertial system, and $m_g\ m_{gi}$ are the (gravitational masses), and G is a constant) such that: $F = ma = Gm_g\sum_i m_{gi}/r_i^2$." This would eliminate some of the Θ-theoretical terms in the circularity alluded to above. On the other hand, it does introduce the notion of higher-order constraints of the form: "All particles have[ing] the same inertial and gravitational masses and the gravity constant G will assume the same models in the theory [of classical mechnics]." (Schmidt, (2003), 3) See also Appendix A.2 below, in this essay.

In the above example involving aspects of classical mechanics, (theory Θ) it becomes readily apparent how some Θ-theoretical terms are linked with other theories, as in the case of classical space-time theory, when determining the acceleration. On the other hand, *some* (but not *all*) cases involving the measurement of *position* are Θ-independent, though still linked to other theories Θ'. For example, certain cases involving the measurement of the positions of stars do not presuppose the entire theory of celestial mechanics (Sneed (1971), 32). Nevertheless such measured values of position would still involve classical space-time geometry, to a certain extent of 'admissible blurring' between those measured values, and the predicted values of the positions given by the theory of classical celestial mechanics or a more specialized rendition thereof, using general relativity.

Hence in this brief discussion one gains some insight into the systematically holistic and contextual nature of Sneed's program. It is holistic, insofar as terms in the theory Θ (whether Θ-independent or Θ-dependent) inevitably involve other theories Θ', Θ", …in the form of linkages and constraints. On the other hand, the study is *contextual*, insofar as Sneed claims that a *particular* set theoretic-structure can successfully provide a metatheoretic formalization of classical mechanics. Moreover, this set-theoretic structure *may* extend to characterize other physical theories. (A set-theoretic extension from Hooke's Law spring systems into general classical mechanics, for instance, is discussed in Appendix A.2 below in this essay) However, "no claim [is] made that it [i.e., the particular set theoretic characterization of classical mechanics] contain *all* scientific theories. There is no claim made about the logical structure of scientific theories in general.[15]" (Sneed (1971), 2-3)

III. A Proposed Modification of Sneed: The "Category-Theoretic" Predicate

> [T]he investigation of…reduction relations between different theories is part of the everyday work of theoretical physicists, but usually they do not adopt a general concept of reduction…the work of the structuralists could lead to a more systematic approach within physics, *although there does not yet exist a generally accepted, unique concept of reduction.* (Schmidt (2003), 4, italics added)

In a previous paper (Kallfelz (2006)) I have sought to specify some necessary conditions comprising a form of "methodological fundamentalism" that has built within it, in my opinion, a concept of reduction more closely tied in with the 'everyday' (multilinear) algebraic structures employed by the physicist. Though Schmidt may in the end prove correct: there may not ever exist a "generally accepted, unique concept" by which one can reduce from one theory to the next. Still, in Sneed's manner of speaking, I submit the claim that the model I propose in my 2006 paper "codifies our intuitions" concerning a method of intertheoretic reduction for physical theories characterizable by classes of multilinear algebras (known as Clifford algebras[16]). It also remains faithful to the nuanced and quasi-empirical manner in which the business of certain theoretical physics tends to get done.[17] Moreover, I show how by way of a counterexample, that Robert Batterman's (2003, 2004) claims of failures of reduction, or singular phenomena,

[15] In this characteristically modest tone, Sneed goes on to write that the normative aim of his logical reconstruction should provide a "tractable and intuitive way of codifying…intuitions [concerning a theory's structure] which could be appealed to as justification for specific claims about the theory [of classical mechanics]…[nevertheless] the enterprise of logical reconstruction…in overall outlook…is descriptive. We presume that the practicing scientists' conception of what [s/]he is doing to be roughly correct…The enterprise is not, for that matter, an epistemological critique of the concepts employed in the existing theory from the viewpoint of some epistemological credo. If a scientist observes things in the laboratory…we are committed to at least deal justly with this claim in our logical reconstruction, rather than denounce it as conceptually confused from the point of view of some external criterion." (Sneed (1971), p 4)

[16] For definitions of Clifford algebra, see Appendix A.3 of this essay

[17] My work draws on the landmark papers of Inonou and Wigner (1952), I Segal (1951), and mostly on some of the recent ground-breaking extensions of the former by David Finkelstein (1996, 2001, 2002, 2003, 2004a, 2004b, 2004c)

in the case of certain critical fluid mechanical phenomena, are based on his reliance of the more standard ("methodologically approximate") mathematical formalisms of differential equations on phase spaces characterized by continuous manifolds.

