

On the formation of a plasma bubble

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[1] A simple self-consistent model is proposed, which describes properties of the tail current sheet after the formation of a small plasmoid on the closed field lines in the magnetotail and its quick tailward retreat. Using a theory of forced magnetic reconnection, it is shown that the flux tube that contained a plasmoid becomes a plasma bubble, with the plasmoid being replaced by a thin current sheet embedded within the original thicker sheet. The model allows for an estimate of the decrease of the plasma entropy per unit magnetic flux within the bubble. It predicts that the bubble structure may also include bifurcated current sheets.
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1. Introduction

[2] Magnetic flux and plasma transport through the tail of the Earth's magnetosphere is bursty in time and localized in space [Baumjohann *et al.*, 1990; Angelopoulos *et al.*, 1992]. Steady-state convection theory, based on the assumptions of frozen-in magnetic flux as well as entropy and mass conservation [Erickson and Wolf, 1980; Schindler and Birn, 1982], predicts unrealistically large plasma pressures for flux tubes arriving from the distant tail to its near-Earth region. To resolve this convection crisis problem, Pontius and Wolf [1990] suggested the existence of underpopulated flux tubes (plasma bubbles) that might convect much faster than the rest of the tail plasmas because of the buoyancy effect. The existence of bubbles and their relation to bursty bulk flows (BBF) was later confirmed by many observations [Angelopoulos *et al.*, 1992; Sergeev *et al.*, 1992, 1996; Nakamura *et al.*, 2004]. Bubbles are believed to appear as a result of a localized reconnection process in the magnetotail [Chen and Wolf, 1993]. However, both formation and properties of bubbles remain insufficiently understood.

[3] The original bubble theory [Pontius and Wolf, 1990] postulated the existence of flux tubes with the lower plasma pressure p and stronger magnetic field than their neighbors. Although these signatures were indeed observed later [e.g., Sergeev *et al.*, 1996], the BBF observations [Angelopoulos *et al.*, 1992] showed rather an increase of the plasma pressure. To resolve this inconsistency, Chen and Wolf [1993] proposed another theory assuming the reduction of the flux tube volume per unit magnetic flux $V = \int ds/B$ (ds is the line element along a field line) in the tubes that had rounder shape and stronger equatorial magnetic field than

their neighbors. However, it remains unclear, which one of these two bubble types appears initially, and what is its specific self-consistent structure. It is also unclear what is the specific difference of the main convection theory parameter pV^γ between the bubble and neighboring flux tubes and how it depends on the preceding reconnection process. In this Letter we propose a model of the bubble formation due to reconnection and subsequent tailward plasmoid retreat. The model is based on the theory of forced magnetic reconnection [Kulsrud and Hahm, 1982; Lakhina, 1992; Schindler and Birn, 1993], which is modified and reinterpreted, invoking stability properties of geomagnetotail plasmas. It describes the properties of young bubbles [Chen and Wolf, 1993], including their self-consistent structure and the reduction of the parameter pV^γ .

2. Model Description and Main Results

[4] Let us consider the magnetotail when a small plasmoid is just formed due to the reconnection of closed field lines (Figure 1). If the plasmoid is small enough it may quickly retreat tailward due to the ballooning-interchange instability [Wu *et al.*, 1998]. This local process should not affect the global state of the current sheet (CS), and in particular the boundary conditions outside the CS. On the other hand, it is known [Kulsrud and Hahm, 1982] that a stationary boundary perturbation results in two different CS equilibria, one with changes of magnetic field topology (plasmoids) and another, containing discontinuities. The removal of the plasmoid from the system should result in the restoration of the magnetic field topology, which existed prior to the plasmoid formation. This new state can be described therefore by the second (discontinuous) solution of the same equilibrium problem, which had the boundary perturbed by the reconnection process that resulted in the plasmoid formation.

[5] To describe this effect quantitatively we consider the solutions of the 2D Grad-Shafranov equation for the dawn-dusk component A of the vector potential of the tail CS [e.g., Schindler and Birn, 1993]

$$\Delta A = \exp(-2A) \quad (1)$$

where $\Delta = \partial^2/\partial x^2 + \partial^2/\partial z^2$ (the X-axis of our coordinate system points in the anti-sunward direction, while the Y- and Z-axes are directed towards dawn and north, respectively). We use here dimensionless variables with the magnetic field B_x being normalized to its lobe value B_0 and the coordinates being normalized to the CS thickness L . The right hand side of (1) corresponds to one of the simplest

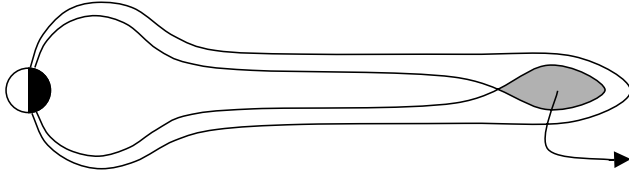


Figure 1. A cartoon of the tailward retreat of a small plasmoid within the closed field line region in the tail.

isotropic plasma equilibria, which transforms in the 1D case ($\partial/\partial x = 0$) into the well-known Harris equilibrium [Harris, 1962].

