

The Character of Asymptotic Thought in the Works of Nicholas of Cusa

Submitted by:

William Kallfelz

Committee for Philosophy and the Sciences

Department of Philosophy

University of Maryland

College Park, MD 20742

wkallfel@umd.edu

September 16, 2005

Department of History and Philosophy

University of Virginia's College at Wise

Medieval-Renaissance Conference XIX

History and Philosophy Panel

Abstract

It is often argued that Nicholas of Cusa used *illustrations*, in the place of *analogies*, in his use of geometric figures in his writings of divine infinity (*De Docta Ignorantia*, 1440, *De Possessio*, 1460.) Such an epistemically weak notion can have the unfortunately adverse connotation of diminishing the depth and subtlety of Cusa's mathematical reasoning. I argue here, that when subjecting Cusa's unique amalgamation and NeoPlatonic and Scholastic thought to a careful analysis, one encounters a radically *constructive* character underlying Cusa's notion of meaning. Having articulated his notions from the *via negativa*, Nicholas of Cusa abandoned any attempt to positively characterize the meaning of his theological and philosophically central points, via referential devices such as the *analogia proportionis*, and the *analogia proportionalis*. However, *because* Nicholas of Cusa centrally relied on the freely constructed character of his state-of-the-art grasp of mathematics, he was able to rigorously articulate a version of NeoPlatonism free from what is often misconstrued from his writings as obscure or arbitrarily speculative. Nowhere is the methodology of Cusanis leant a greater dialectical precision, than in his use of *asymptotic reasoning*. Aside from his subtle grasp of the nature of the differences between discrete and continuous infinities (as discussed, for example, in Book III of *De Docta Ignorantia*) Cusa's manifold illustrations contrasting continuous versus discrete, as well as bounded versus unbounded infinities, continue to offer unique metaphysical and epistemological insights into contemporary debates surrounding the nature of relativism, contingency, and pluralism.

MAJOR PUBLICATIONS

De Concordia Catholica [1433]
De Auctoritate Presidenti in Concilio Generali [1434]
De Docta Ignorantia * (vol. I,II,III) [1440]
De Coniecturis [1442-3]
De Deo Abscondito [1444]
De Quarendo Deum [1445]
De Filiatione Dei [1445]
De Geometricis Transmutationibus ** [1445]
De Arithmetiis Complementis ** [1445]
De Dato Patris Luminum [1445-6]
Coniecturia de Ultimis Diebus [1446]
De Genesi [1447]
Apologia Docta Ignorantiae [1449]
Idiota de Sapientia [1450]
Idiota de Mente [1450]
Idiota de Staticis Experimentis [1450]
De Circuli Quadratura ** [1450]
Quadratura Circuli ** [1450]
Tres Epistolae Contra Bohemos [1452]
De Pace Fidei [1453]
De Visione Dei [1453]
De Mathematicis Complementis ** [1453]
Complementum Theologicum [1453]
De Mathematicis Complementis (II) ** [1454]
Declaratio Rectilineationis Curvae ** [1454?]
De Una Recti Curvique Mensura ** [1457]
Dialogus de Circuli Quadratura ** [1457]
De Beryllo [1458]
De Mathematica Perfectione ** [1458]
De Aequalitatae [1459]
De Principio [1459]
Aurea Proposito in Mathematicis [1459]
De Posse-Est [1460]
Cribation Alkoran [1461]
De Li Non Aliud [1462]
De Ludo Globi [1463]
De Veneratione Sapientiae [1463]
Compendium [1464]
De Apice Theoriae [1464]

* Works cited here explicitly are in boldface.

** Primarily mathematical works.

Regarding mathematical, which proceed from our reason and which we experience to be in us as their source: they are known *by us as our entities and as rational entities...*precisely, by our reason's precision, *from which they proceed...* Without these rational entities reason could not proceed with its work, e.g., with building, measuring, and so on.