In short, Clifford algebras can 'regularize,' i.e. circumvent singularities, in ways that other mathematical formalism relying fundamentally on phase-space methods cannot. This feature makes them an attractive and powerful tool for the working mathematical physicist, as well as for the philosopher sympathetic to structuralism. For "regularizing" means one can *expand* into richer formalisms, of which a subset of them may become the superseding physical theories of the future. Conversely, *contracting* (the reverse of algebraic expansion -- algebraic expansion is an invertible procedure) back down again is likewise assured, hence, there exists a 'paper trail' of superseded theories and a means to 'backtrack,' i.e. a method of intertheoretic reduction, in this algebraic sense.

What I aim to show here is how Sneed's program of metatheoretically characterizing classical mechanics, via set theory, can, in principle, be extended into other domains of study in mathematical physics, (should one replace his set theoretic predicate with a category-theoretic one instead.) My objectives for doing so are two-fold:

1. Joseph Sneed begins his classic work by stating that his focus is on classical mechanics alone, because he does not see how to apply a consistent set-theoretic, measure interpretation to irreducibly probabilistic theories like quantum mechanics and statistical mechanics. Though he sees no principled reason why his program should *not* eventually be extended into those domains.[18]
2. By way of analogy with my claims concerning 'methodologically fundamental' procedures I summarized above, a category-theoretic

[18] "Whether ultimate clarification of the content of these theories [statistical mechanics, quantum mechanics] will reveal that they too conform to my account of the logical structure of theories of mathematical physics, I can not say. However, I can see no compelling reason to think that they will not." (Sneed (1971), xii)

metatheoretic characterization of a Clifford-algebraic mathematically based physical theory applies in the same rigorous manner as Sneed's set-theoretic characterization of a phase-space mathematically based physical theory. Add to that, however, as argued in (Kallfelz (2006)) and summarized above, the former (Category Theoretic -> Clifford algebraic) exhibit regularizable structures, where the latter (Set theoretic -> phase space) do not. This regularizability provides a powerful means of characterizing intertheoretic reduction, for classes of formalisms, that can, in pinciple, be characterized by multilinear algebras.

Sneed ((1971), 16) defines certain minimum conditions by which a physical system referred to by a collection of definite descriptions (*not* proper names[19]) Q can be ascribed a set theoretic predicate. Denote by S_0 the set satisfying such minimal conditions. Then:

- **Defn III.1**: Q is an S_0 *iff* : $Q = \langle D, \tau, \nu \rangle$, where:

a.) There exists a finite non-empty set D (the *domain* of the system)
b.) $\tau: D \to R$
c.) $\nu: D \to R$, where R are the real numbers

In other words, the minimal conditions for a physical system to be described set theoretically exist if there exist a finite non-empty domain D and real-valued mappings τ, ν on D. τ represents the physical system's *theoretical* terms, or the Θ-theoretical terms (for whatever theory Θ applies to Q.) In Appendix A.2, for instance, such terms are elements of M_P , the theory's potential model. Moreover "the τ-function only comes to be considered after all the empirical work has been done…[i]t is a calculational device." (Sneed (1971), 44-45). ν, on the other hand, is the mapping representing the physical system's *non-theoretical* or Θ-independent terms. Because Defn III.1 refers to an

[19] Ibid., p. 17

empirical claim, such mappings just can't list all their members, i.e. their ranges $\tau(D) \subseteq R$, $\nu(D) \subseteq R$ are uncountable.[20]

More set-theoretic predicates, however, must be added to a physical system Q to qualify it as exhibiting any typically (non-trivial) behavior we encounter. For example, consider systems that are *extensive* (like systems characterized by additive quantities like volume, mass, etc.) Sneed ((1971), 18) defines an extensive system by:

- **DefnIII.2**. Any physical system Q is *extensive*, *iff* $Q = \langle D, R, \circ \rangle$ where:
  a.) D is non-empty (though not necessarily finite)
  c.) $R \subseteq D \times D$, where for any a,b,c in D: aRb & $bRc \to aRc$ (i.e., R is transitive)
  d.) $\circ : D \times D \to D$ (i.e. $\circ$ is a binary operation on D) which is associative, with respect to R: $[(a\circ b)\circ c]R[a\circ(b\circ c)]$
  e.) (Closure with respect to R) : $\neg aRb \to$ there exists $c \in D$ such that: $aR(b\circ c)$ and $(b\circ c)Ra$
  f.) $\neg(a\circ b)Ra$ (non-identity)
  g.) $\neg aRb \to$ there exists a positive integer $n \in Z$ such that $aR(nb)$

