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**SCHOOL OF PUBLIC POLICY  
UNIVERSITY OF MARYLAND**

**PUAF Mathematics Immersion Course  
Summer 2011**

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**Problem Set 1 Solutions**

**Question 1: Simplify the following expressions**

1.  $8^3 = 8 * 8 * 8 = 512$
2.  $8^{-3} = 1/(8^3) = 1/(8 * 8 * 8) = 1/512$
3.  $(-8)^3 = (-8) * (-8) * (-8) = -512$
4.  $(-8)^{-3} = 1/[(-8)^3] = 1/(-8) * 1/(-8) * 1/(-8) = 1/(-512) = -(1/512)$
5.  $-(-8)^3 = -(8 * 8 * 8) = -512$
6.  $-(-8)^{-3} = 1/(8^3) = 1/(8 * 8 * 8) = 1/512$
7.  $8^{1/3} = \sqrt[3]{8} = 2$
8.  $(1/8)^3 = (1/8) * (1/8) * (1/8) = 1/512$
9.  $(-1/8)^{1/3} = \sqrt[3]{(-1/8)} = -1/2$
10.  $(-1/8)^{-1/3} = \sqrt[3]{(-1/8)^{-1}} = \sqrt[3]{(-8)} = -2$

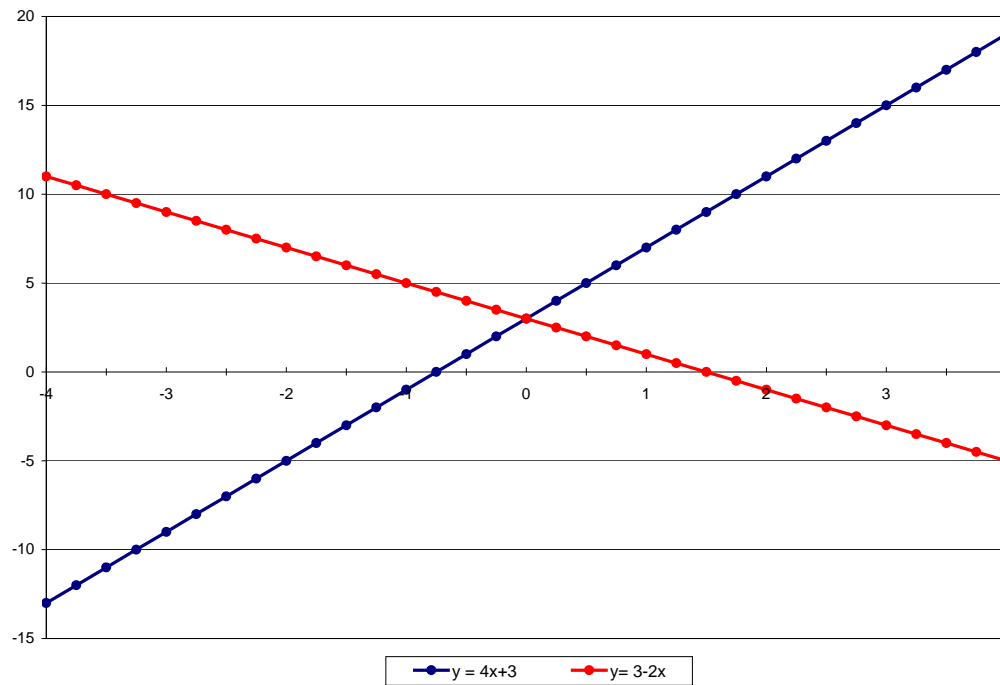
**Question 2: Simplify the following expressions**

1.  $8^2 + 6 * 3 = (8 * 8) + (6 * 3) = 64 + 24 = 88$
2.  $8^2 - 6 * 3 = (8 * 8) - (6 * 3) = 64 - 24 = 40$
3.  $(8^2 - 6) * 3 = [(8 * 8) - 6] * 3 = [64 - 6] * 3 = 58 * 3 = 174$
4.  $8^2 - (6 * 3) = (8 * 8) - (6 * 3) = 64 - 24 = 40$

**Question 3: Solve the following systems of equations**

1.  $y = 4x + 3$  and  $y = 3 - 2x$   
 $\Rightarrow 4x + 3 = 3 - 2x$   
 $\Rightarrow 6x = 0$   
 $\Rightarrow x = 0$  when  $y = 3$   
 $\Rightarrow$  Solution:  $(0, 3)$

Graphically:



