Fundamental Tradeoffs and Constrained Coalitional Games in Autonomic Wireless Networks

Tao Jiang and John S. Baras
Institute for Systems Research and
Department of Electrical and Computer Engineering
University of Maryland, College Park, MD 20742, USA
Email: {tjiang,baras}@isr.umd.edu

***Abstract*— Autonomic networks depend on collaboration between their nodes for all their functionalities. The nodes, even if modeled as selfish, gain from such collaboration, in the sense that they can accomplish functionality and performance that is impossible to achieve without such collaboration. However, such gains from collaboration do not come for free. There are costs for such collaboration incurred by each node (e.g. energy consumption for forwarding other nodes packets). In this paper we use constrained coalitional games (i.e. collaborative dynamic games subject to constraints or costs for collaboration) to investigate several key problems in autonomic wireless networks including: network formation, efficient topologies and stability of coalitions.**

I. Introduction

Autonomic networks rely on the collaboration of participating nodes for almost all their functionalities, for instance, to route data between source and destination pairs that are outside each other's communication range. However, because nodes are resource constrained, we deal with networks composed of users who are trying to maximize their own benefit from participation in the network. In the case of packet forwarding, the fundamental user decision is between forwarding or not forwarding data packets sent by other users. Given the constraints (mostly related to energy) that the user faces, there is a very real cost incurred when choosing to forward. So, all users would like to send their own data packets, but not forward those of other users. Unfortunately, if all users were to do that, the network would collapse.

In this work, we assume that users want to be connected to as many other users as possible, directly (one-hop) or indirectly (multi-hop, through other users). In other words, by activating a communication link towards one of their neighbors, they gain by having access to the users with which that neighbor has activated *his* links, and so on, recursively. In the mean while, activation of links introduces cost. The more neighbors a user is connected to, the more packets he has to forward, which results in higher communication cost. This work is inspired by our interests in studying the fundamental tradeoff between gain and cost in the context of user collaboration in autonomic networks.

The conflict between the benefit from collaboration and the required cost for collaboration naturally leads to game-theoretic studies, where each node strategically decides the degree to which it volunteers its resources for the common good of the network. The players in game theory attempt to maximize an objective function that takes the form of payoff. A user's payoff depends not only on whether he decides to collaborate or not, but also on whether his neighbors will decide to collaborate.

Different with previous work in the literature, we study collaboration based on the notion of *coalitions*. The concept of users being connected to each other, and acquiring access to all the other users that each of them had so far access to, can be well captured by coalitional game theory (also known as cooperative game theory [1]). In coalitional game theory, the central concept is that of coalition formation, i.e., subsets of users that join their forces and decide to act together. Players form coalitions to obtain the optimum payoffs. The key assumption that distinguishes cooperative game theory from non-cooperative game theory is that players can negotiate collectively [2].

A question that has only relatively recently began to attract attention ([3] is the first work in this area) is the actual way in which the coalition is formed. The coalitional game is usually modeled as a *two-phase structure*. Players must first decide whether or not to join a coalition. This is done by pairwise games, in which *both* players have to agree to activate a link between them and thus join the same coalition. In our work, this pairwise game involves, for each node, a comparison between the cost for activating the link towards the other node, and the benefit from joining the coalition that the other player is a member of. In the second step, players in one coalition negotiate the payoff allocation based on the total payoff of the coalition. The central problem is to study the convergence of the iterated pairwise game and whether the dynamics result in a stable solution.

The rest of the paper is organized as follows: Section II reviews literature on collaboration and coalition formation using game theory. In Section III we describe the mathematical framework within which we deal with the concepts just discussed. The two-phase coalitional game is defined in Sec. IV. Section V investigates the dynamics of the iterated pairwise game including its convergence and the network topology at the equilibrium. We discuss the stability of the network at the equilibrium in constrained coalitional games in Sect. VI.

II. Related Work

In recent years, game theory has been widely used to model collaboration enforcement mechanisms in autonomic networks. Srinivasan et al. [4], Urpi et al. [5] and Félegyházi et al. [6] all address the problem of packet forwarding among selfish nodes in wireless ad hoc networks.

In [4], energy constraints are taken into account to describe the packet forwarding game. They show that under the energy constraints, the G-TFT (general tit-for-tat) strategy promotes cooperation if every node conforms it. Urpi et al. [5] propose a general framework for cooperation with energy constraints and traffic patterns. They derive some enforceable policies for cooperation. In [6], the authors prove that an incentive scheme is needed to enforce cooperate in most situations. They introduce the concept of dependency graph, where users' behaviors influence their future throughput.

Game theory has been used in other aspects of wireless networks as well. For instance, Korilis et al. [7] study the problem of allocating link capacities in routing decisions. Roughgarden [8] quantifies the Nash equilibria arising from non-cooperative routing behavior.

