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A PARAMETRIC RADIATIVE TRANSFER MODEL FOR SKY RADIANCE DISTRIBUTION

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Abstract—A parametric model of sky radiance distribution is developed using radiative transfer theory. The radiation field is divided into four components: single-scattering; double-scattering; quasi-multiple-scattering; and surface related component. The first three components are associated with zero surface reflectance. The single-scattering and double-scattering radiance are exactly calculated, and quasi-multiple scattering is fitted statistically from the numerical code DISORT based on the discrete-ordinate algorithm. Surface bidirectional reflectance is incorporated into the last component. Comparisons of this simple model with a numerical code DISORT as well as another parametric sky radiance model are made under different conditions of aerosol optical depth, asymmetry parameter in the phase function and surface reflectance. Results indicate that the present model is very accurate and computationally efficient.

INTRODCTION

Any application to remote sensing which attempts to derive consistent quantitative information of the interaction of shortwave electromagnetic radiation with the Earth's surface and atmosphere needs to consider some form of atmospheric model, particularly parametric models. It is important in such models to be able to capture the directional nature of the sky radiance distribution, as the assumption of an isotropic diffuse irradiance field can often lead to significant errors.[1]

The requirement for such a model in understanding the effect of, and deriving information on atmospheric properties such as aerosol particle size distribution[2] and phase function[3–5] from ground-based measurements of sky radiance distribution is perhaps most readily apparent. The effect of variations in the sky radiance distribution is also of significant importance when attempting to derive intrinsic surface properties of the Earth's surface from remote sensing measurements. Such a model also finds direct application in related fields such as illumination engineering and the evaluation of the performance of solar cells[6].

A number of models of the sky radiance distribution exist, ranging from empirical models, such as that defined by the CIE[7] through to those based on numerical solutions of the radiative transfer equation for a detailed definition of the distribution of atmospheric constituents[8,9]. Somewhere in the middle of this range lie a set of parametric models of sky radiance, in which a relatively small number of parameters are used to define an atmosphere and its coupling to the surface reflectance defining the lower boundary conditions. Such parametric models are of particular use in remote sensing, as they are typically fast to calculate and easy to invert surface properties. These models make a variety of assumptions about the nature of the radiation regime in order to achieve this, and are typically limited in their accuracy and applicability to a given range of conditions. Box and Deepak[10], for example, developed a quasi-single-scattering model of sky radiance, and later added an empirical term to account for multiple-scattering[2]. Their models are however limited to the solar aureole (the region of enhanced brightness within around 20° of the solar disc). Zibordi and Voss[11] developed a sky radiance model based on Sobolev's two-stream approximation to the

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radiative transfer equation[12] which also made the assumption of a Lambertian lower boundary condition. This analytical parametric model was found to compare quite favorably with measured sky radiance data, giving results which were mostly within 10% of the measured values for clear atmosphere. The Z&V model[11] is taken as a useful reference model to compare with the formulation developed here, as the same parameters can be used directly in both models. Results from this comparison suggest that the accuracy of the two-stream approximation can sometimes be rather limited, although the impact of discrepancies between model outputs on the derivation of atmospheric or surface parameters is not investigated here.

A significant difference between previous parametric models and the one developed here is that the new model takes explicit account of both first- and second-order scattering events, rather than including second-order scattering as part of a more approximate solution for orders of scattering greater than one. This work complements recent studies by Liang and Townshend[13,14] which indicate that second-order scattering can account for a significant degree of anisotropy in the radiation field of a soil medium.

The objectives of this study are two-fold: first, to develop a simple but accurate model of sky radiance for parameter inversion and other modeling efforts; and second, to further evaluate accuracy and range of applicability of the Z&V model as an existing sky radiance model which has been used in several previous applications.[15,16]

MODEL DEVELOPMENT

The total sky radiance is divided into four components illustrated in Fig. 1. The first two components are downward singly scattered radiance and doubly scattered radiance, which have the exact solutions to the radiative transfer equation. The third component is due to multiple-scattering (defined here as that due to greater than two scattering events). All three of these components are independent of surface interactions. The last component is composed of the total radiance of the first three components reflected by the surface and scattered back again by the atmosphere.

The atmosphere is assumed to be cloud free and horizontally homogeneous. The total optical depth is τ_0 with zero at the top of the atmosphere. The coordinate system is defined so that zenith angle is positive in the upwelling hemisphere, and negative in the downward hemisphere.