2.  $y = 3x^2 - 11x + 8$  and  $y = -4/x$

$$\Rightarrow 3x^2 - 11x + 8 = -4/x$$

$$\Rightarrow 3x^3 - 11x^2 + 8x = -4$$

$$\Rightarrow 3x^3 - 11x^2 + 8x + 4 = 0$$

$$\Rightarrow (x - 2)(3x^2 - 5x - 2)$$

$$\Rightarrow (x - 2)(3x^2 - 6x + x - 2)$$

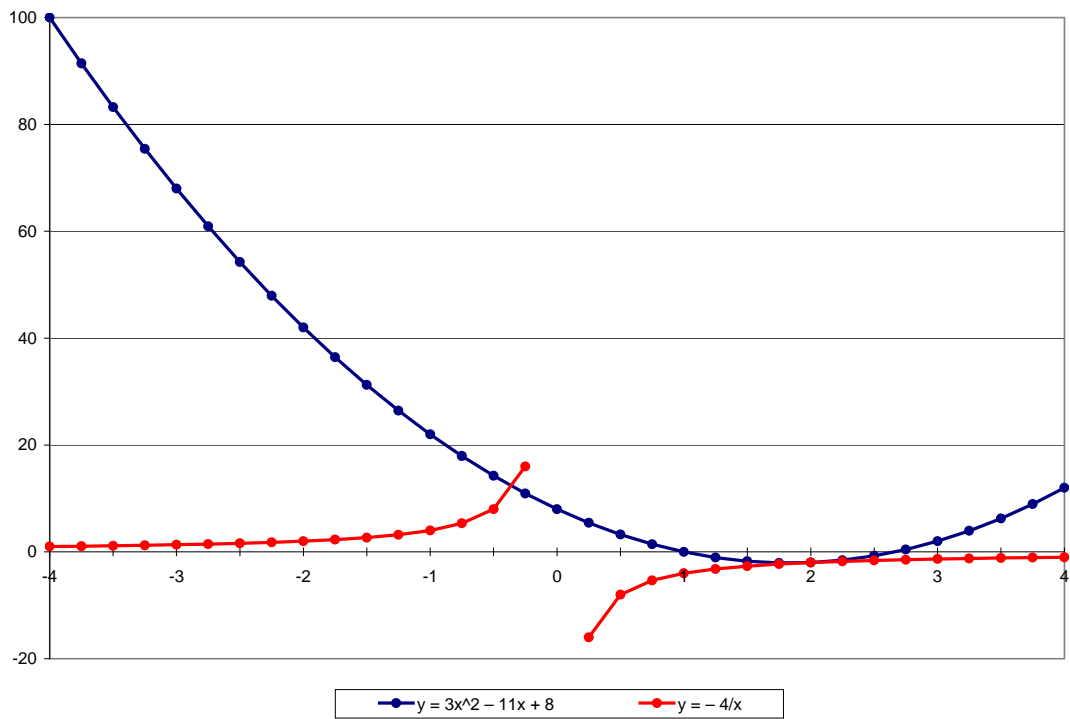
$$\Rightarrow (x - 2)[3x(x - 2) + (x - 2)]$$

$$\Rightarrow (x - 2)(3x + 1)(x - 2)$$

$$\Rightarrow x = 2, -1/3 \text{ and the corresponding values of } y = -2, 12$$

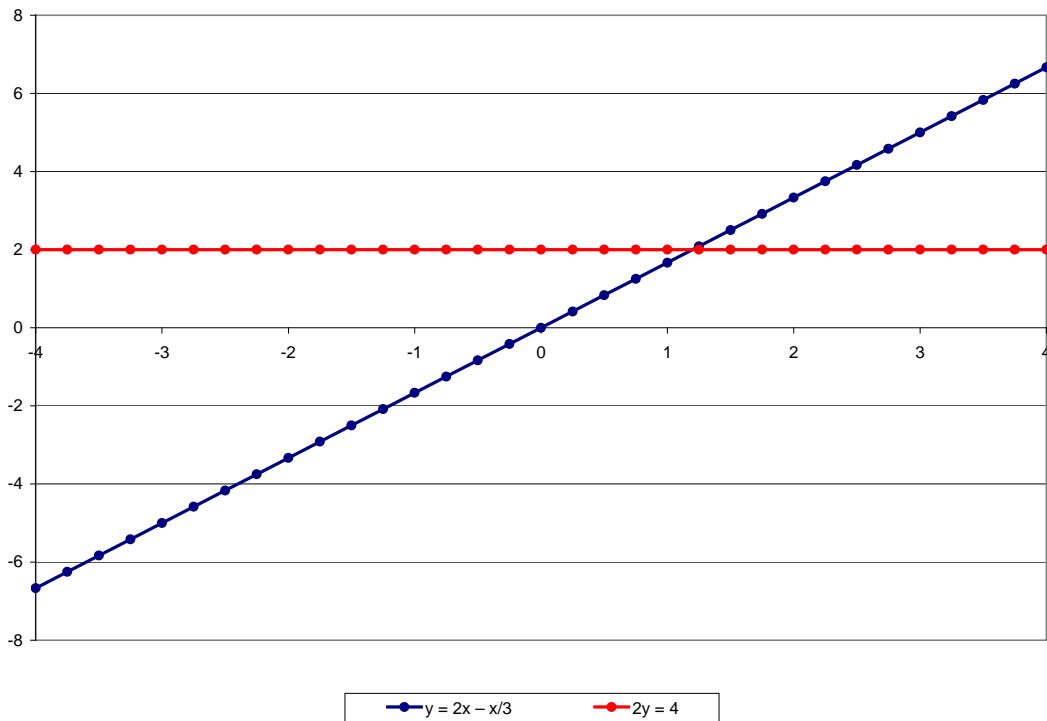
$$\Rightarrow \text{Solutions: } (2, -2), \text{ and } (-1/3, 12).$$

