Numerical Simulations of Busemann Hypersonic Inlets At Finite Flight Angles

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Complete axisymmetric Busemann inlet bodies with a design Mach number of 6.0 are numerically simulated by the solution of the Euler equations at off-nominal pitch/yaw angles ranging up to 9°. The inviscid flow solutions are presented graphically, and performance parameters at the throat including pressure recovery, static temperature ratio, static pressure ratio and adiabatic compression efficiency, are found for each flight condition. Simple statistical measures are used to obtain some indications of distortion level at the throat. Compared to the ideal conceptual Busemann inlet, an inlet with a finite leading edge attached to the Busemann compression surface is approximately 10% lower in adiabatic efficiency. Further, when the inlet is turned by 3°, the compression efficiency decreases again by nearly 10%. At a 6° flight angle, the adiabatic compression efficiency drops to a mere 42%. When the inlet is finally turned to 9°, it ceases to meaningfully function as an inlet and simple computations for the efficiency give unphysical results. Graphical and tabular presentations of the flow solutions for each case are provided and discussed.

Nomenclature

\begin{itemize}
  \item \textit{A} \hspace{1em} \text{Area}
  \item \textit{δ} \hspace{1em} \text{Flow inclination angle at entrance to shoulder shock}
  \item \textit{γ} \hspace{1em} \text{Ratio of specific heats}
  \item \textit{M} \hspace{1em} \text{Mach number}
  \item \textit{\dot{m}} \hspace{1em} \text{Mass flow rate}
  \item \textit{η_c} \hspace{1em} \text{Adiabatic compression efficiency}
  \item \textit{N} \hspace{1em} \text{Number of samples}
  \item \textit{π_c} \hspace{1em} \text{Ratio of post-compression to freestream total pressure}
  \item \textit{ψ} \hspace{1em} \text{Ratio of post-compression to freestream static temperature}
  \item \textit{σ} \hspace{1em} \text{Sample-standard form of standard deviation}
  \item \textit{s_{\text{param}}} \hspace{1em} \text{Sample value of parameter \textit{param}}
  \item \textit{CR} \hspace{1em} \text{Geometric contraction ratio (}\frac{A_{\text{capture}}}{A_{\text{throat}}}\text{)}
\end{itemize}

\text{Subscript}

\begin{itemize}
  \item \textit{t} \hspace{1em} \text{Total or stagnation properties}
  \item \textit{∞} \hspace{1em} \text{Freestream condition}
\end{itemize}

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I. Introduction

The Busemann inlet concept appears to be a promising basis for practical high-performance inward-turning inlet designs in high-speed propulsion applications. It has been the subject of several studies, both recent\textsuperscript{1–4} and further in the past.\textsuperscript{5,6} The benefits of the Busemann flowfield include attributes such as its high pressure recovery, its inward-tuning nature, and that in an inviscid gasdynamic model, its ideal flowfield is wholly known analytically. This latter aspect allows for the creation of streamtraced inlets which capture a specific portion of a “virtual” Busemann inlet flow. Some form of streamtracing will most likely be employed in the design of any real Busemann-derived inlet as was done, for example, in the SCRAM missile program of the 1960s.\textsuperscript{7}

One vexing aspect of supersonic and hypersonic inlet performance is the sensitivity to inflow Mach numbers and flow angularity. Many published design studies focus on design point performance, but any real vehicle employing these inlets will undergo non-negligible periods of operation at off-design conditions. The aim of this work is to serve as a benchmark for the off-nominal flight angle performance of inlets derived from the Busemann flowfield. Since streamtraced inlets can be formed from any star-convex throat or capture shape, designers of such inlets should be aware of the baseline performance level which suitable streamtraced designs should surpass.

In this study, the traditional full Busemann inlet internal geometry is incorporated into a representative full three-dimensional body including a finite semi-circular leading edge. This inlet, depicted in Figure 1 is then simulated through computational solutions of the Euler equations at nominal (design point) conditions as well as at several off-nominal angles at the design point Mach number. Due to the axisymmetry of the Busemann inlet, any flow angularity can be equally construed as either pitch or yaw. The simulation is restricted to the Euler equations, rather than the full Navier-Stokes equations, in order to ascertain the immediate results due to the combination of geometry and flow angularity. Once viscous flow is considered, additional phenomena such as shock-boundary layer interaction and contraction due to the boundary layer play a part in the overall system. Since optimizing a Busemann inlet for viscous flow involves additional complications and tradeoffs,\textsuperscript{3,8} it makes sense when taking a first look at this aspect of off-design behavior to start with a simpler physical system to analyze.