Single-scattering component

The single-scattering component at arbitrary optical depth τ and angle Ω is expressed by

$$I^1(\tau,\Omega)=\begin{cases}\dfrac{\omega F_0 P(\Omega_0\to\Omega)|\mu_0|}{4(|\mu_0|-|\mu|)}\left[\exp\left(-\dfrac{\tau}{|\mu_0|}\right)-\exp\left(-\dfrac{\tau}{|\mu|}\right)\right] & \mu\neq\mu_0\\[2ex] \dfrac{\omega F_0\tau}{4|\mu_0|}P(\Omega_0\to\Omega)\exp\left(-\dfrac{\tau}{|\mu_0|}\right) & \mu=\mu_0,\end{cases} \tag{1}$$

where F_0 is the extraterrestrial irradiance at the top of the atmosphere, ω is the single scattering albedo, $P(\Omega'\to\Omega)$ is the phase function from incident direction Ω' to outgoing direction Ω. The scattering properties of the atmosphere depend on Rayleigh molecules and aerosol particles, so we may further define the scattering phase function as the weighted average of Rayleigh and aerosol scattering phase functions at the specific scattering angle:

$$P(\Psi)=\frac{p_r(\Psi)\tau_r+p_a(\Psi)\tau_{ae}}{\tau_r+\tau_{ae}},$$

subject to the constraint $\frac{1}{2}\int_0^\pi P(\Psi)\sin\Psi\,d\Psi=1$. Here, τ_r and τ_{ae} are the molecular optical depth and aerosol optical depth of the whole atmosphere (i.e. $\tau_r+\tau_{ae}=\tau_0$), respectively. The scattering phase angle, Ψ, is defined as:

$$\cos(\Psi)=\mu\mu_0+\sqrt{(1-\mu^2)(1-\mu_0^2)}\cos(\phi-\phi_0).$$

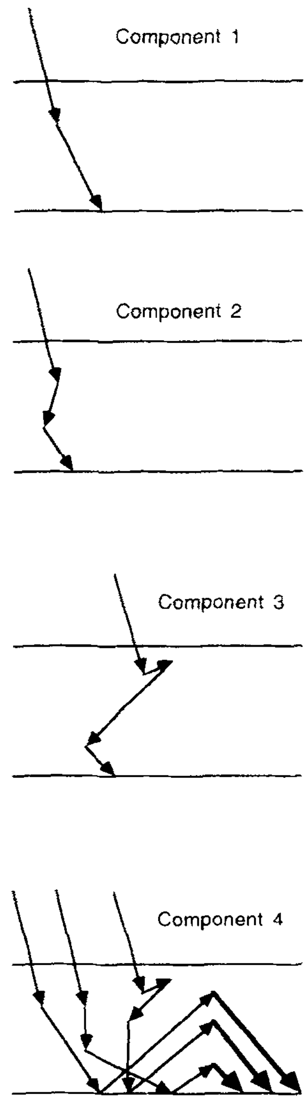

Fig. 1. Illustration of the radiation field decomposition.

The molecular phase function is defined by the classical relation

$$p_r(\Psi) = \tfrac{3}{4}(1 + \cos^2 \Psi),$$

and aerosol phase function can be calculated by Mie theory or in any arbitrary form.

Double-scattering component

The double-scattering component is scattered once more from single-scattering photons in either downward or upwelling direction. Thus, we also need to know the singly scattered radiance in the upward direction $I^{1+}(\tau, \Omega)$

$$I^{1+}(\tau, \Omega) = \frac{\omega F_0 P(\Omega_0, \Omega)|\mu_0|}{4(\mu + |\mu_0|)} \exp\left(-\frac{\tau}{|\mu_0|}\right) - \exp\left[\frac{\tau}{\mu} - \left(\frac{1}{|\mu_0|} + \frac{1}{\mu}\right)\tau_0\right].$$