But the divine works, which proceed from the divine *intellect, remain unknown to us precisely as they are.* If we know something about them, we surmise it by likening a figure to a form. *Hence there is no precise knowledge...*If we had any knowledge, we derive it from the symbolism and the mirror of our mathematical knowledge.

De Posse-Est (italics added), chapter 43, pp. 111-113 [trns. Jasper Hopkins (1978)]

“According to negative theology, ‘there is not found in God anything other than infinity.’ (*De Docta Ignorantia*, vol. I, § 26) “

Jasper Hopkins (1981), p. 14

Infinity as an epistemologically and ontologically *constitutive*, and methodologically *regulative* principle in negative theology: (Kallfelz (1996))

- **Methodologically Regulative:** “In-finity...’enfolds’ learned ignorance... fundamentally conceptual activities involve the identification, differentiation, and the *imputation* of known properties to what is at the outset unknown...thus do all things present themselves as knowable. But when considering infinity as such, one is already germinating a rudimentary negative theology...one starts with *known* limits and ...removes them, proceeding thence to a limitless unknown.” (Kallfelz (1996), 18)
- **Epistemologically and Ontologically Constitutive via the *Metaphysic of Contraction*** (J Hopkins (1983))

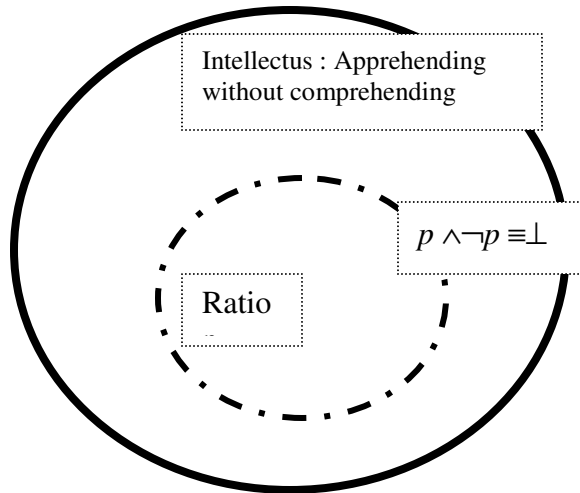
GOD (Maximum Absolutum)	Contraction (⇒)	Universe (Maximum contractum)
Unity	⇒	multiplicity
Infinity	⇒	finitude
Simplicity	⇒	composition
Eternity	⇒	temporality
Necessity	⇒	possibility

The Varieties of ∞

Type	Token	As used and mentioned in Cusa
← Unbounded, Continuous ¹	The Real numbers; i.e.: $(-\infty, \infty)$	Maximum Absolutum (‘The Boundless Infinity, with no comparable measure’)
← Bounded, Continuous	The real interval between 0 and 1, inclusive: $[0, 1]$ 0 1	Maximum Contractum (‘Fully complete unto itself.’)
Bounded, Discrete ²	The rational numbers between 0,1, exclusive: $\{ \frac{n}{m} \mid 0 < n < (m - 1) \}$	Ratio (what can be comprehended)

“In place of...distinctions between *analogia proportionis* and *analogia proportionalis*, we find...the use of geometric figures...true, Nicholas does draw various kinds of distinction...but because analogies do not correspond to any reality to be found in Infinite Being or its relations, they are better called illustrations.”

