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A Modified Hapke Model for Soil Bidirectional Reflectance

Shunlin Liang and John R. G. Townshend**

The Hapke bidirectional reflectance model has been widely used for soil modeling, but gives significant errors when the soil scatters very strongly and anisotropically. In our new model, we decompose the radiation field into three components instead of two: single-scattering radiance, double-scattering radiance, and multiple-scattering radiance. The first two components can be exactly calculated and multiple scattering is equivalently approximated by the original Hapke formula. Comparisons with the numerical code-DISORT based on the discrete ordinate algorithm indicate that this modified model significantly improves the accuracy of the bidirectional reflectance. A numerical calculation for a clay soil using the Mie theory gives reasonable variation in the ranges of the single-scattering albedo and the asymmetry parameter of the phase function where the original Hapke model generally works poorly. An inversion experiment shows that the modified model also improves the inversion accuracy.

INTRODUCTION

The Hapke model (Hapke, 1981; 1986; 1993) is a widely used model of soil bidirectional reflectance. Several inversion experiments have been carried out to retrieve soil physical parameters using this model (e.g., Pinty et al., 1989; Jacquemoud et al., 1992). This model was derived primarily for use with planetary soils, most of which have low single-scattering albedos and are not highly anisotropic. However, researchers have recently begun to question its accuracy when it is applied to terrestrial soils (e.g., Mishchenko, 1994) mainly because of its assumption of isotropic multiple scattering, regardless of the actual phase function of the medium. As a result, this approximation cannot satisfactorily predict the angular pattern, especially when soil scattering is very strong and anisotropic. Moreover, when an inaccurate model is used for inversion, the retrieved parameters may be physically nonsense, although the fit of the model to observations may be almost perfect. This point has been well demonstrated regarding the Hapke model using "theoretical" experiments (Mishchenko, 1994). Mishchenko (p. 103) pointed out that the retrieved parameters may not be reliable for two reasons: ". . . First, several rather crude approximations have been made in the derivation of the Hapke bidirectional reflection function. . . . Second, it is well known that determination of the asymmetry parameter from measurements of the reflected light is an ill-conditioned inverse problem. This means that under certain conditions the reflected intensity can depend on the asymmetry parameter of the phase function rather weakly. As a result, experimental noise and / or approximations like those mentioned above can easily result in absolutely wrong values of the asymmetry parameter."

In the original Hapke model, the total radiance is calculated from the sum of single-scattering radiance and multiple-scattering radiance which is assumed isotropic. If the soil scatters anisotropically, the distribution of the multiple-scattering radiance, which is scattered more than once, is far from isotropic. The strategy in this article is to divide the radiation field of the semiinfinite soil into three components: single-scattering radiance, double-scattering radiance, and multiple-scattering radiance. With this formulation, the first two components can be explicitly determined, and account for most of the anisotropy. The multiple-scattering component is assumed azimuth-independent and calculated by the Hapke approximation excluding azimuth-averaged double-scattering radiance. This decomposition enables us to obtain accurate solutions. The soil bidirectional reflectance distribution function (BRDF) is then easy to

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formulate after creating the explicit formulae for radiance calculations of all components.

Any approximate models should be verified by controlled laboratory experiments in which bidirectional reflectance of the sample as well as all physical parameters in the models (e.g., particle size distribution, refractive index) are measured. However, controlled laboratory experiments are not easily implemented because of practical limitations. An excellent alternative is the use of "theoretical" models. Given the same set of physical parameters, the "exact" solutions generated by theoretical models can be used to verify the accuracy of approximate models. In this study, accuracy verifications are carried out using a numerical code DISORT, which is based on a discrete ordinate algorithm (Stamnes et al., 1988). This code was developed by a group of scientists funded by NASA and extensively verified by different data sources. The DISORT code is "exact" in the sense that it provides accurate solution to the same radiative transfer equation with the same boundary conditions on which the Hapke model is based. Comparisons are made between the modified model, Hapke model, and DISORT code in the third section. To determine what are reasonable soil parameters, a numerical calculation of the clay soil using the Mie theory is given in the fourth section. An inversion experiment is also carried out in the fifth section to figure out the inversion accuracy of the modified model and the original Hapke model using the simulated data from DISORT code.

MODEL DEVELOPMENT

In the absence of polarization, for a plane-parallel homogeneous semiinfinite soil, the radiative transfer equation for the radiance $I(\tau,\Omega)$ in the direction Ω at optical depth τ can be written as

$$\mu\frac{\partial I(\tau,\Omega)}{\partial\tau} = I(\tau,\Omega) - \frac{\omega}{4\pi}\int_{4\pi} P(\Omega',\Omega)I(\tau,\Omega')\,d\Omega', \quad (1)$$

where ω is the single scattering albedo, $P(\Omega',\Omega)$ is the phase function. Ω stands for an azimuth angle φ and a zenith angle $\theta=\cos^{-1}(\mu)$, which ranges from 0° to 90° in the upper hemisphere and from 90° to 180° in the lower hemisphere. The geometric height z has been replaced by optical depth τ with zero at the top of the soil. Note that above equation is only valid for sparsely distributed, independently scattering particles. For soil particles that are densely packed, some modifications may be needed, particularly in the backscattering direction (later we include an empirical hot spot function in this model for this purpose). The exploration of dense-medium radiative transfer is outside the scope of this paper. The objective of this study is to develop a more accurate solution to the same radiative transfer equation with the same boundary conditions on which the Hapke model is based.