Double-scattering radiance as part of the sky radiance can be expressed by

$$I^2(\tau_0, \Omega) = \frac{1}{|\mu|}\int_0^{\tau_0} \frac{\omega}{4\pi}\left\{\int_0^{2\pi_+} I^{1+}(t, \Omega')P(\Omega', \Omega)\,\mathrm{d}\Omega' + \int_0^{2\pi_-} I^1(t, \Omega')P(\Omega', \Omega)\,\mathrm{d}\Omega'\right\}\exp\left(\frac{t - \tau_0}{|\mu|}\right)\mathrm{d}t,$$

where $2\pi_{\pm}$ stands for the upper and lower hemisphere respectively. After some algebraic manipulation, we have

$$I^2(\tau_0,\Omega)=\frac{\omega^2F_0|\mu_0|}{16\pi}\left[\int_0^{2\pi_+}\frac{P(\Omega_0,\Omega')P(\Omega',\Omega)}{|\mu_0|+\mu'}\gamma_1(\mu',\mu)\,\mathrm{d}\Omega'+\int_0^{2\pi_-}P(\Omega_0,\Omega')P(\Omega',\Omega)\gamma_2(\mu',\mu)\,\mathrm{d}\Omega'\right] \tag{2}$$

where

$$\gamma_1(\mu',\mu)=\begin{cases} t1-t2 & |u|\neq|\mu_0| \\ \dfrac{\tau_0}{|\mu|}\exp\left(-\dfrac{\tau_0}{|\mu|}\right)-t2 & |\mu|=|\mu_0| \end{cases}$$

and

$$t1=\frac{|\mu_0|}{|\mu|-|\mu_0|}\left[\exp\left(\frac{-\tau_0}{|\mu|}\right)-\exp\left(-\frac{\tau_0}{|\mu_0|}\right)\right]$$

$$t2=\frac{\mu'}{|\mu|+\mu'}\exp\left(-\frac{\tau_0}{|\mu_0|}\right)\left[1-\exp\left(-\left(\frac{1}{|\mu_0|}+\frac{1}{\mu'}\right)\tau_0\right)\right].$$

If $\mu_0\neq\mu'$, then

$$\gamma_2(\mu',\mu)=\begin{cases} t1-t3 & |\mu|\neq|\mu_0|\ \&\ |\mu|\neq|\mu'| \\ \dfrac{\tau_0}{|\mu|}\exp\left(-\dfrac{\tau_0}{|\mu|}\right)-t3 & |\mu|=|\mu_0|\ \&\ |\mu|\neq|\mu'| \\ t1-\dfrac{\tau_0}{|\mu|}\exp\left(-\dfrac{\tau_0}{|\mu|}\right) & |\mu|\neq|\mu_0|\ \&\ |\mu|=|\mu'| \end{cases}$$

where

$$t3=\frac{|\mu'|}{|\mu|-|\mu'|}\left[\exp\left(-\frac{\tau_0}{|\mu|}\right)-\exp\left(-\frac{\tau_0}{|\mu'|}\right)\right].$$

If $\mu_0=\mu'$, then

$$\gamma_2(\mu',\mu)=\begin{cases} \dfrac{|\mu|}{(|\mu|-|\mu_0|)^2}\left\{\exp\left(-\dfrac{\tau_0}{|\mu|}\right)-\exp\left(-\dfrac{\tau_0}{|\mu_0|}\right)\left[\dfrac{|\mu\mu_0|\tau_0}{|\mu|-|\mu_0|}+1\right]\right\} & \mu\neq\mu_0 \\ \dfrac{\tau_0^2}{2|\mu|}\exp\left(-\dfrac{\tau_0}{|\mu|}\right) & \mu=\mu_0 \end{cases}$$

Quasi-multiple scattering

Extensive calculations show that single scattering and double scattering have captured major features of the downwelling sky radiance field. Instead of deriving complicated formulae for multiple scattering, a statistical formula is fitted from the numerical code DISORT using the discrete-ordinate algorithm [17]. Since Rayleigh optical depth usually is small in the visible and near-infrared spectrum, multiple scattering (scattered more than twice by the atmosphere) is typically not significant. Thus, two major factors are considered in the statistical fitting: solar zenith angle and aerosol optical depth. In generating the test dataset using DISORT, solar zenith angle θ_0 ranges from 5° to 65°, aerosol optical depth τ from 0.1 to 0.7, viewing zenith angle θ from 5° to 70°, relative azimuth angle ϕ from 0° to 225°. The resulting formula fitted to the simulation is:

$$I^3(\theta,\phi)=0.083\omega F_0\tau^{1.5703}\theta_0^{-0.0368}[\theta^2+0.3775\theta\cos(\phi-\phi_0)+0.5418]. \tag{3}$$