All of the work mentioned above model the conflicts in autonomic networks using *non-cooperative* game theory. In this paper, we study cooperation with the concept of coalition. Two users join the same coalition by forming a link between them. We are mainly interested in formation of coalitions modeled as a *cooperative* game. In the cooperative game framework, the collaboration of users are studied as a group behavior rather than stratigies adopted by single users. We believe that the cooperative game theory provides an appropriate model where users may act together to get higher payoff.

In [9], Michiardi and Molva modeled cooperation and coalition formation in mobile ad hoc networks using cooperative game theory. Similar to our work, their cooperative game has two phases as well. In the first phase, agents play N-node prisoner's dilemma (PD) game. Based on the absolute payoff obtained in the non-cooperative PD game, agents can derive their relative payoff compared to the overall payoff of all agents by the ERC theory [10]. The authors show that the ERC preference structure allows for a precise characterization of conditions under which coalitions are enlarged. They also propose to use a reputation based system CORE to encourage collaboration. The main difference between [9] and our work is that we model the game in a more realistic way, where the cost of establishing a link between users is a function of physical distance between users and the gain depends on network topology. The stability of coalitions in [9] is in terms of Nash equilibrium, which is a concept of non-cooperative games. In our work, we investigate new concept of stability: pairwise stability and core stability.

Coalition formation has been extensively studied for social and economic networks in recent years[11] and [12]. The networks we study are wireless communication networks, where the physical locations of nodes are important, and the payoff is in the context of communication network functionalities, such as packet forwarding used in the work. The above two properties of communication networks are not captured in social and economic networks.

III. Problem Formulation

Suppose there are n nodes[1] in the network. Define the set of nodes $N = \{1, 2, \ldots, n\}$. The communication structure of the network is represented by an *undirected* graph g, where a link between two nodes implies that they are able to directly communicate. Because the willingness of both nodes is necessary to establish and maintain a link, we use undirected links to model the symmetric communications between neighboring nodes. The undirected links are also called *pairwise links*. For instance, in wireless networks, reliable transmissions require that the two nodes interact in order to avoid collisions and interference.

Let g^N represent the complete graph, where every node is directly connected to every other node, and let the set $G = \{g | g \subseteq g^N\}$ be the set of all possible graphs with N nodes. If i and j are directly linked in g, we write $ij \in g$. Let $g + ij$ denote the graph obtained by adding link ij to the existing graph g where $ij \notin g$ and $g - ij$ denote the graph obtained by severing link ij from the existing graph g where $ij \in g$(i.e., $g + ij = g \cup \{ij\}$ and $g - ij = g \setminus \{ij\}$). The set of nodes in graph g is $N(g) = \{i | i \in g\}$ and $n(g)$ is the number of nodes in g.

A communication link is established only if two end nodes agree to collaborate with each other, i.e., they are *directly* connected with each other in g. Once the link is added, two end nodes join one coalition and they agree to forward all the traffic from each other. Note that *indirect* communication between two players requires that there is a path connecting them. A path in g connecting i_1 and i_m is a set of distinct nodes $\{i_1, i_2, \ldots, i_m\} \subset N(g)$, such that $\{i_1 i_2, i_2 i_3, \ldots, i_{m-1} i_m\} \subset g$.

The communication structure g gives rise to a partition of the node set into groups of nodes who can communicate with each other. A *coalition* of g is a subgraph $g' \subset g$, where $\forall i \in N(g')$ and $j \in N(g')$, $i \neq j$, there is a path in g' connecting i and j, and $ij \in g$ implies $ij \in g'$.

A. Gain

As we have discussed, users obtain benefits by joining a coalition. A user's gain is explicitly defined on how he is connected to other users in the coalition. In this paper, we assume that nodes always have information sent to other nodes in the network. Thus we assume that each node potentially offers to other nodes benefits V per time unit. For instance, V could be the number of bits per time unit each node could provide, which is a function of the link capacity.

The potential benefit V is an expected value, which may be reduced during transmissions in the network. Following the Jackson-Wolinsky *connections model* [13], the gain of node i

[1] In this paper, the terms node, player and user are interchangeable.

is defined as

$$w_i(g) = \sum_{j \in g} V \delta^{r_{ij}-1}, \quad (1)$$

where r_{ij} is the number of hops in the shortest path between i and j (also known as the geodesic distance in graph theory), and $0 \leq \delta \leq 1$ is the communication depreciation rate. If there is no path between i and j, $r_{ij} = \infty$. The gain function gives higher value to paths with smaller number of hops. It captures the fact that more directly connected nodes gain more than nodes far away in terms of geodesic distance. The depreciation can be explained by communication reliability and efficiency due to transmission failures or delay.

B. Cost

On the other hand, activating links is costly. For instance, the cost for user i to activate his communication link to user j can be equal to the transmission energy (or power) necessary for i to send data to j. According to the wireless propagation model, transmission power consumption depends on the geometric distance between i and j, denoted as d_{ij}[2]. We define the cost c_{ij} as a function of d_{ij}

$$c_{ij} = P d_{ij}^{\alpha},$$

where P is a parameter depending on the transmitter/receiver antenna gain and the system loss not related to propagation, and α is the path loss exponent depending on the specific propagation environment.