To obtain a solution to Eq. (1), appropriate boundary conditions have to be specified. In the upper boundary, the soil is illuminated by a parallel beam in the direction (Ω_0) with net flux $i_0=\pi F_0$, where F_0 is the extraterrestrial solar irradiance. Thus,

$$I(0,\Omega)=\delta(\Omega-\Omega_0)i_0, \quad \mu<0,$$
$$\lim_{\tau\to\infty} I(\tau,\Omega)=0, \quad \mu>0, \quad (2)$$

where $\delta(\Omega-\Omega_0)$ is the Dirac delta function with value unity when $\Omega=\Omega_0$ and zero when $\Omega\neq\Omega_0$.

In radiative transfer modeling, total radiance is usually divided into unscattered and scattered components. Since a soil medium can be considered infinite vertically and no unscattered solar radiance reflected by the bottom boundary in the upward direction can be observed at the top of the soil, only scattered radiance is considered. The radiative transfer equation for scattered radiance and the corresponding conditions become

$$\mu\frac{\partial I(\tau,\Omega)}{\partial\tau} = I(\tau,\Omega) - \frac{\omega}{4\pi}\int_{4\pi} P(\Omega',\Omega)I(\tau,\Omega')\,d\Omega' + \frac{\omega}{4}F_0\,P(\Omega_0,\Omega)\exp\left(-\frac{\tau}{|\mu_0|}\right)$$
$$I(0,\Omega)=0, \quad \mu<0$$
$$\lim_{\tau\to\infty} I(\tau,\Omega)=0, \quad \mu>0 \quad (3)$$

The Hapke Model

For ease of reference, a simple description of the Hapke model representing the solution to (3) is given in this section. In the Hapke model, the single-scattering radiance is modified to account for the hot spot effect, and the multiple-scattering radiance is assumed isotropic and is expressed by the approximate Chandraeskhar H-function (Hapke, 1981; 1986). The formula of the upwelling radiance at the top of the soil is

$$I(\Omega)=\frac{\omega F_0|\mu_0|}{4(|\mu_0|+\mu)}\left\{P(\Omega_0,\Omega)[1+B(\Omega_0,\Omega)] + H(\mu)H(|\mu_0|)-1\right\}, \quad (4)$$

where $H(x)$ is approximated as

$$H(x)=\frac{1+2x}{1+2x\sqrt{1-\omega}}. \quad (5)$$

$B(\alpha)$ is the hot spot correction function (Hapke, 1986)

$$B(\alpha)=\frac{B_0}{1+\tan(\alpha/2)/h}, \quad (6)$$

where α is the phase angle between Ω_0 and Ω, and B_0 and h are two parameters for the height and the angular width of the hot spot peak.

The Modified Model

In order to handle multiple scattering effectively, the radiation field is decomposed into three components: single-scattering radiance $I^1(\tau,\Omega)$, double-scattering radiance $I^2(\tau,\Omega)$, and multiple-scattering radiance $I^M(\tau,\mu)$:

$$I(\tau,\Omega) = I^1(\tau,\Omega) + I^2(\tau,\Omega) + I^M(\tau,\mu). \quad (7)$$

The upwelling single-scattering radiance at the top of the soil can be expressed as

$$I^1(0,\Omega) = \frac{\omega F_0 P(\Omega_0,\Omega)|\mu_0|}{4(|\mu_0| + \mu)}. \quad (8)$$

The upwelling double-scattering radiance at the top of the soil is determined by (Liang and Townshend, 1995)

$$I^2(0,\Omega) = \int_0^{2\pi}\int_{-1}^{0} \gamma_1(\Omega')P(\Omega',\Omega)\,d\Omega' + \int_0^{2\pi}\int_0^{1} \gamma_2(\Omega')P(\Omega',\Omega)\,d\Omega', \quad (9)$$

where

$$\gamma_{u1}(\Omega') = \frac{\omega^2 F_0 P(\Omega_0,\Omega')|\mu_0|\mu}{16\pi(\mu + |\mu_0|)(\mu + |\mu'|)},$$

$$\gamma_{u2}(\Omega') = \frac{\omega^2 F_0 P(\Omega_0,\Omega')\mu_0^2}{16\pi(|\mu_0| + \mu')(|\mu_0| + \mu)}.$$

The calculation of the upwelling multiple-scattering radiance at the top of the soil takes the same approximation as the original Hapke model, but excludes the equivalent double-scattered radiance:

$$I^M(0,\mu) = \frac{\omega F_0|\mu_0|}{4(|\mu_0| + \mu)}\left\{H(\mu) + H(\mu_0) - 1\right\} - I^2(0,\mu), \quad (10)$$

where $I^2(0,\mu)$ is the azimuth-averaged double-scattering radiance so that I^M is the radiance scattered more than twice:

$$I^2(0,\mu) = \frac{1}{\mu}\int_0^{\infty}\int_{-1}^{1} I^1(t,\mu)\,d\mu\,\exp\left(-\frac{t}{\mu}\right)dt,$$

where $I^1(\tau,\mu)$ is the azimuth-averaged single-scattering radiance at different optical depths τ (Liang and Townshend, 1995). After some algebraic deviations, the final formula for $I^2(0,\mu)$ is

$$I^2(0,\mu) = \int_{-1}^{0} \bar{\gamma}_{u1}(\mu')p(\mu',\mu)\,d\mu' + \int_0^{1} \bar{\gamma}_{u2}(\mu')p(\mu',\mu)\,d\mu', \quad (11)$$

where $\bar{\gamma}_{u1}(\mu')$ and $\bar{\gamma}_{u2}(\mu')$ are defined as

$$\bar{\gamma}_{u1}(\mu') = \frac{\omega^2 F_0 p(\mu_0,\mu')|\mu_0|\mu}{8(\mu + |\mu_0|)(\mu + |\mu'|)},$$

$$\gamma_{u2}(\mu') = \frac{\omega^2 F_0 p(\mu_0,\mu')\mu_0^2}{8(|\mu_0| + \mu')(|\mu_0| + \mu)}.$$