Ground related component

The total upward radiance reflected from the surface due to direct sunlight, singly scattered radiance and doubly scattered radiance is

$$L_s(\Omega) = |\mu_0|\pi F_0 \exp\left(-\frac{\tau_0}{|\mu_0|}\right) f(\Omega_0, \Omega) + \int_0^{2\pi} \int_{-1}^{0} [I^1(\tau_0, \Omega') + I^2(\tau_0, \Omega')] f(\Omega', \Omega) \mu' \, d\mu' \, d\phi'$$

where $f(\Omega', \Omega)$ is the surface BRDF. If the surface performs as a Lambertian, then

$$f(\Omega', \Omega) = \frac{R_s}{\pi}.$$

Where, R_s is the surface reflectance. If only one interaction between the atmosphere and the surface is considered, then component four in the sky radiance distribution model becomes

$$I^4(\tau_0, \Omega) = \frac{1}{|\mu|} \int_0^{\tau_0} L_s(\Omega') \exp\left(\frac{t - \tau_0}{\mu'}\right) P(\Omega', \Omega) \, d\Omega' \exp\left(-\frac{\tau_0 - t}{|\mu|}\right) dt.$$

A bit algebraic derivation leads to the following formula:

$$I^4(\tau_0, \Omega) = \frac{\omega}{4\pi} \int_0^{2\pi} \int_0^1 L_s(\Omega') P(\Omega', \Omega) \gamma_3(\mu', \mu) \, d\mu' \, d\phi' \tag{4}$$

where

$$\gamma_3(\mu', \mu) = \frac{\mu'}{|\mu| + \mu'} \left[1 - \exp\left(-\tau_0\left(\frac{1}{|\mu|} + \frac{1}{\mu'}\right)\right)\right].$$

When the atmosphere is not very turbid and surface is not very bright, this assumption of the single interaction is very reasonable. In other cases, multiple interactions may occur. To deal with those extreme cases, we introduce the *interaction factor* (α) for keeping the same formula (4) instead of deriving complicated formula. When multiple interactions occur, more photons will be scattered back to the surface, which is equivalent to the reflectance from a brighter surface with a single interaction. An empirical model based on trial experiments of this factor, which depends only on the surface bihemispherical reflectance (R) is given as:

$$\alpha(R) = \begin{cases} 2.5 - \dfrac{0.15}{R} & \text{if } R \geqslant 0.1 \\ 1 & \text{otherwise.} \end{cases}$$

From this formula, we can see that if the surface reflectance is <0.1, multiple interaction between the surface and atmosphere is negligible. The interaction factor is proportional to surface reflectance. This formula implies that the surface should look approximately twice as bright. Thus we need to multiply the surface BRDF $f(\Omega', \Omega)$ by the interaction factor $\alpha(R)$ in order to account for the multiple interactions while keeping the single interaction formula.

In summary, the total sky radiance is the sum of four components presented in equations (1), (2), (3) and (4). The next section presents an analysis of this model, and a comparison with results from the other models. In the following data analysis, the sky radiance is also normalized sometimes by $|\mu_0|\pi F_0$ and denoted by diffuse transmittance.

MODEL COMPARISONS AND DATA ANALYSIS

To evaluate the accuracy and analyze the behaviors of the analytical sky radiance distribution model, a series of validations for the aerosol atmosphere using the numerical code DISORT based on the discrete-ordinate algorithm[17] have been carried out. Note that datasets previously used for fitting multiple scattering are not employed here for validation. Comparisons with Z&V model are also given. All the following calculations are implemented by setting $F_0 = 1$ for simplicity, and thus downwelling radiance is denoted as a relative quantity. The single-scattering albedo is set as 0.94. For simplicity, the One-Term Henyey-Greenstein (OTHG) function is used as aerosol phase function in the following numerical calculations:

$$p_a(\Psi) = \frac{(1-g^2)}{(1+g^2-2g\cos\Psi)^{1.5}},$$

where g is the asymmetry factor.

Figure 2 compares the three models for different values of optical depth (0.3 and 1.0) two solar zenith angles (30° and 60°) and in two relative azimuth planes (0°–180° and 45°–225°). In all cases, the new model appears to match the results from DISORT very well especially in the principal plane. Differences in large viewing angles can be visually detected. The Z&V model works reasonably good but does not perform that well in some cases. When the solar zenith angle is 60°, the differences in solar aureole between the Z&V model and DISORT are very significant. In all cases, the Z&V model underestimates sky radiance in the forward scattering directions, and overestimates in the backscattering directions.