Notice that in our model, a link can be activated between any pair of nodes by adjusting transmission power. However, a link between two nodes that are faraway introduces very high cost, so the link with high cost will only be activated if the gain of activating it is very high in the coalition formation process.

IV. Coalition Formation Game

The coalition formation is modeled as a two-phase game, which is called *coalition formation game* in the literature [12]. In this section, we give the detailed description of these two phases.

A. Pairwise game

The pairwise game is modeled as an iterated process in which individual nodes activate and delete links based on the improvement that the resulting networks offers them relative to the current network. A link between two nodes can be formed only if both nodes agree to activate the link, while a single node can sever an existing link. Each user receives a payoff based on the network configuration that is in place.

Initially the n nodes are disconnected. The nodes meet over time and have the opportunity to form links with each other. Time, T, is divided into periods and is modeled as a countable, infinite set, $T = \{1, 2, \ldots, t, \ldots\}$. Let $g^{(t)}$ represent the network that exists at the end of period t.

[2] d_{ij} is the geometric distance as opposed to the geodesic distance r_{ij}

A *strategy* of node i is a vector, defined as $\gamma_i = (\gamma_{i,1}, \ldots, \gamma_{i,i-1}, \gamma_{i,i+1}, \ldots, \gamma_{i,n})$, where $\gamma_{i,j} \in \{0, 1\}$ for each $j \neq i$. $\gamma_{i,j} = 1$ is interpreted as saying that node i wants to form a link with node j, while $\gamma_{i,j} = 0$ states that i does not directly communicate with node j. The set of all strategies of node i is denoted by Γ_i. Since node i has the option of forming or not forming a link with each of the remaining $n-1$ nodes, the number of strategies of node i is $|\Gamma_i| = 2^{n-1}$. The set $\Gamma = \Gamma_1 \times \cdots \times \Gamma_n$ is the strategy space of all the nodes. A link ij is formed in network g only if $\gamma_{i,j} = 1$ and $\gamma_{j,i} = 1$. Therefore, a strategy profile $\gamma^{(t)} = (\gamma_1^{(t)}, \ldots, \gamma_n^{(t)})$ at time period t corresponds to the network $g^{(t)}$ at time t. Figure 1 gives an example of the correspondence between the strategy profile and the network. There is a link between 1 and 3 because $\gamma_{1,3} = \gamma_{3,1} = 1$.

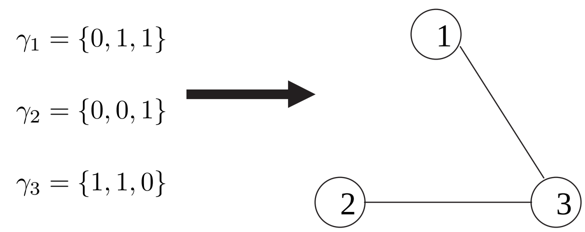


Fig. 1. The correspondence of strategy and network

Define $\mathcal{N}_i(g) = \{j \in N | ij \in g\}$ as the neighbor set of node i. Furthermore, a pair of nodes are *connected* in network g if there is a path between i and j, denoted as $i \overset{g}{\longleftrightarrow} j$. We define $\mathcal{C}_i(g) = \{j \in N | i \overset{g}{\longleftrightarrow} j\}$ as the set of all nodes with whom i communicates, either directly or through other nodes. The payoff of node i from the network g is defined as

$$v_i(g) = w_i(g) - c_i(g) = \sum_{j \in \mathcal{C}_i(g)} V \delta^{r_{ij}-1} - \sum_{j \in \mathcal{N}_i(g)} P d_{ij}^{\alpha}. \quad (2)$$

Now we describe the dynamic process of the iterated pairwise game. The game is assumed to be repeated in each time period $t = 1, 2, \ldots$. Define p_{ij} as the probability that the node pair ij is selected, in each time period, to play the pairwise game. Notice that there may be multiple pairs selected in the same time period. These pairs can play simultaneous as long as they do not contain the same node. More specifically, if both ij and ik are selected, i cannot play two games simultaneously. Thus i will not play any of the two games and it will inform its neighbors j and k as well. Therefore, there is no game played on links ij and ik in the current time period. Figure 2 shows a scenario where node pair 12, 13 and 56 are selected. Since 1 is selected to play two games, it will not play any of them. No game takes place between either 12 or 13. On the other hand, 56 will play the game. This dynamic process requires no communication or synchronization for selecting node pairs and playing games. Each pair of nodes tosses a coin to decide whether they need play the game. If a node is selected to play the game, he first checks if he plays two or more games simultaneously. If yes, it stops all of the games and informs

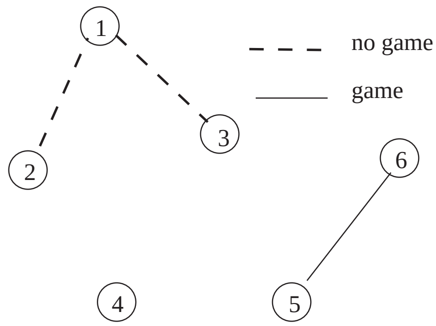


Fig. 2. Games between selected node pairs

its neighbors. Therefore, the dynamic pairwise game is purely distributed.