Sneed uses Defn III.2 (ibid, 20) to supply the necessary set-theoretic conditions for an extensive system to be *quantifiable* which he does by setting up a notion of a *numerical extensive system*:

- **DefnIII.3**. Any system X is a numerical extensive system (NES), provided: $x = \langle N, \leq, + \rangle$ where: a.) $N \subseteq R^{+}$ (the positive real numbers) , b)for any x and $y \in N$, then $x + y \in N$

Sneed uses definitions III.2, and III.3 to show that any extensive system is homomorphic to an NES, i.e. there aways exists some mapping φ: $Q \to X$ such that : $\varphi(x\circ y) = \varphi(x)+\varphi(y)$ for any $x,y \in Q$.

The purpose of presenting DefnI., DefnIII.2, DefnIII.3 (and there are far more in his book) is to indicate the categorical-theoretic structure lurking beneath them. As was

[20] A reasonable stipulation, given theoretical underdetermination, and 'admissible blurs' (See Appendix A.2)

shown in I., every set with an associated mapping is naturally expressible as a category, hence DefnIII.1 naturally translates as a categorical-theoretic predicate. Define then, S_0 categorical representation as C_0.(where the domain D are C_0's objects, and the mappings τ, ν are C_0's morphisms.) On the other hand, DefnIII.2 is more restrictive, since the mapping doesn't allow for identities (III.2.c). However, the NES, which is a homorphic image of the extensive system, is a partially ordered set, which *is* category, as discussed in section I. above.

So, not surprisingly (since category theory is more expressive than set theory, as argued in I.)) one can, with some patience, tweak the definitions throughout Sneed (1971) to embed them in category theory. But this in itself will not extend Sneed's work outside his domain of study (classical mechanics), it will only translate his set theoretic predicates into categorical-theoretical ones.

To effect the extension, consider the following example: Finkelstein (2001) characterizes quantum theory via a multilinear algebra (Clifford algebra). Clifford algebras form a category. Hence, by adopting Geroch's "partially" forgetful functor (page 6, Fig. I.1) we abstract to the level of a simple category:

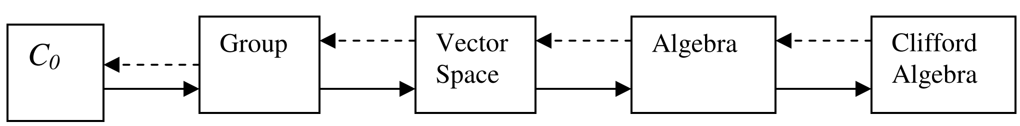

Fig III.1

Hence, reading in the opposite direction (from the standpoint of free creative functors or the solid arrows) Sneed's minimal set theoretic predicate, translated in terms of a category, has been shown to apply in principle to physical theories characterized by a Clifford algebra (as in the case of quantum theory.)

But the procedure applies not just to quantum theory per se, but to *any* physical formalism in principle characterizable by a Clifford algebra. These include versions of electromagnetism (both classical and quantum (Snygg (1997)), computational fluid mechanics (Scheuermann (2000)), etc.

IV.Conclusion

As indicated above, category theory greatly extends the range of applications of Sneed's set-theoretic characterization of physical formalisms. To those sympathetic to structuralism in the philosophy of physics, category theory indicates a fruitful venue to extend Sneed's research program. Also, category-theory naturally corresponds to structuralist characterizations of multilinear algebras. The latter are regularizable, i.e., they provide, through the action of algebraic contraction,[21] an effective machinery applicable to intertheoretic reduction in a manner that phase space methods cannot. The implications for this in the philosophy of physics, include, at the very least, a caveat for those who would hastily dismiss Sneed's structuralism as not being applicable to areas outside very refined reconstructions of classical mechanics.

APPENDIX A.1 Category Theory: Basic Definitions and Motivation

Category theory provides a unique insight into the general nature, or universal features of the construction process that practically all mathematical structures share. For example, consider the following three fundamental classes of mathematical structure, differing substantially in terms of their characteristic properties:

I.a) Set	(by Principle of Extension) $S_\Phi = \{x \mid \Phi(x)\ \}$ for some property Φ
I.b) Cartesian Product	For any two sets $X, Y : X\times Y = \{(x,y) \mid x \in X,\ \ y \in Y\}$

[21] As discussed in detail in pp11-16, ibid. (2006))