One of the most noticeable features of the dense-medium radiative transfer is the enhanced backscattering effect, which is also called the hot spot effect. Hapke (1986) accounted for this effect using shadowing theory. Later studies indicate that the hot spot effect may result from the coherent backscattering (Tsang and Ishimaru, 1985; Ishimaru, 1990). No matter what the explanations, the empirical function developed by Hapke (1986) is very suitable to characterize the hot spot in terms of peak height and its angular width. The hot spot effect can be easily incorporated into our modified model by simply modifying the upwelling single-scattering radiance (8)

$$I^1(0,\Omega) = \frac{\omega F_0 P(\Omega_0,\Omega)[1 + B(\Omega_0,\Omega)]|\mu_0|}{4(|\mu_0| + \mu)}, \quad (12)$$

where $B(\Omega_0,\Omega)$ is the hot spot correction function defined in (6).

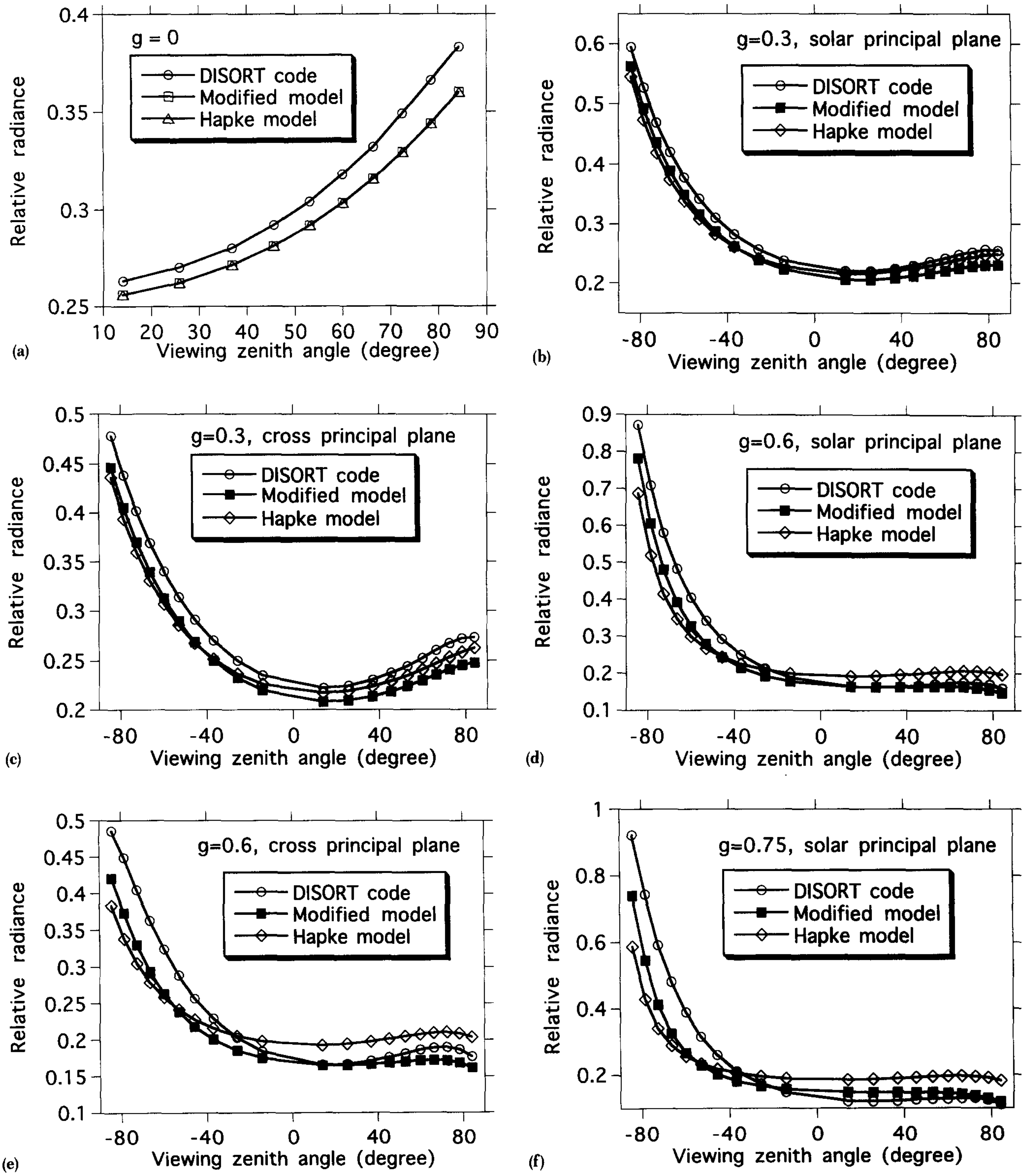
Having obtained all components of the model, the bidirectional reflectance distribution function (BRDF) $f(\Omega_0,\Omega)$ can be easily calculated from