We assume that each node is myopic. Given that nodes i and j play the game, if the link ij is already in the network, then the decision is whether to sever it, and otherwise the decision is whether to activate the link. The nodes involved act myopically, activating the link if it makes each at least as well off and one strictly better off, and deleting the link if its deletion makes either player better off. Mathematically speaking, if only node pair ij is selected in time period t, then the network $g^{(t+1)}$ has either

- $g^{(t+1)} = g^{(t)} - ij$ if $v_i(g^{(t)} - ij) > v_i(g^{(t)})$ or $v_j(g^{(t)} - ij) > v_j(g^{(t)})$, or
- $g^{(t+1)} = g^{(t)} + ij$ if $v_i(g^{(t)} + ij) > v_i(g^{(t)})$ and $v_j(g^{(t)} + ij) \geq v_j(g^{(t)})$, or $v_i(g^{(t)} + ij) \geq v_i(g^{(t)})$ and $v_j(g^{(t)} + ij) > v_j(g^{(t)})$, or
- $g^{(t+1)} = g^{(t)}$ if none of the above satisfies.

If more than one pairs are selected to play the game, each pairwise game could be considered separately.

In this paper we do not consider networks with nodes that have the ability of foreseeing the evolution of the game. Such networks may be important when there are relatively small numbers of forward-looking players who are well-informed about the value of the network and the motivations of others. However, in large autonomic networks where nodes' information is local and limited, myopic behavior is a more natural assumption, and a reasonable starting point of our work.

If after some time period t, no additional links are formed or severed, then the network formation process has reached a *stable state*. Thus a coalition or coalitions are formed at the stable state. Then the coalition formation game moves to the second phase, in which users act together to achieve maximum payoffs.

B. Coalitional game

A coalition is a subset of nodes that is connected in the subgraph induced by the active links. In other words, two nodes are in the same coalition, if and only if there exists a path of active links between them. If two nodes of separate coalitions join, then the two coalitions *merge* into one. In this paper, we are interested in the total productivity of the coalition formed by selfish nodes, how this is allocated among the individual nodes and the stability of the coalition. These notions are captured by coalitional games.

Coalition formation has been widely studied in economics and sociology in the context of coalitional game [12], [11]. In our game, some nodes are not directly connected with each other; therefore the game we consider has to take the communication constraints into consideration. Myerson [14] was the first to introduce a new game associated with communication constraints, the *constrained coalitional game*, which incorporates both the possible gains from cooperation as modeled by the coalitional game and the restrictions on communication reflected by the communication network.

An important concept in coalitional games is the characteristic function v [15]. Since the game we study has communication constraints, the characteristic function v is defined on a particular network rather than on a set of nodes in general coalition games, i.e., $v : g^N \rightarrow \mathbb{R}$ defined on all subsets of G with the convention: $v(\emptyset) = 0$. Notice that in our work the empty set $\emptyset$ represents a graph where there is no link between any two nodes in the graph. Given $g \subset g^N$, $v(g)$ is interpreted as the maximum payoff the network g can get given the network structure.

In our case, the value of v is the maximum aggregate of the payoffs from all nodes in the graph

$$v(g) = \sum_{i \in g} v_i(g) \quad (3)$$

A payoff allocation rule $x : g \rightarrow \mathbb{R}^n$ describes how the value $v(g)$, associated with each network, is distributed to the individual nodes. $x_i(g)$ is the payoff of node i from network g and under the characteristic function v. For a graph g', which is a subgraph of g, define

$$x(g') = \sum_{i \in g'} x_i(g).$$

The payoff allocation is *feasible* if $x(g) \leq v(g)$ and *efficient* if $x(g) = v(g)$. In our case, the payoff may not be transferable, so the payoff allocation rule represents the payoff that each node receives from the network, i.e., $x_i(g) = v_i(g)$. It is easy to show that such a payoff allocation rule is feasible and efficient. We will discuss the stability of the constrained coalitional game in details in Sect. VI.

V. Dynamics of the Iterated Pairwise Game

A. Convergence

Having described the iterated pairwise game in Sec. III, we study the convergence of such a game. In particular, we are interested in the conditions under which all nodes in the network are connected, i.e., $\mathcal{C}_i = N, \forall i \in N$. The coalition that contains all the nodes is called the "grand coalition".

To study the convergence, we first define a concept of stability: *pairwise stability*.