I.c) Mapping	For any two sets *X, Y*, where $f \subseteq X \times Y$, *f* is a *mapping* from *X* to *Y* (denoted $f : X \to Y$) *iff* for $x_1 \in X$, $y_1 \in Y$, $y \in Y$, if $(x_1, y_1) \in f$ (denoted: $y_1 = f(x_1)$) $(x_1, y_2) \in f$ then: $y_1 = y_2$.
I.d) Bijection (set equivalence)	For any two sets *X, Y*, where $f : X \to Y$ is a mapping, then *f* is a bijection *iff*: a) *f* is onto (surjective), i.e. $f(X) = Y$ (i.e., for any $y \in Y$ there exists a $x \in X$ such that: $f(x) = y$, b) *f* is 1-1 (injective) *iff* for $x_1 \in X$, $y_1 \in Y$, $y \in Y$, if $(x_1, y_1) \in f$ (denoted: $y_1 = f(x_1)$) $(x_1, y_2) \in f$ then: $y_1 = y_2$.
II.a) Group	I.e., a group $\langle G, \circ \rangle$ is a set *G* with a binary operation $\circ$ on *G* such that: a.) $\circ$ is *closed* with respect to *G*, i.e.: $\forall (x, y) \in G : (x \circ y) \equiv z \in G$ (i.e., $\circ$ is a *mapping into G* or $\circ : G \times G \to G$, or $\circ(G \times G) \subseteq G$)). b.) $\circ$ is *associative* with respect to *G*,: $\forall (x, y, z) \in G$: $(x \circ y) \circ z = x \circ (y \circ z) \equiv x \circ y \circ z$, c.) There (uniquely) exists a (left/right) identity element $e \in G$: $\forall (x \in G)\, \exists! (e \in G) : x \circ e = x = e \circ x$. d.) For every *x* there exists an *inverse element* of *x*, i.e.: $\forall (x \in G)\, \exists (x' \in G)$: $x \circ x' = e = x' \circ x$.
II.b) Direct product	For any two groups *G, H*, their *direct product* (denoted $G \otimes H$) is a group, with underlying set is $G \times H$ and whose binary operation * is defined as, for any $(g_1, h_1) \in G \times H$, $(g_2, h_2) \in G \times H$: $(g_1, h_1) * (g_2, h_2) = ((g_1 \circ h_1), (g_2 \bullet h_2))$, where $\circ$, $\bullet$ are the respective binary operations for *G,*and *H*.
II.c) Group homomorphism	Any *structure-preserving mapping* φ from two groups *G* and *H*. I.e. $\varphi : G \to H$ is a homomorphism *iff* for any $g_1 \in G$, $g_2 \in G$: $\varphi(g_1 \circ g_2) = \varphi(g_1) \bullet \varphi(g_2)$ where $\circ$, $\bullet$ are the respective binary operations for *G,*and *H*.
II.d) Group Isomorphism (group equivalence)	Any *structure-preserving bijection* ψ from two groups *G* and *H*. I.e. $\psi : G \to H$ is an isomorphism *iff* for any $g_1 \in G$, $g_2 \in G$: $\psi(g_1 \circ g_2) = \psi(g_1) \bullet \psi(g_2)$ (where $\circ$, $\bullet$ are the respective binary operations for *G,*and *H*) *and* ψ is a *bijection* (see I.d above) between group-elements *G* and *H*. Two groups are *isomorphic* (algebraically equivalent, denoted: $G \cong H$) *iff* there exists an isomorphism connecting them $\psi : G \to H$.)
III.a) Topological Space	Any set *X* endowed with a collection τ_X of its subsets (i.e. $\tau_X \subseteq \wp(X)$, where $\wp(X)$ is *X*'s power-set, such that: 1) $\emptyset \in \tau_X$, $X \in \tau_X$ 2) For any $U, U' \in \tau_X$, then: $U \cap U' \in \tau_X$. 3) For any index (discrete or continuous) γ belonging to index-set Γ: if $U_\gamma \in \tau_X$, then: $\bigcup_{\gamma \in \Delta \subseteq \Gamma} U_\gamma \in \tau_X$. *X* is then denoted as a *topological space,* and τ_X is its *topology*. Elements *U* belonging to τ_X are denoted as *open* sets. Hence 1), 2), 3) say that the empty set and all of *X* are always open, and finite intersections of open sets are open, while arbitrary unions of open sets are always open. Moreover: 1) Any collection of subsets $\Im$ of *X* is a *basis* for *X's* topology *iff* for any $U \in \tau_X$, then for any index (discrete or continuous) γ belonging to index-set Γ: if $B_\gamma \in \Im$, then: $\bigcup_{\gamma \in \Delta \subseteq \Gamma} B_\gamma = U \in \tau_X$ (i.e., arbitrary unions of basis elements are open sets.) 2) Any collection of subsets Σ of *X* is a *subbasis* if for any $\{S_1, \ldots, S_N\} \subseteq \Sigma$, then $\bigcap_{k=1}^{N} S_k = B \in \Im$ (I.e. finite intersections of sub-basis elements are basis elements for *X*'s topology.)
III.b) Topological product	For any two topological spaces *X, Y*, their *topological product* (denoted $\tau_X \otimes \tau_Y$) is defined by taking, *as a sub-basis*, the collection: $\{(U,V) \mid U \in \tau_X , V \in \tau_Y \}$. I.e., $\tau_X \times \tau_Y$ is a subbasis for $\tau_X \otimes \tau_Y$. This is immediately apparent since, for U_1 and U_2 open in *X*, and V_1 and V_2 open in *Y* : since: $U_1 \times U_2 \cap V_1 \times V_2 = (U_1 \cap V_1) \times (U_2 \cap V_2)$ this indeed forms a basis.
III.c) Continuous	Any mapping from two topological spaces *X* and *Y,* preserving openness. I.e. *f* :