![Figure 1: Isometric views of a Busemann inlet.](image)

II. Dimensional Inlet Model

The selected design freestream Mach number and isentropic flow turning angle for this study are 6.0, and 5°, respectively. Design point performance and physical characteristics are detailed in Table 1. An isolator/combustor entrance pressure of approximately half an atmosphere is generated by this inlet at its design Mach number when flying at an altitude of 80,000 feet.

The traditional Busemann inlet design procedure, as described in the literature,\textsuperscript{1,4–6} yields the flow properties at the inflow plane, shoulder entrance, and throat, as well as the wall curve and internal contraction ratio. However, the Busemann inlet model assumes inviscid flow and an isentropic entrance plane. Even if the gasdynamic model is restricted to an inviscid one, an isentropic leading edge would require infinitesimal thickness to achieve since any streamline inside of the stagnation point is captured. As the purpose of this
Table 1: Physical and ideal thermodynamic performance parameters for internal Busemann flowfield.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freestream Mach number</td>
<td>( M_\infty )</td>
<td>6.000</td>
</tr>
<tr>
<td>Throat Mach number</td>
<td>( M_{throat} )</td>
<td>3.583</td>
</tr>
<tr>
<td>Static pressure ratio</td>
<td>( \frac{p_{throat}}{p_\infty} )</td>
<td>18.208</td>
</tr>
<tr>
<td>Static temperature ratio</td>
<td>( \psi )</td>
<td>2.298</td>
</tr>
<tr>
<td>Pressure recovery</td>
<td>( \frac{p_{throat}}{p_{t\infty}} )</td>
<td>0.989</td>
</tr>
<tr>
<td>Contraction Ratio</td>
<td>( CR )</td>
<td>7.172</td>
</tr>
<tr>
<td>Length per unit of throat radius</td>
<td>–</td>
<td>20.087</td>
</tr>
<tr>
<td>Shoulder shock entrance Mach number</td>
<td>( M_s )</td>
<td>3.937</td>
</tr>
<tr>
<td>Flow inclination angle at shoulder shock entrance</td>
<td>( \delta )</td>
<td>5°</td>
</tr>
</tbody>
</table>

When considering the ideal, internal flowfield alone, no specific length scale need be chosen, and the inlet can be scaled “photographically.” Once a finite leading edge is defined, the length scale of the overall inlet must be fixed as well. Thus the throat radius is chosen as 1 ft., and the leading edge radius is set to 0.0025 in. This yields an inlet approximately 20.09 ft. long with a throat area of \( \pi \) ft.\(^2\). Compared to the ideal contraction ratio of 7.172, the finite leading edge results in an actual contraction ratio of 7.186, an increase of approximately 0.19%. The physical characteristics of the finite inlet body are detailed in Table 2.

Figure 2: Close-up of leading edge region used to form cross-section of three-dimensional wetted inlet.
III. Flight Conditions, Performance Metrics, and Simulation Details

Due to the inherent sensitivity of high-speed vehicles to flow angularity, as well as the large forces and loads involved, real flight vehicles will tend to be limited in their off-nominal flight angle envelope. As a realistic upper bound angle, \(9^\circ\) is chosen. Simulation results are provided for nominal, \(3^\circ\), \(6^\circ\), and \(9^\circ\) flow angles. These angles were imposed in the inlet coordinate system depicted in Figure 1 as positive yaw values, i.e., rotation about the Z axis in the direction of positive Y values. The net velocity vector was always prescribed to give a Mach number of 6.0, corresponding to the design speed.

Gauging the performance of the inlet at each flight condition is done through several metrics. Predictably, pressure recovery, i.e., the ratio of the total pressure of the throat flow to the freestream flow, is a principal figure of merit. An efficiency metric is also computed. In this work the adiabatic compression efficiency which is defined as:

\[
\eta_c = \frac{\psi - \left(\frac{1}{\pi_c}\right)^{\frac{\gamma - 1}{\gamma}}}{\psi - 1}
\]  

is used. Here \(\psi\) is the static temperature ratio between freestream and throat, \(\pi_c\) is the total pressure ratio between the freestream and throat, and \(\gamma\) is the familiar ratio of specific heats at the throat. For throat flows which are non-uniform however, as occurs at off-nominal flow angles, it is necessary to use averaged quantities for the thermodynamic properties. In the present analyses, area-averaging is used for static pressure and mass-averaging is used for all other properties.