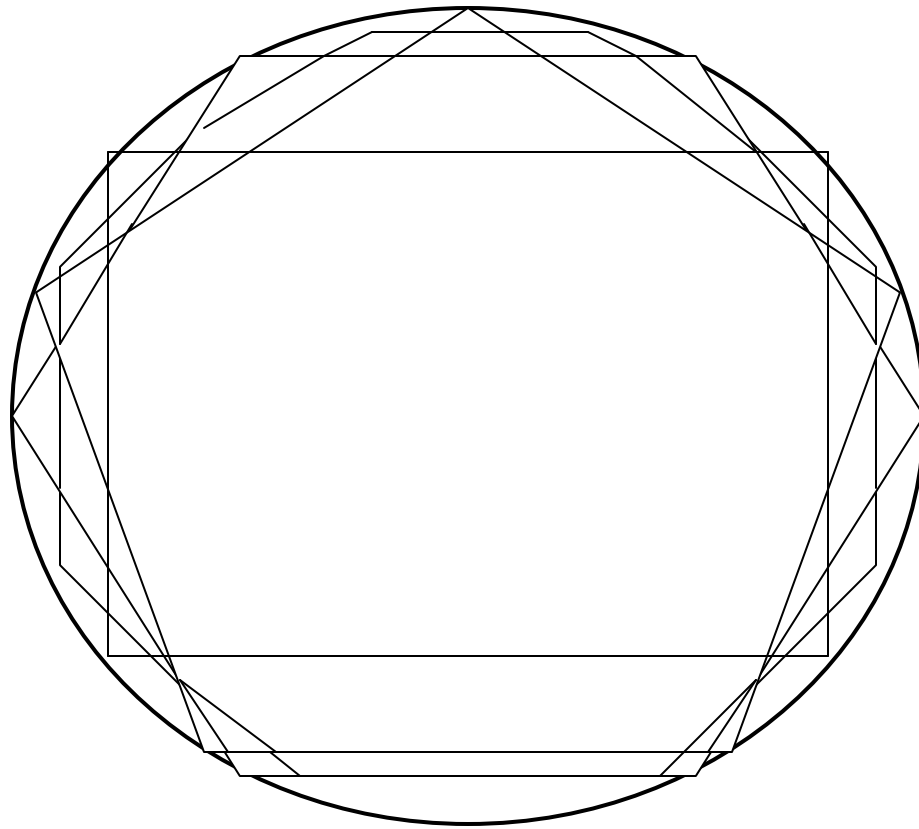
(Jasper Hopkins [1980], pp. 15-16.)



“With respect to the *intellectus*, reason (ratio) is at the horizon...but with respect to the sense, reason is at the zenith, things that are within time and things that are beyond time coincide in reason.” (*De Docta Ignorantia*, vol. III, ch. 6, § 216)

¹ Continuous means uncountable (there are so many points, there’s no way to list them). Continuous sets have cardinality denoted by \aleph_1 .

² Discrete sets, though they may have infinitely members, are nevertheless *countable*. That is to say, one can in principle all their points. Their cardinality is denoted by \aleph_0 . One can prove in modern mathematics, the (*prima facie* paradoxical) inequality: $\aleph_1 > \aleph_0$



CIRCLE \equiv TRUTH
INSCRIBED POLYGONS \equiv ATTEMPTS BY THE *RATIO* TO COMPREHEND

As the number of sides of the polygons $N \rightarrow \infty$:
“The polygons never become equal to the circle...we know truth not to be precisely comprehensible as it is.” (DDI: ch. 3, § 11)

(In contemporary mathematical terms: $\aleph_0 < \aleph_1$)

In other words, we get a singularity: What goes on *at the limit*, i.e., the circle, does *not* equal what goes on *in the limit* (a polygon with \aleph_0 sides, i.e., *countably* many sides.)

ASYMPTOTIC REASONING (Robert Batterman (2002))