However, for most applications in remote sensing, we are not typically concerned with very hazy atmospheres, and an optical depth of 1.0 can be taken as an extreme case. We can see from this analysis that both models perform relatively well when compared with results from DISORT, but that at high optical depths and at high solar zenith angles, the Z&V model performs less well than the new model.

It is clear from equation (1) that the phase function greatly affects the magnitude and angular distribution of the sky radiance. Figure 3 compares results from the three models for two different values of asymmetry parameter (0.5 and 0.8). When the asymmetry parameter is set to 0.8, there is a very high peak in the solar aureole. Both parametric models predict this trend quite well, but the new model performs much better than the Z&V model in both cases.

Figure 4 illustrates the effect of surface reflectance on diffuse transmittance in two azimuth planes with the modestly turbid atmosphere. In all cases, the new model matches DISORT very well. Although multiple interactions dominate when the surface reflectance is 0.8, the new model works

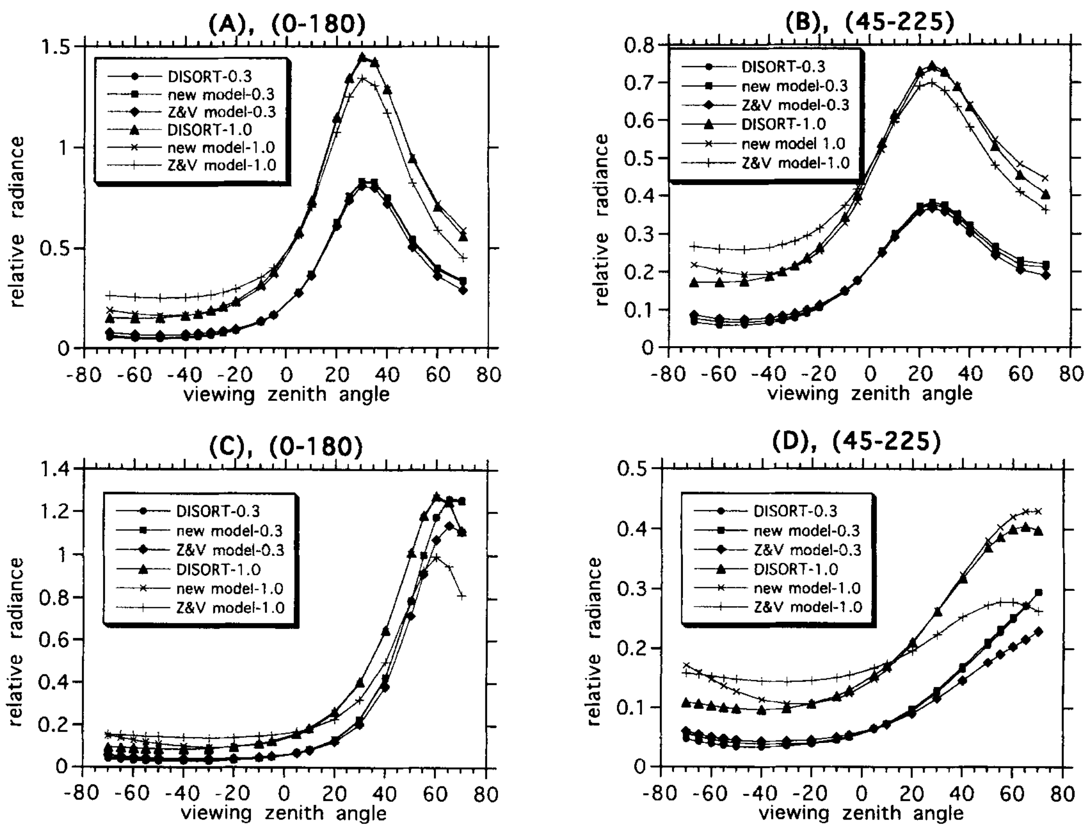

Fig. 2. Comparison of the new developed sky radiance distribution model with DISORT as well as Z&V model with different aerosol optical depth (0.3 and 1.0). (A) and (B) for solar zenith angle 30°, (C) and (D) for 60°. Asymmetry parameter 0.65, surface reflectance 0.0.

very well because of the adjustment using the interaction factor. The Z&V model displays the same kind of error trend as observed in Fig. 1, but works reasonably well.