Definition 1: A network g is *pairwise stable* if

- for all $ij \in g$, $v_i(g) \geq v_i(g - ij)$ and $v_j(g) \geq v_j(g - ij)$, and

- for all $ij \notin g$, if $v_i(g) < v_i(g+ij)$ then $v_j(g) > v_j(g+ij)$ or if $v_j(g) < v_j(g+ij)$ then $v_i(g) > v_i(g+ij)$.

We first give a simple fact on the dynamics of the pairwise game:

Lemma 1: The iterated pairwise game converges to a pairwise stable network or a cycle of networks.

Sketch of Proof (Informal) If in a certain time period, the network is not pairwise stable, there must exist at least one link that can be formed or severed to improve the payoffs of the two end nodes. As long as such a link is selected, the network changes to another network. This procedure either stops at the pairwise stable network or it changes back to a network that has been met due to the limited number of possible networks $|g^N|$. In the later case, the procedure forms a cycle. ■

Figure 3(a) is a network of 6 nodes starting from no links. Take $c_{12} = c_{23} = c_{34} = c_{45} = c_{56} = c_{61} = 1$, where $c_{ij} = c_{ji}$ for all $i, j \in N$ and the cost of other links are much greater than 1, $V = 0.9$ and $\delta = 0.3$. We observe that a first link's costs exceed its payoff, while thereafter links are valuable. If users follow the myopic strategy defined in Sec. IV, no link could be formed at all. However, it is obvious that the network shown in Figure 3(b) provides better payoffs than the empty network, where $v_i = 0.421$ for all $i = 1, 2, \ldots, 6$, and it is easy to verify that the network is pairwise stable. Therefore,

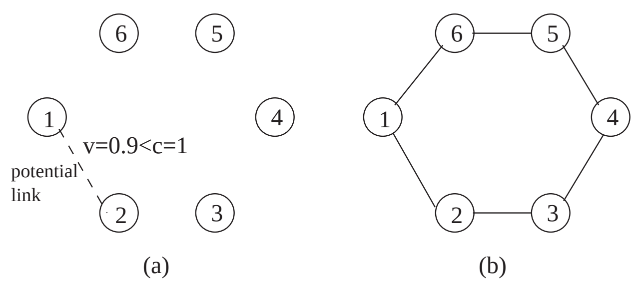


Fig. 3. A network where the game converges to an inefficient network.

some random events are needed to help the network jump out of the inefficient stable network.

In evolutionary games, mutations are introduced such that the evolution of the game is modeled as a Markov chain, where the states of the Markov chain are the strategy profiles γ. Given nonzero mutations for each state of the Markov chain, we have that the Markov chain is irreducible and aperiodic. Therefore, it has a unique corresponding stationary distribution. The work of Harsanyi and Selten [16] and Kardori, et al, [17] show that by letting the mutation probability go to 0 in a certain way, the game converges to a unique Pareto equilibrium. The mutations for network formation mean that when two nodes agree to form a link, with a probability ε, the link is not formed, or when a link is to be deleted because one of the two nodes choose to sever it, the link is not deleted with probability ε. Such mutations may result from transmission failures or noise. Thus by using mutations, the pairwise game converges to a stable network.

One of the main differences our model has compared to other game models [18] is that the cost is not a constant. In our model, the cost is a function of the distance between two nodes. Therefore, the physical locations of nodes in the network are important for the coalition formation. We consider the network as a random network where nodes are placed according to a uniform Poisson point process on the $[0, 1] \times [0, 1]$ square with the periodic boundary (i.e. the square is replicated throughout space to form an infinite lattice), where the boundary effects are not taken into consideration. We are mainly concerned with results that occur with high probability (w.h.p.), that is probability tending to one as $n \to \infty$.

Based on the analyses on connectivity of the continuous percolation model in [19] and [20], we have the following theorem:

Theorem 2: The coalition formation at the stable state depends on the parameter V for gain and α for cost.

1) Given $\delta = 0$, $V = P\left(\frac{\ln n}{n}\right)^{\frac{\alpha}{2}}$ is a *sharp* threshold for establishing the grand coalition.
   - If V is greater than the threshold, with high probability, all nodes collaborate with at least one of their neighbors.
   - If V is less than the threshold, with high probability, the network is partitioned into small coalitions.

2) For $0 < \delta \leq 1$, the threshold is less than $P\left(\frac{\ln n}{n}\right)^{\frac{\alpha}{2}}$.