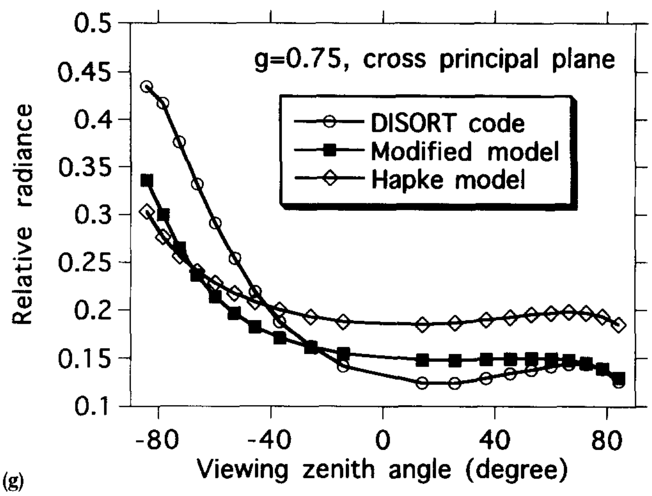
$$f(\Omega_0\Omega) = \frac{I(0,\Omega)}{|\mu_0|\pi F_0}. \quad (13)$$

Notice that the usual bidirectional reflectance is equivalent to the BRDF $f(\Omega_0,\Omega)$ multiplied by π.

ACCURACY VERIFICATION

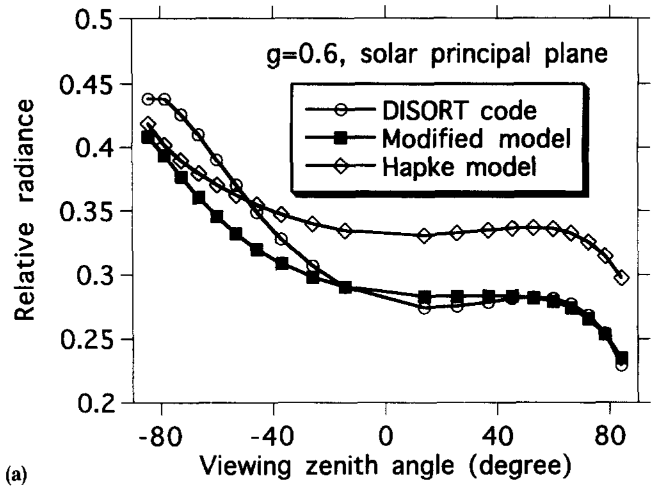
To evaluate the accuracy of the present model and the original Hapke model, the numerical code DISORT based on the discrete ordinate algorithm (Stamnes et al., 1988) has been employed to calculate upwelling radiance at the top of the soil. The DISORT code has been well developed and validated by a group of scientists funded by NASA. Their results are taken as benchmark in this paper. The soil is assumed to be illuminated by the monodirectional solar radiance with irradiance π (i.e., $F_0 = 1$), and thus the calculated radiance is denoted as relative radiance in the following figures. For simplicity, it is assumed that the phase function can be characterized by the one-term Heney–Greenstein function (Lenoble, 1985), although any arbitrary function is accepted by this model. The hot spot function is excluded in the following calculations because we mainly try to illustrate the difference between the new model and the original Hapke model. Thus, two parameters characterizing the inherent soil properties control the magnitude and angular distribution of the upwelling radiance at the top of the soil: namely, the single-scattering albedo ω and asymmetry parameter g for the phase function which is defined as $g = 1/2\int_0^{\pi} d(\cos\theta)P(\theta) \times \cos\theta$, where θ is the phase angle.


g = 0
DISORT code
Modified model
Hapke model
Relative radiance
Viewing zenith angle (degree)
(a)
g=0.3, solar principal plane
DISORT code
Modified model
Hapke model
Relative radiance
Viewing zenith angle (degree)
(b)
g=0.3, cross principal plane
DISORT code
Modified model
Hapke model
Relative radiance
Viewing zenith angle (degree)
(c)
g=0.6, solar principal plane
DISORT code
Modified model
Hapke model
Relative radiance
Viewing zenith angle (degree)
(d)
g=0.6, cross principal plane
DISORT code
Modified model
Hapke model
Relative radiance
Viewing zenith angle (degree)
(e)
g=0.75, solar principal plane
DISORT code
Modified model
Hapke model
Relative radiance
Viewing zenith angle (degree)
(f)

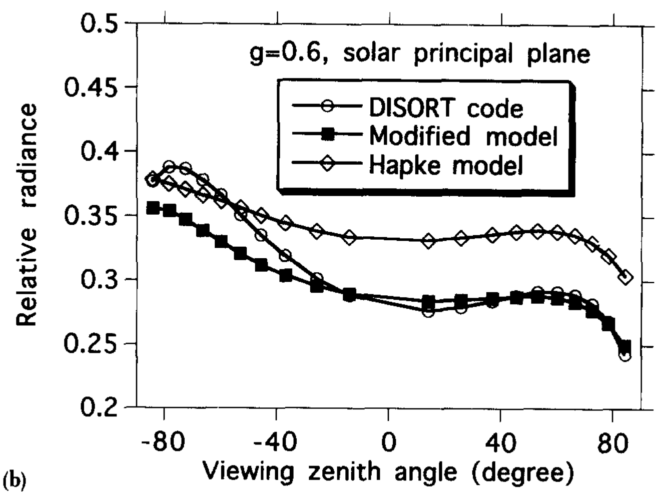


(g)

Figure 1. Comparisons of the Hapke model and the modified model with the numerical code DISORT with different asymmetry parameters. The solar zenith angle is 60°, the solar azimuth angle 0°, and single-scattering albedo 0.94. The solar principal plane is denoted for azimuth angles 0° and 180°, the cross-principal plane for 45° and 225°. Negative angles for azimuth planes 180° and 225°.