As noted above, the new model takes into account an arbitrary surface BRDF, while the original Z&V model does not. No further analysis of this aspect of the model is undertaken, since it is apparent that the sky radiance is only a weak function of the surface reflectance. In some cases however, where the atmosphere is clear but the surface reflectance is high (e.g. vegetation canopies in the near infrared, ice, snow, and bright sandy surfaces) the assumption of a Lambertian surface is likely to lead to significant errors.

BRIEF SUMMARY

A simple but very accurate sky radiance distribution model has been developed. The new model consists of four components: single-scattering; double-scattering; quasi-multiple-scattering; and surface related component. The first two components can be calculated exactly. The third component is small compared with other components in most of cases and fitted statistically from datasets simulated by the numerical code DISORT. The last component considers once interaction between the atmosphere and the surface but take into account arbitrary surface BRDF.

Comparisons between the numerical code DISORT based on the discrete ordinates method and the new model indicate that the new model is generally very accurate for a wide range of conditions in forward modeling. It would appear therefore that this parametric model is well-suited to modeling sky radiance distributions, and that the promising results, combined with the fact that it can be coupled to an arbitrary surface reflectance function, indicate that it may be a useful model for a variety of remote sensing applications.

Comparisons with the Z&V model indicate that the Z&V model is a useful model of sky radiance, which is appropriate to a range of remote sensing applications, although the accuracy of the model

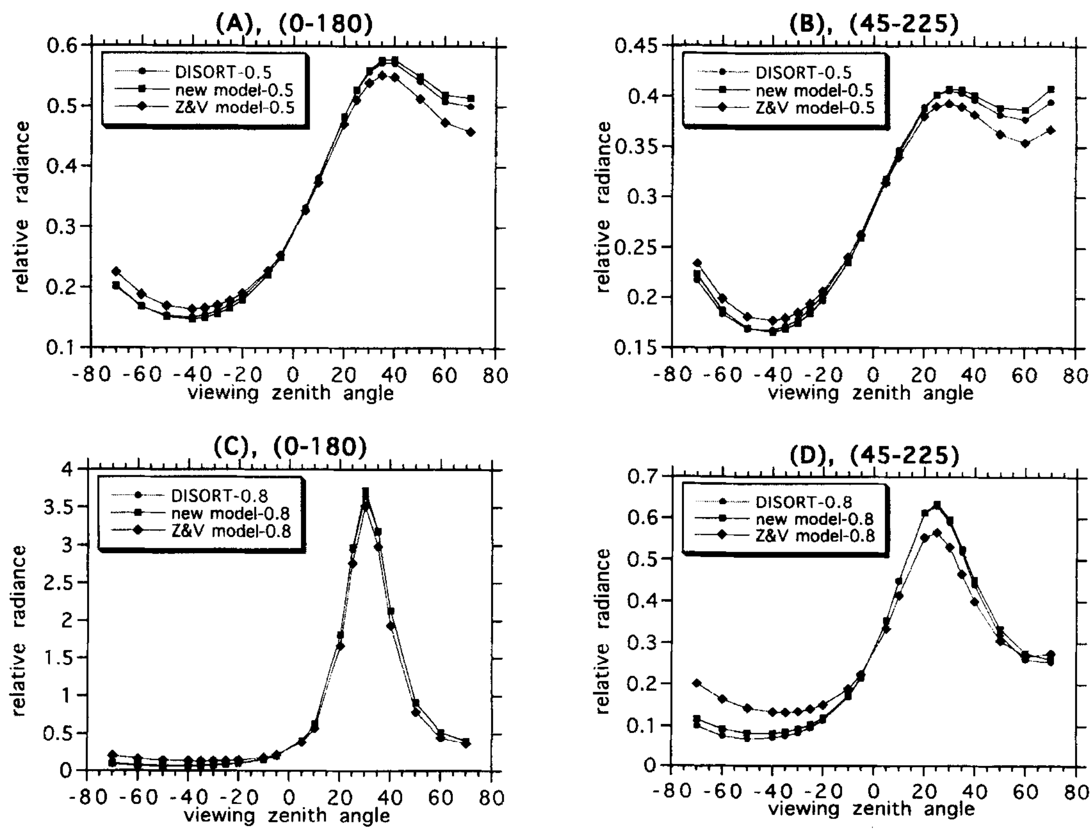

Fig. 3. Evaluation of the effect of asymmetry parameter on sky radiance distribution. (A) and (B) for asymmetry parameter 0.5, and (C) and (D) for 0.8. Aerosol optical depth is 0.5, surface reflectance 0.3, solar zenith angle 30°.