mapping	$X \rightarrow Y$ is continuous *iff* for any $U \in \tau_X$: $f(U) = V \in \tau_Y$
III.d) Homeomorphism (topological space equivalence)	Any *continous bijection h* from two topological spaces *X* and *Y*. I.e. $h : X \rightarrow Y$ is a homeomorphsim *iff* : a) *h* is continuous (see III.c), b) *h* is a bijection (See I.d). Two spaces X and *Y* are *topologically equivalent* (i.e., homeomorphic, denoted: $X \cong Y$) *iff* there exists a homeomorphism connecting them, i.e. $h : X \rightarrow Y$

Table A.1

Now the classes of mathematical objects exhibited in Table A.1 comprising sets, groups, and topological spaces, all exhibit certain common features:

- The concept of *product* (I.b, II.b, III.b) (or concatenating, in 'natural manner' property-preserving structures.) For instance, the Cartesian (I.b) product preserves the 'set-ness' property for chains of objects formed from the class of sets, the direct product (II.b) preserves the 'group-ness' property under concatenation, etc.
- The concept of 'morphing' (I.c, II.c, III.c) from one class of objects to another, in a property-preserving manner. For instance, the continuous map (III.c) respects what makes spaces *X* and *Y* 'topological,' when morphing from one to another. The homomorphism respects the group properties shared by *G* and *H*, when 'morphing' from one to another, etc.
- The concept of 'equivalence in form' (isomorphism) (I.d, II.d, III.d) defined via conditions placed on 'how' one should 'morph,' which fundmantally should be in an *invertible* manner. One universally necessary condition for this to hold, is that such a manner is modeled as a bijection. The other necessary conditions of course involve the particular property structure-respecting conditions placed on such morphisms.

Hence, from the above observations obtained from Table A.1, we may surmise that *any* mathematical system, or general class of mathematical entities will share, across-the-board, the following aspects, exhibited below as:

Aspect	**Underlying Notion**
Object	The primitive entities of the system.
Product	How to 'concatenate' in a natural manner to form chains of such objects.
Morphsim	How to 'morph' from one object to another.
Isomorphism (structural equivalence)	How such objects, relative to the system, are understood to be equivalent.

Table A.2

Category Theory provides a such a systematic account and universal language for dealing with such universal notions as object, product, (iso/)morphism. Some of its derivative notions, alluded to above, include what it means for an object to be 'natural.' (Herrlich & Strecker (1979), 3)

Appendix A.2: A Systematic Set-Theoretic Characterization of a Hooke's law Spring System (Scheibe, Sneed):