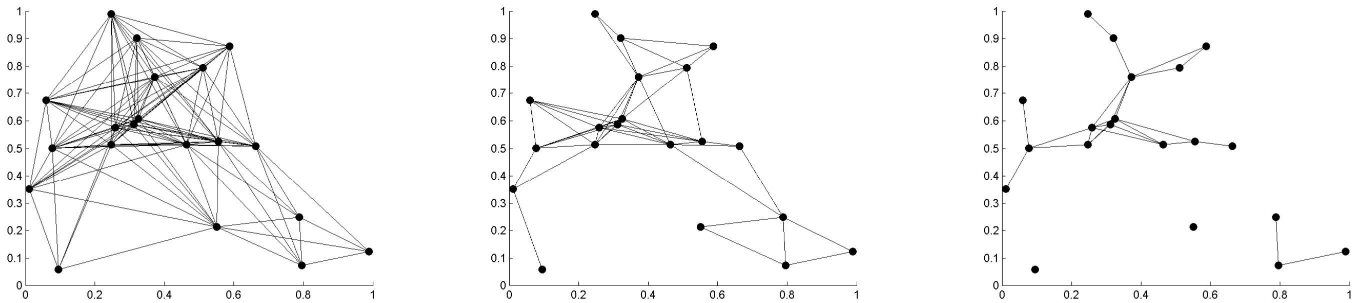
The appendix gives the proof of Theorem 2.

We run simulations to study the games at the stable state. In the simulations, 20 nodes are randomly placed on a $[0, 1] \times [0, 1]$ square. Two nodes are selected to play the pairwise game according to a fixed probability $1/n(n-1)$, where $n = 20$. Figure 4 shows the number of coalitions when the network reaches the pairwise stable state. The threshold predicted by our analytic results does exist for different δ's. When $\delta = 1$, the phase transition happens very sharply, because the gains for nodes to join a coalition linearly increases with the size of the coalition. When the size of the coalition is large enough, the gains the coalition provides are greater than any link cost. Thus a grand coalition is formed. On the other hand, for $\delta = 0$, only nodes closer to each other may form a link. Similarly, we give the maximum size of coalitions in Fig. 5.

Notice that we observe phase transition phenomena. Understanding phase transitions is very important for network design, because a slight change in the parameters controlling a phase transition, may induce dramatic changes in network performance, i.e., network collaboration in our case.

B. Topology

Three figures in Fig. 6 give the topology of a network formed with different cost parameters. As we can observe, when cost is low, the network forms as a complete graph. The other extreme end is when the cost is high, which results in a partitioned network. The most valuable topology is shown in Fig. 6(b). This figure represents the most common scenario in real life. Interestingly, the topology shows the small-world property: most links are connected between neighbors with



(a) P = 0.5 (low cost); complete graph (b) P = 2 (middle cost); small world topology (c) P = 4 (high cost); partitioned network

Fig. 6. Topology of the network with various cost parameters. $V = 1, \delta = 0.2, \alpha = 2$ for all three figures.

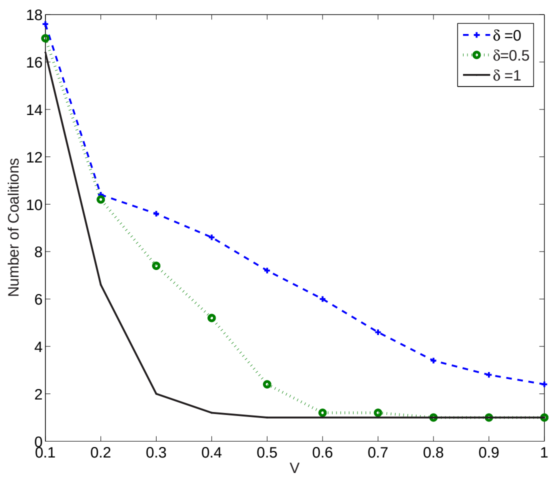


Fig. 4. Number of coalitions *vs* payoff V. $P = 10, \alpha = 2$.

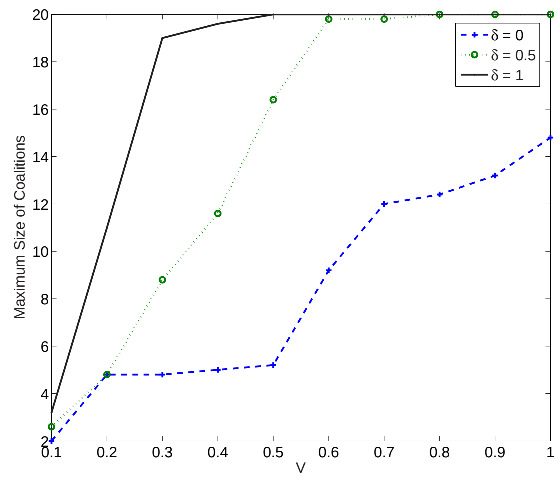


Fig. 5. Maximum size of coalitions *vs* payoff V. $P = 10, \alpha = 2$.

few long-range shortcuts. In the past five years, there has been substantial research on the small-world model in various complex networks, such as the Internet and biological systems. Our formation game converges to a small world network as well. This further proves that the small world model is an efficient communication structure.

VI. Stability of Coalitions

Having formed coalitions in the network, we are interested in studying the stability of these coalitions. We have defined pairwise stability, and showed that by introducing mutations, the pairwise game converges to a pairwise stable network. Pairwise stability is a relatively weak stability, where the network is stable if at most one node pairs choose to form or sever their link in one time period.