1. Given a theory \mathfrak{S}' that supersedes theory \mathfrak{S} , *asymptotic reasoning* involves the attempt to characterize a reducing relation between \mathfrak{S}' , \mathfrak{S} in terms of a *mathematical* maneuver. This maneuver involves finding some characteristic parameter ξ in \mathfrak{S}' and investigating the behavior of \mathfrak{S}' in the $\xi \rightarrow \infty$ limit.
2. In the (typically exceptional but philosophically uninteresting) case when \mathfrak{S}' smoothly converges to \mathfrak{S} , (i.e., $\lim_{\xi \rightarrow \infty} \mathfrak{S}' = \mathfrak{S}$), we say the reducing relation between \mathfrak{S}' , \mathfrak{S} is *regular*. On the other hand, in the far more usual and philosophically interesting case when this does *not* occur (*either* because: $\lim_{\xi \rightarrow \infty} \mathfrak{S}'$ fails to exist, *or* because $\lim_{\xi \rightarrow \infty} \mathfrak{S}'$ exists, but: $\lim_{\xi \rightarrow \infty} \mathfrak{S}' \neq \mathfrak{S}$) we call this a case exhibiting a *singularity*.
3. Phenomena often classified as *emergent* (i.e., when their upper-level properties do not supervene on, nor can they be derived from their fundamental properties –epistemologically *or* ontologically³) are characterized by a *singularity* (in the above-mentioned sense.)
4. Moreover, in such emergent cases, the superseded theory \mathfrak{S} *plays an essential role* in the depiction of the emergent phenomena. In other words, such phenomena can only be adequately depicted by invoking the ontologies of *both* superseding and superseded theories \mathfrak{S}' , \mathfrak{S} in a seemingly *paradoxical* manner. Conversely, the superseding theory \mathfrak{S}' is inadequate for the job of characterizing the emergent phenomena alone. This suggests that \mathfrak{S} is somehow “deeply contained” in \mathfrak{S}' .

³ Michael Silberstein (1999) distinguishes weak emergence versus strong emergence in terms of *epistemological* versus *ontological* emergence. In the former case, the emergence is weak because it is only due to our cognitive and computational limitations that we fail to derive upper-level from fundamental properties. Whereas in the latter case, there exists a bone fide case of emergence, insofar as it's independent of any of our cognitive processes and their associated inherent limitations.

CONCLUSIONS

CLAIM: Asymptotic reasoning characterizes an important (if not fundamental) aspect of Nicholas of Cusa's thought. Though his metaphysical and epistemological milieu is late-Scholastic/neoPlatonist (thus making his thought "a central part of ... the medieval world"⁴) his *methodology of intertheoretic reduction is asymptotic*:

I.1) \mathfrak{S}' : Theory of the *Ratio*'s activity fundamentally constituted by mathematical activity ("Without these rational activities, reason could not proceed with its work, e.g., with building, measuring, and so on."), \mathfrak{S} : Metaphysical theory of truth. $\xi = N$.

Question/problem : How do we understand truth as superseded by ratio's activity, to complete the schema? Tentative answer: Though "absolute Truth belongs only to God"⁵ ...[cataphatic or symbolic theology] characterizes our *perspectives* of truth. Thus reason's activity can act as a "superseding theory" insofar as it can inexhaustibly generate new perspectives.

I.2) *De Docta Ignorantia* (as explicitly depicted in the inscribed n-gon illustration) characterizes *Ratio*'s engagement with truth as a relationship exhibiting a singularity ("The polygons never become equal to the circle...we know truth not to be precisely comprehensible as it is.")

I.3) & I.4) The *emergence* is inextricably interwoven with *paradox*. This is because (contrary to the *physical* systems Batterman investigates) the "system" is *our very faculty of comprehension [Ratio], which can only be described "from the inside."* We see the *emergence* occur in the *Intellectus* "apprehending without comprehending" outside *Ratio*'s limitations, bounded by the Law of non-Contradiction. In other words, what the *Intellectus* apprehends can *never* be reduced to *comprehensible* terms: this is so because *Ratio*'s very constituent principle, upon which all else comprehensible issues ($\neg(p \wedge \neg p)$) is violated. Moreover, the paradoxical coexistence of such ontologies is evidenced throughout the character of Cusa's writings. Consider:

"I, an insignificant human being, would not be content with You my God if I knew You to be comprehensible...On the other hand, that which the intellect does not at all understand cannot fully satisfy it, either. Rather [it is satisfied] only [by] that which it understands by not understanding."