Graphically:



3.  $y = 2x - x/3$  and  $2y = 4$   
 $\Rightarrow 2x - x/3 = 2$   
 $\Rightarrow 6x - x = 6$   
 $\Rightarrow 5x = 6$   
 $\Rightarrow x = 6/5 = 1.2$  at which point  $y = 2$   
 $\Rightarrow$  Solution:  $(1.2, 2)$

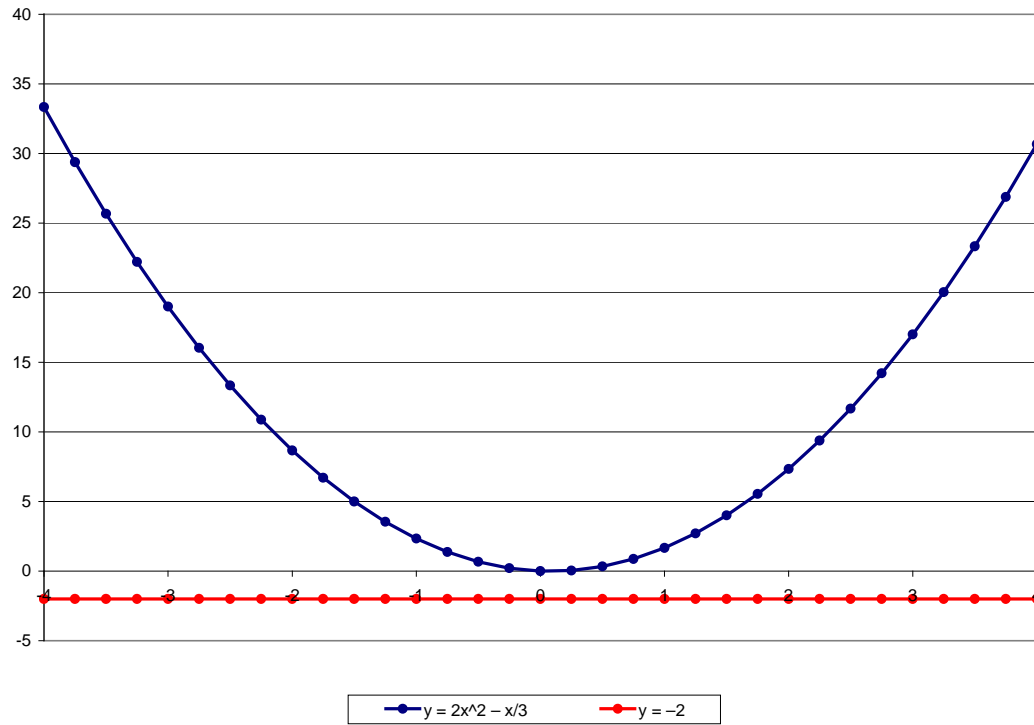
Graphically:



4.  $y = 2x^2 - \frac{x}{3}$  and  $y = -2$   
 $\Rightarrow 2x^2 - \frac{x}{3} = -2$   
 $\Rightarrow 6x^2 - x + 6 = 0$   
 $\Rightarrow 6x^2 - x + 6 = 0$   
 This has no real solution.

We would have arrived at the same conclusion if we had graphed the functions. The solutions would have been the values of  $x$  where the two functions/curves intersect. But since the two functions never intersect, there is no real value of  $x$  for which both equations are simultaneously satisfied.