[6] The unperturbed tail-like solution of (1) can be taken in the form [e.g., Lembege and Pellat, 1982]

$$A_0(x, z) = \log\{\cosh[F(x)z]/F(x)\} \quad (2)$$

with $F(x) = \exp(-\varepsilon x)$, where $\varepsilon \ll 1$ is a small parameter giving the ratio of the equatorial magnetic field ($z = 0$) to the field B_0 .

[7] We consider the perturbation A_1 of the potential A_0 due to the following boundary perturbation at $z = 1$

$$A_1|_{z=1} = \delta_0 f(x - x_0) \quad (3)$$

with the Fourier transform of the function $f(x)$

$$f_k = \left(k^2/\sqrt{2}k_0\right) \exp(-k^2/4k_0^2) \quad (4)$$

corresponding to the simplest wavelet form of the perturbation along the X-direction. In contrast to earlier works [Kulsrud and Hahm, 1982; Lakhina, 1992] considering pure sinusoidal perturbations of 1D CSs, our perturbation is localized at $x = x_0$ with the scale $\Delta x \sim 1/k_0$. Also, distinct from these works, the perturbation does not necessarily drive reconnection. It is rather a part of the global self-consistent tearing mode structure, either forced or spontaneous.

[8] The solution of (1) with the equilibrium (2) being perturbed according to (3)–(4) can be found by linearizing (1) (a rigorous asymptotic theory valid for arbitrary boundary perturbations is given by Tur et al. [2001])

$$\Delta A_1 + \frac{2F_0^2}{\cosh^2(F_0 z)} A_1 = 0 \quad (5)$$

Here we assume that $F(x) \simeq F(x_0) = F_0$, due to the local character of the perturbation (3)–(4). For each wavenumber k , equation (5) has two solutions [Lakhina, 1992] $\cos(kx) a_k^{(p,b)}(z)$ with

$$a_k^{(p)} = \tanh(\bar{z}) \sinh(kz) - k \cosh(kz), \quad (6)$$

$$a_k^{(b)} = \tanh(\bar{z}) \cosh(kz) - k \sinh(kz), \quad (7)$$

where $\bar{z} = F_0 z$. With symmetry condition $A(-z) = A(z)$, the first of these solutions describes the perturbations with magnetic topology changes. The other solution has a jump

in the derivative $\partial A/\partial z$ at $z = 0$, corresponding to a surface current. The solution of (1) with the boundary conditions (3)–(4) can now be presented in the form $A_0 + A_1$ with these two choices for the perturbed potential A_1 marked below by the indices (p) and (b)

$$A_1^{(p,b)}(x, z) = \frac{\delta_0}{2\pi} \int_{-\infty}^{\infty} f_k \frac{a_k^{(p,b)}(z)}{a_k^{(p,b)}(1)} \cos(kx) dk \quad (8)$$

[9] The two solutions for parameters $\varepsilon = 0.01$, $x_0 = 7$, $k_0 = 2$, and $\delta_0 = 0.06$ are shown in Figure 2. (The specific choice of these parameters reflects the scale ratios characteristic of the geomagnetotail and plasmoids and will be further discussed in section 3.) According to this figure, the flux tube, which contained the plasmoid (Figure 2 (top)) transforms into a tube, which has no constriction near the X-line. Knowing this new state of the system, allows one to estimate the change of entropy per unit flux pV^γ caused by the loss of the plasmoid as $\delta(pV^\gamma) \approx -\gamma pV^{\gamma-1} \delta V$. The reduction of the flux tube volume δV is given by the integral over the plasmoid separatrix (the small loop shown in Figure 2 (top))

$$\delta V \sim \int_{\text{plasmoid}} \frac{ds}{B} \approx 4 \int_0^{h/2} \frac{dz}{B_z} \sim \frac{2h}{B_n} \quad (9)$$

Here h is the thickness of the plasmoid along the Z-direction, and B_n is the equatorial magnetic field in the non-perturbed tail (2).