1. $M_P = \langle P, S, m, DYN \rangle$ is the class of *potential models* of the theory (of Hooke's Law spring systems), where: S ={springs with their spring constants k}, P = {particles attached by the spring(s)}, m = {the masses of each particle in P}. DYN is the set of all dynamical laws (or equations of motion in the theory Θ of classical particle dynamics, e.g. Newton's Second Law for the system of N particles, where, for any particle k: $m_k \ddot{\vec{r}}_k = \sum_{\substack{i,j \\ i \neq j}} k_{ij} |\vec{r}_i - \vec{r}_j| \hat{r}_{ij} + \vec{D}_k$, where k_{ij} is the spring constant of the spring connecting the ith with the jth particle, $\hat{r}_{ij}$ is a unit vector connecting the ith with the jth particle, $\vec{D}_k$ are dissipative forces acting on *kth* particle, like friction , etc.)connecting the positions and forces on ach of the particles.
2. The class of *actual* models (M) expresses the *empirical content* of the theory. (Clearly $\varnothing \subseteq M \subseteq M_P$) M
3. M_{PP} = {partial potential models} = $\{P, \{\vec{r}_k \mid 1 \leq k \leq N\}\}$. Partial models do not contain any Θ-theoretical terms, which in the case of classical mechanics, would omit spring constants, masses, and forces, leaving just the position set of the collection of particles.
4. C = {constraints} Constraints connect different models to one and the same theory. Examples here would include: token particles of the same particle-type have the same masses, token springs of the same spring type have the same spring constants.)
5. L = {links} $\subseteq M_\Theta \times M_{\Theta'}$, where M_Θ ,$M_{\Theta'}$ are models of theories Θ, Θ' respectively. Examples would of elements of L include the classical theory of spacetime (as is presupposed in the determination of the positions $\vec{r}_k$ and inertial masses m_k of the of the kth particle), the theory of elasticity (for determining the values of the spring constants k_{ij}), etc. Links are conditions connecting different models of different theories.
6. A = {admissible blurs}, which are degrees of approximation admitted between different models. For instance, the measured value $\vec{r}_k^{(meas)}$ of the position of the kth particle (an element of M_{PP}) would fall within a suitable error-bar range centered around the exact value of $\vec{r}_k$ as predicted by DYN (I.e. $|\vec{r}_k^{(meas)} - \vec{r}_k| < \varepsilon$)

7. Hence, the core formal theoretical part of Θ is the collection: $K_\Theta = \langle M_P, M, M_{PP}, C, L, A \rangle$
8. I_Θ = {domains of intended application of Θ}. This is an open class consisting of, for instance, examples like small rigid bodies connected by coiled springs, rubber bands, mechanical systems vibrating with oscillations of small amplitude, etc.
9. Hence the theory-element $T_\Theta = \langle K_\Theta, I_\Theta \rangle$ is the "smallest unit to be regarded as a theory." (Schmidt (2003), 8)
10. The specialization relation $\sigma \in \tau \times \tau$, where τ is a collection of theory-elements, would, for example, modify NSL (in *DYN* defined in 1.) to include time-dependent force terms.
11. ***N*** is then a Poset (partially ordered set) of theory-elements $T_\Theta = \langle K_\Theta, I_\Theta \rangle$, ordered according to the specialization relation (defined in 10.) Classical mechanics would be such an example in this case, as it certainly is an example of such a network of $T_\Theta = \langle K_\Theta, I_\Theta \rangle$ "essentially ordered by the degree of generality of its force laws." (ibid.)
12. ***E* = *N*(*t*)** then represents the evolution of classical mechanics, propagating through historical time.

Appendix A.3 Definition of A Clifford Algebra

A *vector space* is to a structure $\langle V, F, *, \cdot \rangle$ endowed with a (commutative) operation (i.e. $\forall(x,y) \in V : x*y = y*x$, denoted, by convention, by the "+" symbol, though not necessarily to be understood as addition on the real numbers) such that: i) $\langle V, * \rangle$ is a commutative (or Abelian) group. ii) Given a field of scalars F (i.e. a an algebraic structure $\langle F, +, \times \rangle$ endowed with two binary operations such that $\langle F, + \rangle$ and $\langle F, \times \rangle$ form commutative groups and $+, \times$ are connected by left (and right, because of commutativity) distributivity, i.e., $\forall(\alpha,\beta,\gamma) \in F : \alpha\times(\beta+\gamma) = (\alpha\times\beta) + (\alpha\times\gamma)$), the *scalar multiplication* mapping into $V \cdot$: $F \times V \rightarrow V$ obeys distributivity (in the following two senses): 1) $\forall(\alpha,\beta) \in F \ \forall (\varphi \in V)$: $(\alpha+\beta)\cdot\varphi = (\alpha\cdot\varphi) + (\alpha\cdot\varphi)$ 2) $\forall(\varphi, \phi) \in V \ \forall \gamma \in F : \gamma\cdot(\varphi+\phi) = (\gamma\cdot\varphi) + (\gamma\cdot\phi)$.)