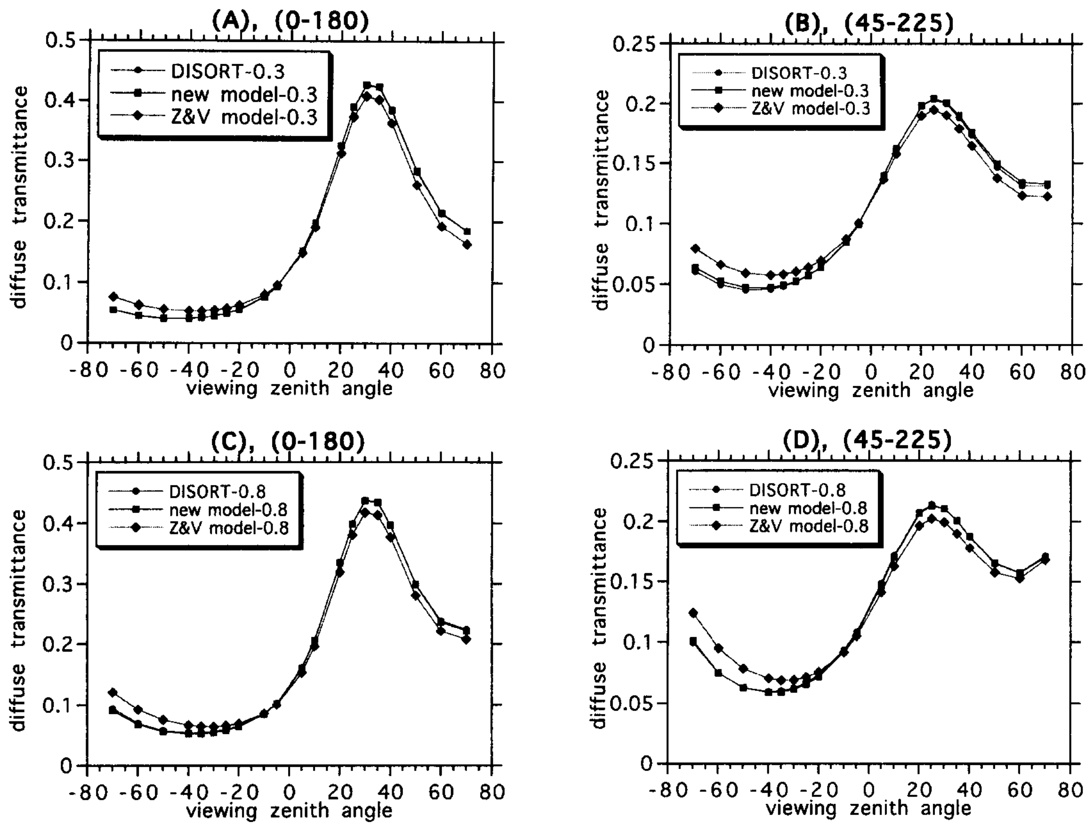

Fig. 4. Evaluation of the effect of different surface reflectance on sky radiance distribution. (A) and (B) for surface reflectance 0.3, (C) and (D) for 0.8. Asymmetry parameter 0.65, aerosol optical depth 0.5, solar zenith angle 30°.

seems to be poorer than the new model, especially for atmospheres with a high optical depth and for high solar zenith angles.

In conclusion, both the new model and the Z&V model are quick to calculate, and, on the whole, provide reasonable estimates of sky radiance, although the range of parameters for which the latter is accurate appears to be more limited than that of the former. Further work is required on the validation of the new model, especially in relation to comparing model predictions with measured sky radiance data. Futher work is also required on assessing the impact of inaccuracies in the sky radiance distribution of the derivation of atmospheric parameters and on atmospheric correction algorithms for remote sensing data, as well as on the use of the model in deriving aerosol size distributions from measured sky radiance data.

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