The impact of flow angularity also merits the examination of the mass flow ratio, i.e., the ratio of the mass flow at the throat at a given flight condition to the design point throat mass flow. Additionally, an attempt is made to quantify the “distortion,” or degree of nonuniformity at the throat. This is done by examining the equal-area sampled standard deviation of mass flux, static pressure ratio, total pressure ratio, and Mach number. In order to compute these values, the circular throat shape was sampled such that each sampling location was the center of an equal area annular sector segment. Twenty radial points and ninety circumferential points were used in the sampling grid, for a total of 1,800 sample values per parameter. The sample-standard formula for standard deviation was used:

\[
\sigma(\text{param}) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (s_{\text{param}} - \bar{s}_{\text{param}})^2}
\]  

where \(N\) is the number of samples (again, 1,800 here).

The flow simulations presented were performed with Lockheed Martin’s Splitflow CFD code\(^a\).\(^1\) Splitflow is a finite-volume flow solver with up to second-order spatial accuracy, and uses an implicit time marching scheme. Fluxes are upwind differenced for inviscid terms, and Roe’s flux difference splitting scheme is employed. Splitflow operates on unstructured hexahedral meshes which are adaptively refined over the course of a solution run. All flow simulations were performed at the same atmospheric conditions and Mach number, with only the inflow angularity varied. All simulations were performed on whole inlet bodies, with no computational symmetry planes. Table 3 lists the final approximate cell counts for the various cases. Each case required approximately two to three days of computational time on four AMD K8 model 246 processors, linked in pairs by bonded full-duplex gigabit ethernet, to reach convergence. Convergence was determined by examining residual values and the rate and locations of grid refinement. More aggressive optimization of the grid parameter control values over the course of the run would likely have resulted in shorter run times.

<table>
<thead>
<tr>
<th>Inflow Angle</th>
<th>Total Cell Count, millions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>1.31</td>
</tr>
<tr>
<td>(3^\circ)</td>
<td>1.5</td>
</tr>
<tr>
<td>(6^\circ)</td>
<td>2.726</td>
</tr>
<tr>
<td>(9^\circ)</td>
<td>2.942</td>
</tr>
</tbody>
</table>

Table 3: Number of cells in CFD simulations.

\(^a\)Splitflow, ©Lockheed Martin Corporation. Used by permission.
IV. Performance Results

IV.A. Finite Busemann Inlet at Nominal Conditions

The flow simulation result obtained for the on-design finite Busemann inlet compares favorably with inviscid theory. Figures 3a and 3a are plots of the magnitude of the density gradient from the Euler flow solution for the on-design case. They are similar to Schlieren image diagnostics performed in supersonic flow experiments, and will be used to illustrate the steep density changes occurring within each configuration discussed. From Figures 3d - 3e, it can be seen that the throat flow is not completely uniform – rather there is an outer annular region of lower performance surrounding a core whose parameters are close to the ideal values. This is attributable to the leading edge shock, as well as as the additional mass capture forced by the finite leading edge. Note that, as indicated in Tables 4 and 5, the nominal inlet configuration also suffers reduced efficiency and has small, but non-negligible non-uniformity in the flow properties at the throat. Compared to previous simulations of this configuration with an isentropic entry into the internal compression surface at identical flight conditions, the inclusion of the leading edge is clearly responsible for the additional deviations from ideal values.

IV.B. Flight Angle of 3°

For the first off-nominal case of 3° inflow angle, it is immediately visible that the leading side of the leading edge causes a “blanking” effect, resulting in a low-pressure region immediately behind the leading side. On the other hand, the trailing side is now impacted with flow at a steeper incidence angle than in the nominal case. The high-incidence angle flow must then be turned through a larger angle in order to conform to the inlet walls. As a result, the throat shock becomes skewed so that its tip now points toward the trailing side and the cone opens up toward the leading side. The pointed cone shock then intersects the inlet wall upstream of the throat on the leading side and reflects, causing a strong pressure rise where the flow passes through two shock surfaces as it proceeds axially. In addition, due to the imperfect cancellation of the flow turning shock at the shoulder, an additional rearward-pointing conical shock surface is generated. However, since the upstream conditions entering the shoulder vary, so does the strength and curvature of this additional shock surface. Figure 4c shows the results of the multiple varying-strength shock fronts. The net results as seen at the throat are clearly evident upon examination of Figures 4d - 4e.

The initial flow distortion which skews the initial conical shock leads to increasingly greater downstream intensity of circumferential variation. Correspondingly, there is a significant increase in shock-induced static temperature and pressure rises, which also serve to reduce the Mach number at the throat. Compared with the nominal case, the areal nonuniformity takes a steep jump upward, especially in the mass flux and pressure ratio. Adiabatic efficiency drops to approximately 72%.