(a)



(b)

Figure 2. Comparison of the Hapke model and the modified model with the numerical code DISORT with asymmetry parameter 0.6. Other parameters are the same as Figure 1 except the solar zenith angle is 30°.

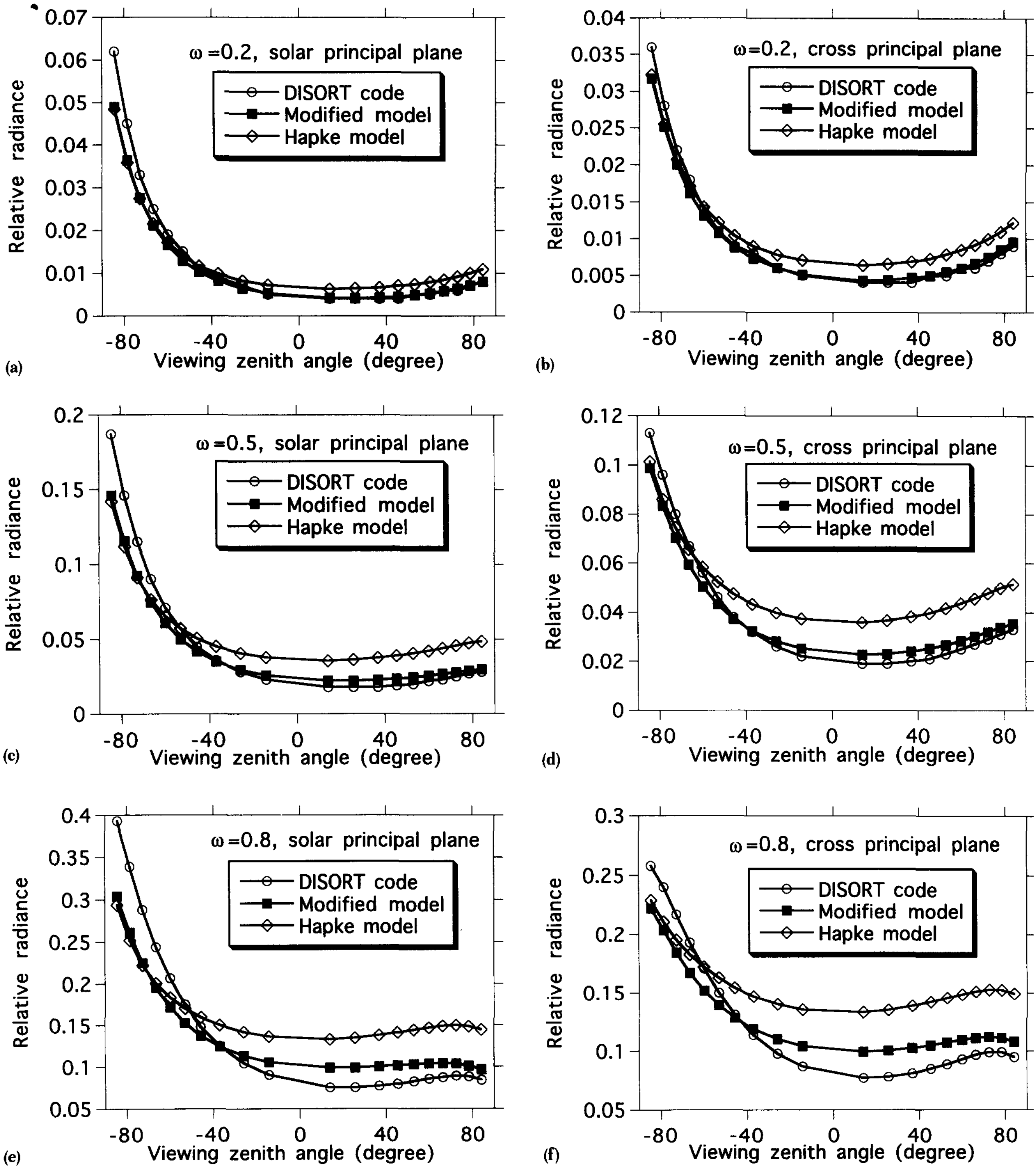
Figure 1 compares three model outputs with different asymmetry parameters in the principal plane (0–180°) and another azimuth plane (45–225°). Negative angles are for azimuth planes 180° and 225°, indicating forward scattering direction. Larger g implies greater anisotropy of multiple scattering. When $g=0$, the soil scatters isotropically, and the modified model produces the same results as the original Hapke model. When g becomes larger, both the Hapke model and the modified model underestimate the radiance in the forward scattering direction and overestimate in the backscattering direction. Overall, the modified model yields much better angular patterns. When the viewing angle is smaller than 40°, the modified model predicts upwelling radiance quite accurately. The original Hapke model does not work well at large values of the asymmetry parameter, and becomes worse when the solar zenith angle is smaller (Fig. 2).