[10] Though the estimate (9) looks quite natural since the plasmoid removes some plasma from the flux tube, it becomes feasible because we have found the specific self-consistent final state of the CS, which replaces the plasmoid after its retreat (Figure 2 (bottom)). As seen in Figure 3, this state is rather unusual. It requires a system of singular currents, localized at the neutral plane. The positive peak of the singular current $-I_y$ enhances the non-singular

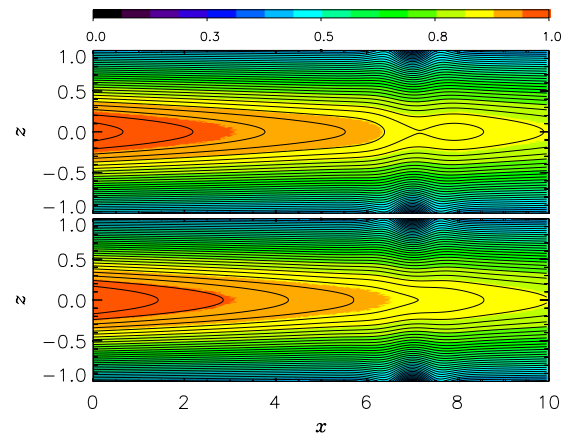


Figure 2. Two different solutions of the Grad-Shafranov equation (1) with the same boundary perturbation (3)–(4). Contour plots show the magnetic field lines for solutions assuming (top) changes of topology and (bottom) continuity, respectively. The corresponding non-singular current densities, normalized by their maximum values, are color-coded.

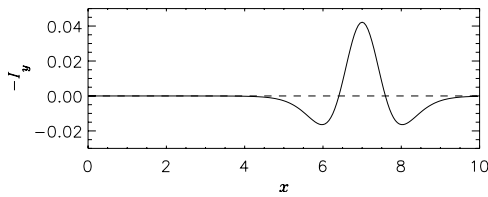


Figure 3. Profile of singular current flowing in the neutral plane for the case shown in Figure 2 (bottom).

current, which is negative in the adopted coordinate system and whose absolute value is color-coded in Figure 2. It represents a thin CS embedded within the original Harris-like CS (2). The negative singular currents split or “bifurcate” the original tail current.

[11] Such unusually thin CSs with a thickness of the order of a few thermal ion gyroradii in the lobe field, both embedded and bifurcated ones, are indeed found in the magnetotail [McComas *et al.*, 1986; Sergeev *et al.*, 1993, 2003; Nakamura *et al.*, 2002; Runov *et al.*, 2003; Asano *et al.*, 2005]. On the other hand, studies of the plasma bubbles and BBFs revealed the associated CS thinning and bifurcation [Sergeev *et al.*, 1992; Nakamura *et al.*, 2002, 2004]. Of particular interest is the set of current density profiles inferred by Nakamura *et al.* [2002] from the multi-probe Cluster observations of the earthward fast flow event on August 12, 2001, and shown in the bottom panel of their Figure 5. It reveals a combination of thin CSs, with an embedded sheet sandwiched between two bifurcated sheets along the Sun-Earth direction, which strongly resembles our Figure 3.

[12] The corresponding steady-state kinetic models of thin non-Harris sheets [Sitnov *et al.*, 2000, 2003; Schindler and Birn, 2002; Birn *et al.*, 2004] show that embedding and bifurcation can readily be possible in thin CSs in the presence of small plasma anisotropy and non-gyrotropy.

[13] Our model also shows (Figure 4) that similar changes of the magnetic field outside a CS may correspond to drastically different structures inside the CS. In particular, some of the earthward traveling compression regions [Slavin *et al.*, 2003], which are usually interpreted as perturbations arising from the small plasmoids or flux ropes dragged by a more global reconnection process toward the Earth, may be manifestations of plasma bubbles rather than plasmoids, particularly, close enough to the Earth ($X_{GSM} > -25R_E$), where the plasmoids are not likely to occur [Ieda *et al.*, 1998]. This is consistent with the statistics of convective fast flows inside CSs [Ohtani *et al.*, 2004], which shows no change of sign in the B_z component for earthward fast flows, in contrast to a clear sign reversal for tailward flows.