IV.C. Flight Angle of 6°

At an inflow angle of 6°, the skew of the initial conical shock increases further, causing it to impact the internal wall further forward of the throat. The reflected shock is also steeper, inciting a greater pressure rise. Again, a non-axisymmetric rearward-pointing conical shock surface is generated at the shoulder. For sections of the flow that have passed through three shock surfaces, the pressure ratio becomes nearly 81. Unsurprisingly, the distorted shocks cause significant increases in temperature and pressure ratio, and drop the efficiency to a very poor value of roughly 42%. Similarly, the areal nonuniformity climbs by all measures as well.

IV.D. Flight Angle of 9°

With the flight angle further increased to 9°, now far from the nominal condition, the inlet stops functioning meaningfully. The low pressure region behind the leading side of the inlet becomes separated. This leads to less nonuniformity, as can be seen in Figures 6d - 6e, and from the measures provided in Table 5. However, the aerothermodynamic performance of the inlet as an efficient supersonic diffuser is completely ruined. Even though the conservation laws are satisfied in this operating condition, use of the mass-averaged thermodynamic properties at the throat yields an unphysical adiabatic efficiency figure. On an areal basis the majority of the throat has a pressure ratio of less than unity, hence the resultant efficiency value. Additionally, there are very high total pressure losses, especially concentrated in the multiply-shocked region. Obviously
this flight angle is outside the viable envelope for this inlet. In a real, viscous inlet, shock-induced boundary layer separation and possibly unstart would most likely occur at or before this point.

Figure 3: Busemann Inlet at $M = 6.0$, nominal, no yaw.
(a) Cutaway: Shock structure at plane $Z = 0$. Yawed upward in this view.

(b) Cutaway: Shock structure at plane $Y = 0$. Yawed out of page in this view.

(c) $\frac{p}{p_\infty}$ at several axial stations. Yawed about $Z$ axis, toward right in this view.

(d) Throat pressure recovery. Yawed right in this view.

(e) Mach number at throat. Yawed right in this view.

(f) $\frac{p}{p_\infty}$ at throat. Yawed right in this view.

Figure 4: Busemann Inlet at $M = 6.0$, yawed $+3^\circ$ about $Z$ axis.
(a) Cutaway: Shock structure at plane $Z = 0$. Yawed upward.

(b) Cutaway: Shock structure at plane $Y = 0$. Yawed out of page in this view.

(c) $\frac{p}{p_\infty}$ at several axial stations. Yawed about Z axis, toward right in this view.

(d) Throat pressure recovery. Yawed right in this view.

(e) Mach number at throat. Yawed right in this view.

(f) $\frac{p}{p_\infty}$ at throat. Yawed right in this view.

Figure 5: Busemann Inlet at $M = 6.0$, yawed $+6^\circ$ about Z axis.
Figure 6: Busemann Inlet at $M = 6.0$, yawed $+9^\circ$ about Z axis.

V. Conclusions and Future Work

The simulations performed and analyzed illustrate important aspects of the standard Busemann inlet. Even with no imposed inflow angle, by simply incorporating the ideal Busemann compression surface into a practically realizable inlet with minimal deviation, the compression efficiency immediately decreases by a substantial amount. When this is compounded with a finite flight angle, the performance degradation is severe. At an angle of $9^\circ$, the inlet completely fails to serve its intended purpose. While the basic results follow intuition, it is perhaps surprising how little deviation is required from the nominal flight condition to
see a marked decrease in measurable performance.

Two key design considerations are suggested by these results. The first is the need for careful tailoring of the leading edge. In addition to the commonly held wisdom of trying to use as sharp a leading edge as feasible, it seems meaningful to consider modifying the initial portion of the internal compression wall to mitigate the effects of the leading edge shock, perhaps by a small controlled expansion. Additionally, shaping the leading edge in a manner different than the standard semicircular arc may also be worth exploring, so long as the shape is still manufacturable.

The other design implication is a clear need to avoid using the prototypical Busemann inlet as-is. Rather, streamtracing should undoubtedly be performed in order to take advantage of the Busemann flowfield while ameliorating or overcoming the problems due to the completely unswept and unrelieved leading edge. Again, while this may be intuitive, these simulations make readily apparent just how important it is to use streamtracing or a comparable method to develop the net three-dimensional inlet body shape.

Future designs and design studies will need to consider viscous flow, which leads to additional complexity. Questions on the optimal approach to boundary layer correction, as well as the need and strategy for boundary layer bleeds will require thoughtful consideration. Additionally, depending on the orientation and flight angle of the inlet, the outer shell shape could also influence the off-design behavior. The analyses performed in this work with simple equal-area sampled standard deviation of some typical inlet figures of merit should be used as a starting point to formalize appropriate distortion metrics for ramjet/scramjet inlets. In this work, these measures trended well with inlet performance as long as the inlet was not operated in a grossly off-design way.

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### References