Figure 3 compares three models with different single-scattering albedos. When the single-scattering albedo is small, both the Hapke model and the modified model produce accurate results. However, when the single scattering albedo becomes larger (Fig. 3d–h), where multiple scattering dominates, the modified model predicts radiance much better than the original Hapke model, especially in the backscattering directions and with smaller viewing angle in the forward scattering direction.

MIE CALCULATION FOR CLAY SOIL

In the previous section, it has been shown that if the single-scattering albedo and the asymmetry parameter are small, the original Hapke model can predict the soil upwelling radiance well. But, for other parameter values, the Hapke model works less satisfactorily, and the proposed modification of the model significantly improves its accuracy. The question is what ranges of these two parameters does the soil medium possess in reality. This is a very difficult problem, and there exists no satisfactory answer to this question. The major reason is that soil particles are usually densely packed and inhomogeneously distributed. Theoretical investigations of these realistic scenarios are urgently required.

Although an irregular and densely packed particle may have smaller single-scattering albedo and asymmetry parameter (Hapke, 1993; Mishchenko, 1994; McGuire and Hapke, 1995), the Mie theory is still valuable to illustrate the reasonable ranges of these two parameters in the ideal situations. Assume that the soil medium consists of spherical particles with radius r. The particle refractive index and the absorption coefficient constitute



0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
ω =0.2, solar principal plane
DISORT code
Modified model
Hapke model
Relative radiance
-80
-40
0
40
80
Viewing zenith angle (degree)
(a)
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
ω=0.2, cross principal plane
DISORT code
Modified model
Hapke model
Relative radiance
-80
-40
0
40
80
Viewing zenith angle (degree)
(b)
0.2
0.15
0.1
0.05
0
ω =0.5, solar principal plane
DISORT code
Modified model
Hapke model
Relative radiance
-80
-40
0
40
80
Viewing zenith angle (degree)
(c)
0.12
0.1
0.08
0.06
0.04
0.02
0
ω =0.5, cross principal plane
DISORT code
Modified model
Hapke model
Relative radiance
-80
-40
0
40
80
Viewing zenith angle (degree)
(d)
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
ω =0.8, solar principal plane
DISORT code
Modified model
Hapke model
Relative radiance
-80
-40
0
40
80
Viewing zenith angle (degree)
(e)
0.3
0.25
0.2
0.15
0.1
0.05
ω =0.8, cross principal plane
DISORT code
Modified model
Hapke model
Relative radiance
-80
-40
0
40
80
Viewing zenith angle (degree)
(f)

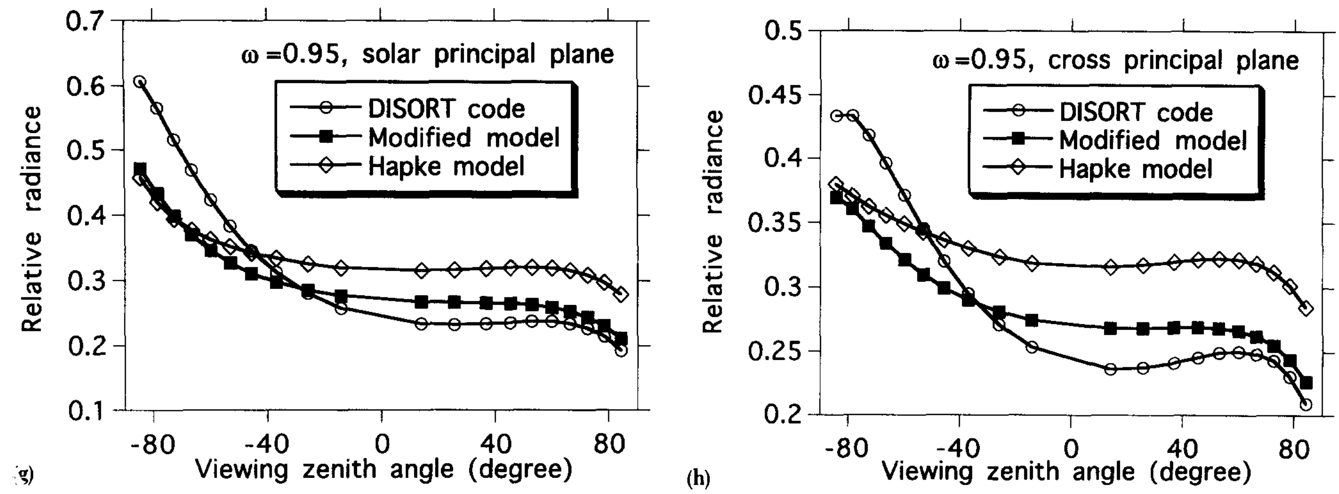
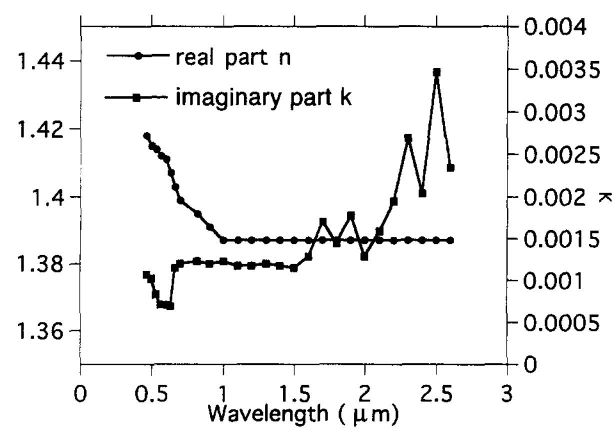


Figure 3. Comparisons of the Hapke model and the modified model with the numerical code DISORT with different single-scattering albedos. The solar zenith angle is 40°, the solar azimuth angle 0°, and asymmetry parameter 0.65.

the real and imaginary parts of the optical complex index. Given those parameters, a code of the Mie theory, kindly provided by Dr. W. Wiscombe at NASA / GSFC (Wiscombe, 1980), was used to calculate the single-scattering albedo and the asymmetry parameter.