3. Discussion and Conclusion

[14] We have shown that the plasma bubble, which appears after the formation and tailward retreat of a small plasmoid, can be described as the alternative solution of the same Grad-Shafranov equation that described the perturbed CS containing the plasmoid. Our results are consistent with observations of BBFs, bubbles and thin CSs in the tail of Earth’s magnetosphere as well as some of the earlier bubble models. They also have some distinctive features. In par-

ticular, Chen and Wolf [1993] suggested the reduction of entropy notwithstanding the higher plasma pressure for mature bubbles. In contrast, our model predicts a similar reduction for young bubbles. This is consistent however with observations [Sergeev *et al.*, 1996], which suggest that the “classical” bubbles exhibiting decreased internal pressures occur when the plasma sheet is relatively thick and background field lines are relatively round. On the other hand, our model is limited to the bubble formation process and it does not address the issue of its further acceleration and possible transformation into a BBF. However, it predicts another group of phenomena associated with young bubbles, namely the effects of thin CS embedding and bifurcation. Although both plasma bubbles and thin CSs were extensively studied in the past, no systematic study of their possible connection has been performed so far.

[15] The core part of the theory of forced magnetic reconnection [Kulsrud and Hahn, 1982; Lakhina, 1992], which our model is based upon, is the equilibrium theory of a field reversal with boundary perturbations. Then additional stability arguments are invoked to determine which of the two equilibria should occur in practice. In particular, Kulsrud and Hahn [1982] studied collisional plasmas with considerable resistivity and zero normal component of the magnetic field $B_n = 0$. They found that a thin CS (discontinuous solution), which develops as a result of the boundary perturbation in a tearing-stable non-singular CS, becomes tearing-unstable and it transforms into a state with different magnetic topology (plasmoid-containing solution). In contrast, the geomagnetotail CS is characterized by collisionless plasmas and finite B_n field, which drastically change its stability properties. According to Sitnov *et al.* [2002], to be unstable such a sheet must be not only thin enough but also sufficiently long to overcome the stabilization effect of trapped electrons [Lembege and Pellat, 1982]. Its length L_t must exceed some critical value $L_c \gg \lambda \sim L_p$, where λ is the tearing wavelength and L_p is the length of the plasmoid. (Note that this is fully consistent with the plasmoid statistics [Ieda *et al.*, 1998] showing that the critical distance from the Earth, where plasmoids start to appear is about $6L_p \gg L_p$.) These stability properties suggest two possible scenarios of the tail dynamics. In the case of a short tail ($L_t < L_c$) the boundary perturbation should result in the formation of a thin CS, which may remain stable, having however no bubble properties. If the tail is long enough ($L_t > L_c$) it becomes tearing-unstable and a plasmoid may be formed,

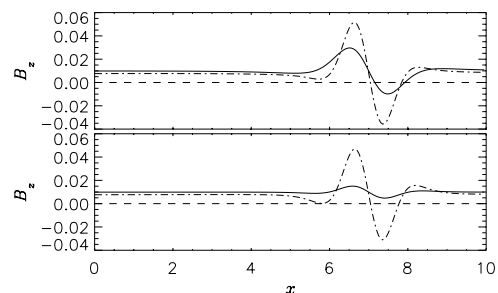


Figure 4. Profiles of B_z component of the magnetic field along X-axis for the (top) plasmoid and (bottom) bubble cases close to the neutral plane ($z = 0.1$, solid lines) and relatively far from it ($z = 0.5$, dash-dotted lines).

either spontaneously or due to boundary perturbations. However, the resulting configuration with the tailward gradient of the equatorial magnetic field profile is unstable to ballooning-interchange modes [Wu *et al.*, 1998], which allow for the escape of the plasmoid. The final state in this scenario, a tail CS with an additional “mini-tail” comprised of the bubble thin CSs has stability properties different from the original tail. The mini-tail associated with the bubble is rather short ($\sim L_p$). Hence, it is either tearing-stable or assumes the formation of a small sub-plasmoid with the length $\sim L_p(L_p/L_c) \ll L_p$ at its tailward end.

[16] It is important to note that both instabilities discussed above develop on the time scales well below the MHD scales necessary to modify the global configuration, including the boundary conditions. In particular, according to Brittnacher *et al.* [1995], the tearing growth rate γ_t for the lobe field 30nT and the parameters $\varepsilon = 0.01$ and $k_0 \sim 1$ can be estimated as $\gamma_t \sim 0.1\text{--}0.3\text{ s}^{-1}$, while the growth rate γ_i of the interchange mode with the shortest wavelength in the Y-direction, considered by Wu *et al.* [1998], is $\gamma_i \sim 0.1\text{ s}^{-1}$. Thus, the tail CS stability theory fully supports the sequence of transitions between different possible equilibrium states considered in section 2, although this sequence is different from the one, considered in earlier models of forced magnetic reconnection.

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