An *algebra A,* then, is defined as a *vector space* $\langle V, F, *, \cdot \rangle$ endowed with an associative binary mapping • into A (i.e., $\bullet : A \times A \rightarrow A$, such that $\forall(\psi, \varphi, \phi) \in G$: $(\psi \bullet \varphi) \bullet \phi = \psi \bullet (\varphi \bullet \phi) \equiv \psi \bullet \varphi \bullet \phi$ denoted, by convention, by the "×" symbol, though not necessarily to be understood as ordinary multiplication on the real numbers) I.e, this can be re-stated by saying that $\langle A, \bullet \rangle$ forms a *semigroup* (i.e. a set A closed under the binary associative product •)

Clifford algebras (or geometric algebras) remain an attractive candidate for algebraicizing any theory in mathematical physics (assuming the Clifford product and sum can be appropriately operationally interpreted in the theory *T*.) There exist a variety of different axiomatic characterizations of Clifford algebra, one of the most well-known in Hestenes and Sobczyk's (1984) treatment. They define a Clifford algebra *CL* as a graded (multilinear) algebra endowed with a quadratic antisymmetric (adjoint) form

$\uparrow$ and grade operator $< >_r$ such that, for any Clifford elements (mutlivectors) A,B: $<AB>^{\uparrow}_r = (-1)^{C(r,2)} <B^{\uparrow}A^{\uparrow}>_r$ (where: $C(r,2) = {}^{r!}/_{(2!(r-2)!)} = r(r-1)/2$.) (1984, p6). A general Clifford element (multivector) A of Clifford algebra CL of maximal grade N is expressed by the linear combination: $A = \alpha^{(0)}A_0 + \alpha^{(1)}A_1 + \alpha^{(2)}A_2 + \ldots + \alpha^{(N)}A_N$ where $\{\alpha^{(k)} \mid 1 \le k \le N\}$ are the elements of the scalar field (expansion coefficients) while $\{A_k \mid 1 \le k \le N\}$ are the *pure* Clifford elements associated with closed Clifford subspaces $CL_{(k)}$ of grade k, i.e. $CL = CL_{(0)} \oplus CL_{(1)} \oplus \ldots \oplus CL_{(N)}$. For example, $CL_{(0)}$ (the Clifford subspace of grade 0) are the Real numbers R. $CL_{(1)}$ (the Clifford subspace of grade 1) are the Complex numbers $\boldsymbol{C}$. $CL_{(2)}$ (the Clifford subspace of grade 2) are the Quaternions $\boldsymbol{H}$. $CL_{(3)}$ (the Clifford subspace of grade 3) are the Octonions $\boldsymbol{O}$, etc.

V. References

Adamek, et. al. (1990) *Abstract and Concrete Categories: The Joy of Cats,* New York: Wiley & Sons.

Batterman, Robert (2002). *The Devil in the Details: Asymptotic Reasoning in Explanation, Reduction and Emergence.* New York, NY: Oxford University Press.

Batterman, Robert (2003). "Response to Belot's 'Whose Devil? Which Details?'" *Philosophy of Science.* Forthcoming. *Philosophy of Science Archives: Physics* http://philsci-archive.pitt.edu/archive/00001623/

Batterman, Robert (2003). "Intertheory Relations in Physics" *Stanford Encyclopaedia of Philosophy* http://plato.stanford.edu/entries/physics-interrelate/

Batterman, Robert (2004) . "Critical Phenomena and Breaking Drops" *Philosophy of Science Archives: Physics* http://philsci-archive.pitt.edu/archive/00001622/

Batterman, Robert (2005) . "'Fundamental Physics' Molecular Dynamics versus Hydrodynamics" *Philosophy of Science Archives: Physics* http://philsci-archive.pitt.edu/archive/00000582/

Baylis, ed.(1995) *Clifford (Geometric) Algebras*. Birkhauser .

Bredon, Glen (1993) *Topology and Geometry,* New York: Springer-Verlag.

Belot, Gordon (2003). "Whose Devil? Which Details?" *Philosophy of Science.* Forthcoming. http://philsci-archive.pitt.edu/archive/00001515/

Bishop, Robert (2004). "Patching Physics and Chemistry Together" *PSA* 2004 http://philsci-archive.pitt.edu/archive/00001880/

Carruthers, et. al., eds. (2002) *The Cognitive Basis of Science.* Cambridge: Cambridge University Press.

Finkelstein, David (1996). *Quantum Relativity: A Synthesis of the Ideas of Einstein and Heisenberg.* Berlin: Springer-Verlag.

Finkelstein et al., (2001) Clifford Algebra as Quantum Language, *J. Math. Phys* 42, 1489. http://www.physics.gatech.edu/people/faculty/finkelstein/CliffLang2001.pdf
Finkelstein, David (2002) *Post Quantum Relativity* (manuscript), School of Physics, Georgia Institute of Technology, Atlanta, Georgia.