It is beyond the scope of this present research to calculate those two parameters for various different kinds of soil. An example is given below for the clay soil because of the ready availability of its optical properties. The particle radius is assumed as 1.5 μm, the optical complex index and their wavelength dependence of clay soil are taken from the measurements (Egan and Hilgeman, 1979) and displayed in Figure 4 from the visible to mid-infrared spectrum region. The calculated single-scattering albedo and the asymmetry parameters are displayed in Figure 5. In this specific example, the single-scattering albedo varies from 0.96 to 0.99, and the asymmetry parameter from 0.55 to 0.85. In these cases, the widely used Hapke model will not predict upwelling radiance very accurately, as demonstrated in Figures 1–3.

Figure 4. Measured complex optical index of clay soil from Egan and Hilgeman (1979).



INVERSION EXPERIMENT

Understanding of the relative merits of the two models can be also aided by inversion experiments. In most situations, a physically based BRDF model always fits experimental data very well if enough free parameters exist. However, if the model is not sufficiently accurate, the fitted physical parameters may be far away from their true values. This point can be well demonstrated in the following inversion experiment.

The inversion experiment is carried out to estimate parameters through minimizing the residual function

$$\sum_{i=1}^{N}(I_i - \hat{I}_i)^2$$

Figure 5. Calculated single-scattering albedo and asymmetry parameter using the Mie theory based on optical parameters given in Figure 4. The clay particle radius is assumed to be 0.0015 mm.

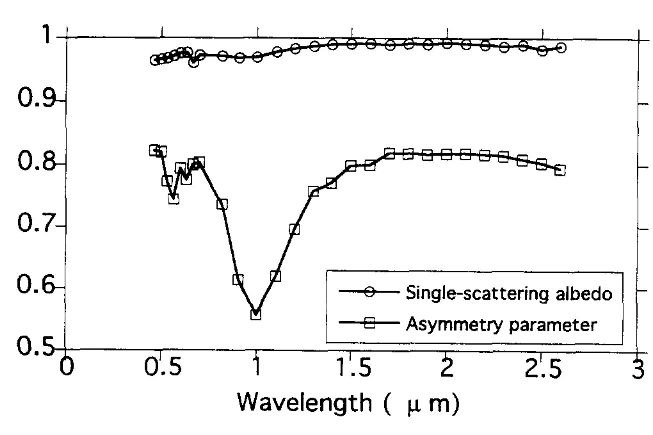


Table 1. Retrieved Single-Scattering Albedo ω and Asymmetry Parameter g Using the Modified Model and the Original Hapke Model from Three Simulated Datasets[a]

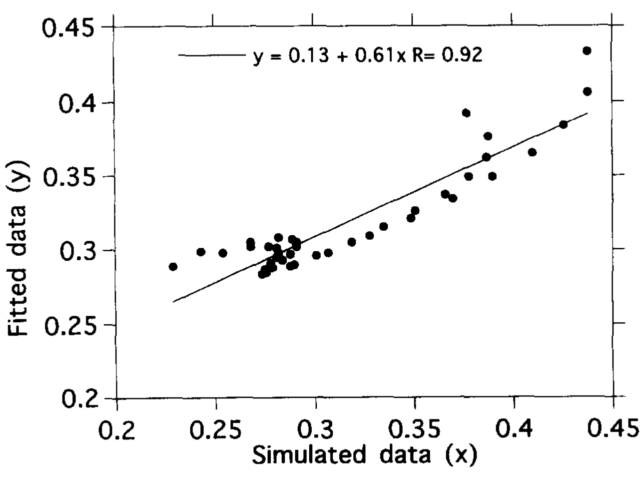
		Dataset-1		*Dataset-2*		*Dataset-3*	
		ω = 0.5	*g = 0.65*	*ω = 0.94*	*g = 0.6*	*ω = 0.94*	*g = 0.6*
Hapke model	P	0.4349	0.4882	0.9580	0.5471	0.9099	0.4026
	C	0.3758	0.4512	0.9312	0.4125	0.9132	0.4571
	P+C	0.4512	0.4877	0.9508	0.5259	0.9110	0.4247
	ε (%)	15.9	26.8	1.3	17.5	3.0	26.4
Modified model	P	0.5114	0.5952	0.9658	0.6124	0.9331	0.4870
	C	0.4593	0.5664	0.9482	0.4994	0.9341	0.5138
	P+C	0.4977	0.5958	0.9621	0.5894	0.9335	0.4950
	ε (%)	3.6	9.9	1.9	6.9	0.7	16.9

[a] P stands for inversion from data only in the principal plane (0–180°), C for inversion from data only in the cross-principal plane (45–225°) and ε for averaged relative error.

where I_i is the simulated radiance using the DISORT code, $\hat{I}_i$ is the predicted radiance using the original Hapke model or the modified model, and N is the number of samples. In order to find optimal estimates of these parameters, an iteration process is needed. At each iteration, the iteration length and iteration direction need to be determined. To date, one of the most successful direction set algorithms is the method due to Powell (1964), especially with the modifications suggested by Zhangwill (1967) and Brent (1973). The program is taken from *Numerical Recipes in C* (Press et al., 1988). This method has been successfully applied in other inversion studies (Liang and Strahler, 1993; 1994).