Next we give the definition for stronger stability: a *core stable* network [21]. A network g is core stable if there is no subset of nodes $S \in N$ who prefer another network $\hat{g}$ to g and who can change the network from g to $\hat{g}$ without the cooperation from the rest of the set of nodes $N \setminus S$.

Definition 2: A network $g \subset g^N$ is *core stable* if there does not exist any set of nodes $S \subset N$ and $\hat{g} \subset g^N$ such that:

- $x_i(\hat{g}) \geq x_i(g)$ for all $i \in S$ and there is at least one node with strict inequality,
- if $ij \in \hat{g}$ but $ij \notin g$, then $i, j \in S$,
- if $ij \notin \hat{g}$ but $ij \in g$, then either $i \in S$ and/or $j \in S$.

We are particularly interested in core stability. Core stability allows that a node is able to interact and coordinate with any other node in the same coalition. We believe this stronger stability is very useful in real networks, where users in the network can act together to achieve better payoffs.

It is obvious that many pairwise stable networks are not core stable. We present a case where a pairwise stable network is core stable as well.

Corollary 3 (Core stability): Given that $\delta = 1$, $\alpha = 0$ and the pairwise network g is connected, g is core stable.

The appendix gives the proof of Corollary 3.

VII. Conclusions and Future Work

In this paper, we study autonomic networks which rely on the collaboration of participating nodes. There are fundamental tradeoffs between the benefit from collaboration and the required cost for collaboration. This conflict naturally leads us to game-theoretic studies, where each node attempts to maximize its payoff which is a function of the gain and the cost of collaboration.

We model the games played by users as a two-phase coalition formation game. In the first phase, users play pairwise games to decide whether to form or sever a link between them based on their payoffs. By playing the pairwise games, coalitions are formed. In the second phase of the game, users in the same coalition interact to maximize the total payoff. We study the convergence of the iterated pairwise games. The conditions on which a grand coalition is formed are given. Topology of the converged network is investigated. We also study the stability of the formed coalition in the sense of core stability.

There are some problems that we are working on, or plan to work on, in the future. In our model, we simply assume that users always have information to transmit and the transmission failures are modeled by one parameter: the depreciation rate. We use this abstract model to simplify our analytic results. However, it is more realistic to model gains and costs based on specific network traffic patterns and certain traffic intensity.

When the depreciation rate $\delta > 0$, we only gave a very loose bound on the threshold for the grand coalition. It is our interest to investigate tighter bounds on the threshold.

Appendix

Sketch of Proof on Theorem 2

In the iterated pairwise game, a link is formed only if the gain is greater than the cost. Assume that the relative gain of user i due to a link ij is w_{ij} and the cost of the link is c_{ij}. Then i agrees to activate the link ij if the following inequality is satisfied

$$w_{ij} > c_{ij} = P d_{ij}^{\alpha},$$

which is equivalent to

$$d_{ij} < \sqrt[\alpha]{\frac{w_{ij}}{P}}.$$

Therefore, i agrees to link with j if j is in the circle centered at i whose radius satisfies the above inequality.

A fundamental result in the theory of the independent Erdos-Renyi random graph $G(n,p)$ states that if $p \gg \frac{\ln n}{n}$, the random network is connected, which gives the mean degree $np = \ln n$. In our case, the networks are modeled as random geometric graphs [20]. n nodes are placed according to a homogenous Poisson point process $[0,1] \times [0,1]$ square. Assume that the density of the Poisson point process is λ, then $\lambda = n$. The degree of a particular node is equal to the number of nodes in the circle centered at the node with radius equal to $d = \sqrt[\alpha]{\frac{w_{ij}}{P}}$. We have that the mean degree of the node is equal to $\pi d^2 \lambda$. Similarly, we want to find that the mean degree of the random geometric graphs is much greater than $\ln n$ in order to form a connected graph. Therefore, we have that $\pi d^2 \lambda \gg \ln n$, which is to say that w_{ij} is asymptotically equal to $P\left(\frac{\ln n}{n}\right)^{\frac{\alpha}{2}}$.

Given $\delta = 0$, we have that the threshold value for V is equal to $P\left(\frac{\ln n}{n}\right)^{\frac{\alpha}{2}}$.

For $\delta > 0$, $w_{ij} \geq V$, so we have that the critical value of V is less than $P\left(\frac{\ln n}{n}\right)^{\frac{\alpha}{2}}$.

Proof of Corollary 3

Since the network is connected, $x_i(g) = (n-1)V - |\mathcal{N}_i|P, \forall i \in N$. Furthermore, $\delta = 1$, so the pairwise network g has no cycles, i.e., g is a tree, whose total number of links is $n-1$; otherwise, by deleting a link in a cycle, the gains of the two end nodes of the deleted link stay the same and their costs are reduced.