De Visione Dei ch 16 §71:7-15 [trns. Hopkins (1985), p. 61],

⁴ Hopkins (1985), pp. 92-93

⁵ Karsten Harries (1990) p. 96

CODA

The claim that Cusa's methodology is characterized in some important way by asymptotic reasoning is lent further credence when considering the following analogy:

- The contemporary philosophical tradition represented by Batterman is best typecast *not* as a break from the overall analytic-philosophical paradigm. Rather, Batterman, et. al., give accounts of notions like 'unity' and 'reduction' stripped of 'top-down' *apriori* metaphysical and epistemological baggage, seeking, rather, a 'bottom-up' account of the use such notions in their detailed *study* of the fields in the mathematical sciences of the twentieth and twenty-first centuries.
- Likewise, Cusa's thinking does *not* break substantially from the late Medieval intellectual tradition. "Cusa makes no attempt to hold on to a medieval world...[because] he does not perceive it as threatened...he is too centrally a part of it...[he] neither breaks with the medieval world nor attempts to save it." (Hopkins [1985] 92-93.) Cusa, however, in rejecting traditionally 'top-down' scholastic notions of analogy, attempts to give, from the 'bottom-up' intuitions concerning transcendence faithful to his articulation of the *via negativa*. He succeeds primarily in doing this insofar as drawing on notions of infinity and asymptotes in his detailed *study* of his contemporary mastery of the fields of fifteenth-century mathematical sciences.

REFERENCES

Ashworth, Jennifer. "Medieval Theories of Analogy," *The Stanford Encyclopedia of Philosophy*, <http://plato.stanford.edu/entries/analogy-medieval/>, 2004.

Batterman, Robert (2002). *The Devil in the Details: Asymptotic Reasoning in Explanation, Reduction and Emergence*. New York, NY: Oxford University Press, 2002.

Cartwright, Nancy. *The Dappled World*. Cambridge: Cambridge U Press, 1999.

Collins, James. *God in Modern Philosophy*, Chicago: Henry Regnery Co., 1959.

Copleston, F., *A History of Philosophy*, vol. III, Westminster, MD: Newmann Press, 1953.

Dupre, Louis, ed., *Nicholas of Cusa*, *American Catholic Philosophical Quarterly*, vol. LXIV, no. 1, Winter, 1990.

Hopkins, Jasper. *A Concise Introduction to the Philosophy of Nicholas of Cusa*, Minneapolis: University of Minnesota Press, 1978.

_____, *Nicholas of Cusa on Learned Ignorance*, Minneapolis: Arthur Banning Press, 1981.

_____, *Nicholas of Cusa's Metaphysic of Contraction*, Minneapolis: Arthur Banning Press, 1983.

_____, *Nicholas of Cusa's Dialectical Mysticism*, Minneapolis: Arthur Banning Press, 1985.

Kallfelz, William, "Nicholas of Cusa's Theology of the Infinite Reflected in His *Docta Ignorantia*, Master of Theological Studies major research paper, Atlanta, Georgia: Candler School of Theology, Emory University, 1996.

McGinn, Bernard. "Evil-Sounding, Rash, and Suspect of Heresy': Tensions Between Mysticism and Magisterium in the History of the Church," *The Catholic Historical Review*, vol. XC, No. 2, April, 2004.

Morrison, Margaret (2000a) "Unity and the Limits of Science," in Carrier, Massey, Ruetsche, eds., *Science at Century's End: Philosophical Questions on the Progress and Limits of Science* (Pittsburgh-Konstanz Series in the Philosophy and History of Science, University of Pittsburgh Press, 2000) 217-233.

_____. (2000b) *Unifying Scientific Theories*, Cambridge: Cambridge U. Press, 2000.

Petry, Ray, ed., *Late Medieval Mysticism*, Philadelphia: The Westminster Press, 1957.