The simulated radiance by the DISORT code is used to retrieve the single-scattering albedo and asymmetry parameter by means of the original Hapke model and the modified model. Three sets of data are simulated by the DISORT code. Dataset-2 and dataset-3 have the same parameters except that the solar zenith angles are 60° and 30°, respectively. The solar zenith angle of dataset-1 is 40°. Inversion accuracy varies depending on which azimuth plane the data is generated. In all inversion experiments presented below, 10 observations are simulated at each azimuth angle. It is interesting to find that this algorithm is very robust for the selection of initial values. The inversion results are presented in Table 1. The relative errors for the two parameters based on the absolute difference using the Hapke model are (15.9%, 26.8%) for dataset-1, (1.3%, 17.5%) for dataset-2, and (3.0%, 26.4%) for dataset-3, where the first number in the bracket is for the single-scattering albedo and the second for the asymmetry parameter. The corresponding relative errors using the improved Hapke model are (3.6%, 9.9%) and dataset-1, (1.9%, 6.9%) for dataset-2, and (0.7%, 16.9%) for dataset-3. The modified model produces much better inversion results of both single-scattering albedo and asymmetry parameter than the original Hapke model. It is clearly that the inversion accuracy of the single-scattering albedo is much better than that of the asymmetry parameter.

Figure 6. Relation between the fitted radiance using the Hapke model and the original simulated radiance. The true parameters are $\omega = 0.94$ and $g = 0.6$. The retrieved parameters $\hat{\omega} = 0.9110$ and $\hat{g} = 0.4247$.



Although estimated parameters are not very accurate using the original Hapke model, the fitted radiance statistically correlate well with the simulated radiance. An example is given in Figure 6 where the fitted parameters $\omega = 0.911$ and $g = 0.4247$ (see Table 1). Figure 7 illustrates the angular distribution of the error which is described by the difference between the fitted radiance and the simulated radiance. Clearly the errors result from the inaccuracy of the original model.

SUMMARY

The Hapke model for soil bidirectional reflectance has been modified in this study. The radiation field is divided into three rather than two components. The multiple-scattering component is approximated by the original formula with appropriate modification, while single-scattering and double-scattering components are exactly calculated.

A numerical code DISORT based on the discrete-

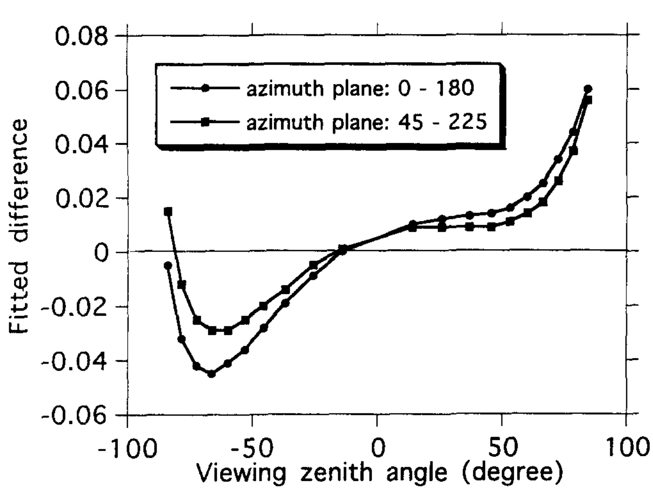


Figure 7. Angular distribution of the difference of the fitted radiance and the original simulated radiance displayed in Figure 6.

ordinate algorithm is used for accuracy verification. When the soil asymmetry parameter is small, where scattering by a soil is close to isotropic or when the single-scattering albedo is small, implying that multiple scattering is weak, both the modified model and original Hapke model work quite well. When the soil scatters anisotropically or multiple scattering dominates, the modified model works much better than the original Hapke model.

A numerical calculation of the clay soil using Mie theory shows that from the visible to mid-infrared spectrum, the asymmetry parameter of the soil particles is larger than 0.55 and single-scattering albedo is larger than 0.96. In those cases, the modified model is much better than the Hapke model, and the latter does not work well. Mie theory applies to sparsely distributed spherical particles, whereas densely packed and irregular soil particles may have smaller single-scattering albedo and asymmetry parameter, which may reduce the errors inherent in the original Hapke model for soil bidirectional reflectance. Further investigations of realistic ranges of those physical parameters for different types of soil are required.

If the parametric model is not accurate enough, estimated parameters may be far from "true" values. An inversion experiment shows that the modified model can yield more accurate estimation of parameters than the Hapke model. It is also demonstrated that a good satisfactory fit between the simulated radiance or reflectance does not mean that the model can consistently retrieve correct physical parameters.

Present soil radiative transfer models still cannot account for soil moisture content, organic matter, and other biochemical substances that may have significant effects on soil bidirectional reflectance. Those models also assume that monodirectional solar radiance illuminates the top of the soil. Thus no sky radiance is explicitly incorporated. Some of those issues are addressed in a subsequent article (Liang and Townshend, 1995).

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