Finkelstein, David (2004a) *Finite Quantum Relativity*, URL=http://www.physics.gatech.edu/people/faculty/finkelstein/FQR02.pdf

Finkelstein et al.,(2004b) *Quantum Binary Gravity* URL=http://www.physics.gatech.edu/people/faculty/finkelstein/QBGravity031215.pdf

Finkelstein and Shiri-Garakani (2004c) *Finite Quantum Harmonic Oscillator* URL=http://www.physics.gatech.edu/people/faculty/finkelstein/FHO0410082.pdf

Friedman, Kenneth (1982) "Is Intertheoretic Reduction Possible?" *British Journal for the Philosophy of Science*, vol. 22, no. 1 (17-40).

Geroch, Robert (1985) *Mathematical Physics* (Chicago Lecture Series in Physics). Chicago: University of Chicago Press.

Giere, Ronald. (1988) *Explaining Science: A Cognitive Approach.* Chicago: University of Chicago Press.

Gruenbaum, Adolf & Salmon, Wesley, eds. (1988) *The Limitations of Deductivism*. Berkeley: University of California Press.

Hempel, Karl (1988) "Provisos: A Problem Concerning the Inferential Function of Scientific Theories," *Erkenntnis* XXVIII: 147-164. (rep. in Gruenbaum and Salmon (1988) 19-36.

Herrlich, Horst & Strecker, George (1979) *Category Theory* (Sigma Series in Pure Mathematics) Berlin: Heldermann Verlag.

Hestenes and Sobczyk (1984) *Clifford Algebras to Geometric Calculus: A Unified Language for Mathematics and Physics*, (Fundamental Theories of Physics), Dordrecht: D. Reidel.

Hofmann & Mislove, eds. (1996) *Semigroup Theory and Its Applications,* London Mathematical Society Lecture Notes Series (vol 231). Cambridge: Cambridge University Press.

Hofweber, Thomas (2004). "Logic and Ontology," *Stanford Encyclopedia of Philosophy* (on-line) , <URL = http://plato.stanford.edu/entries/logic-ontology/ >

Howard, Don (2005) "Reduction and Emergence in the Physical Sciences: Some Lessons from the Particle Physics-Condensed Matter Debate" http://www.nd.edu/~dhoward1/Reduction%20and%20Emergence.pdf

Howie, John M. (1976) *An Introduction to Semigroup Theory*, London: Academic Press.

Howie, John M. (1995) *Fundmanetals of Semigroup Theory,* Oxford: Clarendon Press.

Inonu and Wigner (1952) On the Contraction of Groups and their Representations. *Proceedings of the National Academy of the Sciences* 39, 510-524.

Kallfelz, William (2005) "Contracting Batterman's 'No-Man's Land:' Reduction Rejoins Explanation" *Philosophy of Science Archives: Physics* http://philsci-archive.pitt.edu/archive/00002402/

Kallfelz, William (2006) "Methodological Fundamentalism" (manuscript, May 11, 2006) Department of Philosophy, University of Maryland, College Park, MD.

Lenhard, Johannes (2004) "Nanoscience and the Janus-Faced Character of Simulations," in Baird, et. al., eds. *Discovering the Nanoscale*. Amsterdam: IOS Press, 93-100.

Mikhalev &. Pilz. (2002) *The Concise Handbook of Algebra.* Kluwer Academic Publishers, Dordrecht.

Morreau, Michael (1997) "Fainthearted Conditionals," *Journal of Philosophy* 94:187-211.

Morreau, Michael (1999) "Other Things Being Equal," *Philosophical Studies* 96, 163-182.

Nersessian, Nancy (2002). "The cognitive basis of model-based reasoning in science," in Carruthers, et. al., eds. (2002) 133-153.

Okniński, Jan (1991) *Semigroup Algebras*. NY: Marcel Dekker.

Scheuermann, Gerik (2000) "Topological Field Visualization with Clifford Algebra" http://www.informatik.uni-leipzig.de/TopologicalVectorField.pdf

Schmidt, Hans-Juergen (2002), "Structuralism in Physics." *Stanford Encyclopaedia of Philosophy,* http://plato.stanford.edu/entries/physics-structuralism/

Segal, I. E. (1951) A Class of Operator Algebras which are Determined by Groups. *Duke Mathematical Journal* 18, 221.

Sneed, Joseph (1971) *The Logical Structure of Mathematical Physics*, (Synthese Library) D Reidel.

Torretti, Roberto (1999) *The Philosophy of Physics*. Cambridge: Cambridge University Press

Wilson, Jessica (2005) "Non Reductive Physicalism and Degrees of Freedom"