Suppose that there exists a set of nodes $S \subset N$ and $\hat{g} \in g^N$ which satisfy the three conditions in Definition 2. There must exist a pair of nodes $i \in S$ and $j \notin S$ where $ij \in g$ and $ij \notin \hat{g}$. Thus $\hat{g}$ is disconnected because g is a tree. Therefore, for any $i \in S$, $x_i(\hat{g}) = (|S|-1)V - |\hat{\mathcal{N}_i}|P$, where $\hat{N}_i$ is the set of i's neighbors in $\hat{g}$. Now consider $\sum_{i\in S} x_i(g)$ and $\sum_{i \in S} x_i(\hat{g})$. We have

$$\sum_{i\in S} x_i(g) \geq |S|(n-1)V - 2(|S|-1)P,$$

$$\sum_{i\in S} x_i(\hat{g}) = |S|(|S|-1)V - 2(|S|-1)P.$$

Therefore, $\sum_{i\in S} x_i(g) > \sum_{i\in S} x_i(\hat{g})$, which contradicts with the first condition in Definition 2.

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References

[1] M. J. Osborne and A. Rubinstein, *A Course in Game Theory*. MIT Press, 1994.

[2] R. B. Myerson, *Game Theory: Analysis of Conflict*. Harvard University Press, 1991.

[3] R. J. Aumann and R. B. Myerson, “Endogenous formation of links between players and of coalitions: An application of the shapley value,” in *The Shapley Value: Essays in Honor of Lloyd Shapley*, A. Roth, Ed. Cambridge University Press, Cambridge, UK, 1988, pp. 175–191.

[4] V. Srinivasan, P. Nuggehalli, C. F. Chiasserini, and R. R. Rao, “Cooperation in wireless ad hoc networks,” in *Proceedings of IEEE INFOCOM’03*, vol. 2, San Francisco, March 30 - April 3 2003, pp. 808–817.

[5] A. Urpi, M. Bonuccelli, and S. Giordano, “Modelling cooperation in mobile ad hoc networks: a formal description of selfishness,” in *Proceeding of WiOpt*, 2003.

[6] M. Félegyházi, J.-P. Hubaux, and L. Buttyán, “Nash Equilibria of Packet Forwarding Strategies in Wireless Ad Hoc Networks,” *IEEE Transactions on Mobile Computing*, vol. 5, no. 5, pp. 463–476, May 2006.

[7] Y. Korilis, A. Lazar, and A. Orda, "Architecting noncooperative networks," *IEEE Journal on Selected Areas in Communications (JSAC), Special Issue on Advances in the Fundamentals of Networking*, vol. 13, p. 7, September 1995.
[8] T. Roughgarden, *Selfish Routing and the Price of Anarchy*. The MIT Press, 2005.
[9] P. Michiardi and R. Molva, "Analysis of coalition formation and cooperation strategies in mobile ad hoc networks," *Ad Hoc Networks*, vol. 3, no. 2, pp. 193–219, March 2005.
[10] G. E. Bolton and A. Ockenfels, "ERC: a theory of equity, reciprocity, and competition," *The American Economic Review*, vol. 90, pp. 166–193, 2000.
[11] B. Dutta and M. O. Jackson, Eds., *Networks and Groups: Models of Strategic Formation*, ser. Studies in Economic Design. Springer-Verlag Berlin Heidelberg, 2003.
[12] M. Slikker and A. v. d. Nouweland, *Social and Economic Networks in Cooperative Game Theory*. Kluwer Academic Publishers, 2001, vol. 27.
[13] M. O. Jackson and A. Wolinsky, "A strategic model of social and economic networks," *Journal of Economic Theory*, vol. 71, no. 1, pp. 44–74, October 1996.
[14] R. B. Myerson, "Graphs and cooperation in games," *Mathematics of Operations Research*, vol. 2, pp. 225–229, 1977.
[15] F. Forgo, J. Szep, and F. Szidarovszky, *Introduction to the Theory of Games: Concepts, Methods, Applications*. Kluwer Academic Publishers, 1999.
[16] J. C. Harsanyi and R. Selten, *A General Theory of Equilibrium in Games*. Cambridge MIT Press, 1988.
[17] M. Kandori, G. J. Mailath, and R. Rob, "Learning, mutation, and long run equilibria in games," *Econometrica*, vol. 61, no. 1, pp. 29–56, 1993.
[18] V. Bala and S. Goyal, "A noncooperative model of network formation," *Econometrica*, vol. 68, no. 5, pp. 1181–1129, September 2000.
[19] R. Meester and R. Roy, *Continuum Percolation*. Cambridge University Press, 1996.
[20] M. Penrose, *Random Geometric Graphs*. Oxford University Press, 2003.
[21] M. O. Jackson and A. Watts, "The evolution of social and economic networks," *Journal of Economic Theory*, vol. 106, pp. 265–295, 2002.