Graphically:



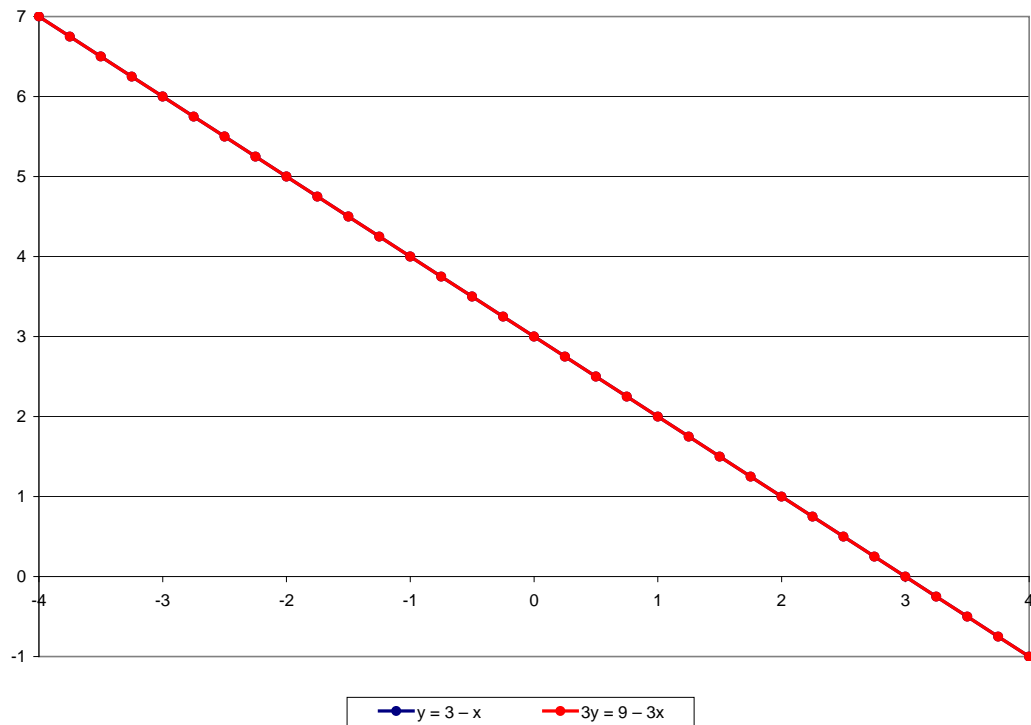
5.  $y = 3 - x$  and  $3y = 9 - 3x$

$$\Rightarrow 3 - x = 3 - x$$

$$\Rightarrow 0 = 0$$

$\Rightarrow$  Indeterminate since they are both the same line. And hence there are infinite solutions.

Graphically:



6.  $y = 2x - 3z + 4$  and  $y = x + 2z$

$$\Rightarrow 2x - 3z + 4 = x + 2z$$

$$\Rightarrow x = 5z - 4$$

$\Rightarrow$  Which is one equation with two variables, and hence the solution is indeterminate.

**Question 4: Factorise the following expressions**

1.  $x^2 - 3x + 2$

$$= x^2 - x - 2x + 2$$

$$= x(x - 1) - 2(x - 1)$$

$$= (x - 2)(x - 1)$$

2.  $x^2 - 10x + 21$

$$= x^2 - 7x - 3x + 21$$

$$= x(x - 7) - 3(x - 7)$$

$$= (x - 7)(x - 3)$$

3.  $x^2 + 4x - 12$

$$= x^2 - 2x + 6x - 12$$

$$= x(x - 2) + 6(x - 2)$$

$$= (x - 2)(x + 6)$$

4.  $4x^2 - 9$

$$= (2x + 3)(2x - 3)$$

5.  $4x^2 + 9$

Cannot be factorised.

**Question 5: Solve the following without using a calculator. Round the solution to two decimal points**

- $2/7 + 3/7$   
 $= (2 + 3)/7 = 5/7 = 0.71$
- $2/7 - 3/14$   
 $= 4/14 - 3/14 = (4 - 3)/14 = 1/14 = 0.07$
- $1/5 + 3/4 - 5/2 + 7/8$   
 $= (8 + 30 - 100 + 35)/40 = (73 - 100)/40 = -27/40 = -0.675$
- $(1/5 + 3/4)*(-5/2 + 7/8)$   
 $= [(4 + 15)/20]*[(-20 + 7)/8] = 19/20 * -13/8 = -247/160 = -1.54$
- $(1/5 - 3/4)*(-5/2 + 7/8)$   
 $= [(4 - 15)/20]*[(-20 + 7)/8] = -11/20 * -13/8 = 143/160 = 0.89$

**Question 6: Evaluate the following algebraic equations**

- $(5x^2y + y^2) + (2x^2y + y)$   
 $= (5x^2y + 2x^2y) + y^2 + y = 7x^2y + y^2 + y$
- $(5x^2y + y^2) - (2x^2y + y)$   
 $= (5x^2y - 2x^2y) + y^2 - y = 3x^2y + y^2 - y$
- $(5x^2y + y^2)*(2x^2y + y)$   
 $= 10x^4y^2 + 5x^2y^2 + 2x^2y^3 + y^3$
- $[-(5x^2y + y^2)]*[-(2x^2y + y)]$   
 $= 10x^4y^2 + 5x^2y^2 + 2x^2y^3 + y^3$