Current-driven instabilities in forced current sheets

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Received 8 July 2003; revised 4 December 2003; accepted 31 December 2003; published 11 March 2004.

[1] The nonlocal kinetic linear stability analysis of the non-Harris thin current sheet equilibrium, namely the thin current sheet embedded in a thicker anisotropic plasma sheet [Sitnov et al., 2000a, 2000b], with respect to current-driven instabilities is performed using the finite element technique. In contrast to the Harris sheet, the new equilibrium becomes possible due to the plasma anisotropy outside the sheet caused by two warm counterstreaming field-aligned beams and complex ion orbits that cannot be described in such thin current sheets in terms of the conventional magnetic moment. It is found that in contrast to the case of the Harris sheet, the analogs of the drift-kink instability in these current sheets can have significant growth rates for the realistic ion-to-electron mass and temperature ratios. The unstable modes share the properties with both the lower-hybrid and drift-kink modes. In particular, the unstable modes resemble the lower-hybrid drift modes as they are more highly structured across the sheet than the drift-kink instability (DKI) and assume both odd (DKI-like) and even parity solutions. On the other hand, in contrast to the lower-hybrid drift instability (LHDI) and like the DKI, the unstable modes have much larger wavelength, electromagnetic component, and significantly perturb the central current region. The possible role of the current-driven instabilities in magnetic reconnection and magnetic annihilation as well as the geophysical implications such as the current disruption in the geomagnetotail during substorms are also discussed. INDEX TERMS: 2772 Magnetospheric Physics: Plasma waves and instabilities; 2744 Magnetospheric Physics: Magnetotail; 3230 Mathematical Geophysics: Numerical solutions; 2788 Magnetospheric Physics: Storms and substorms; 7827 Space Plasma Physics: Kinetic and MHD theory; KEYWORDS: current-driven instability, thin current sheet, reconnection onset, magnetic annihilation, current disruption


1. Introduction

[2] Current sheets are the key structure elements of many magnetized plasma formations. They maintain oppositely directed magnetic fields created by the dynamo process. Their filamentation or disruption provide the reverse processes, which transform the magnetic field energy into the particle energy and are known as magnetic reconnection and magnetic annihilation. In collisional plasmas these processes are controlled by plasma resistivity. However, in high temperature plasmas typical for fusion devices, Earth’s magnetosphere, and solar corona collisions are negligible. They cannot explain energy transformation processes and in particular their characteristic time scales [e.g., Biskamp, 2000]. The corresponding collisionless mechanisms involve excitation of plasma waves, wave-particle interaction, and plasma turbulence. This is why the stability problem of the current sheets in collisionless plasmas is crucial for determining the onset conditions and time scales of magnetic reconnection and magnetic annihilation. In particular the stability of the current sheet in the tail of Earth’s magnetosphere determines the onset of magnetic reconnection and current disruption phenomena, which are the main mechanisms of magnetospheric substorms [Baker et al., 1996; Lui et al., 1992; Lui, 1996]. Instabilities also play an important role in the magnetic reconnection experiments involving weakly collisional plasmas [Carter et al., 2002; H. Ji et al., 2004], hereinafter referred to as Ji et al., submitted manuscript, 2004].

[3] However, in spite of the long research efforts, the main unstable modes of the current sheets responsible for the onset of the explosive energy release remain poorly understood. One of the most impressive examples is the mechanism of the substorm onset in the Earth’s magnetotail, the main reservoir where the magnetic field energy is accumulated and then suddenly released during substorms. Originally, this explosive release of energy was explained by the onset of the reconnection with the X-line pattern due to the collisionless tearing mode [Laval et al., 1966; Coppi et al., 1966]. However, as was shown later, the presence of even a very small component $B_n$ of the magnetic field...
normal to the sheet plane, which magnetizes plasma electrons, results in strong stabilization of the tearing mode [Lembege and Pellat, 1982; Pellat et al., 1991]. Further studies [Sitnov et al., 1998, 2002] clarified that the onset of the X-line reconnection via tearing modes becomes possible when the tail current sheet is long enough to allow for kinetic response of the electron species resulting from different motions of trapped and passing particles. In fact, recent Geotail measurements showed that the formation of the near-Earth neutral line and plasmoisds during substorms usually starts in the premidnight sector of the magnetotail between $X_{GSM} = -20R_E$ and $X_{GSM} = -30R_E$ [Nagai et al., 1998; Ieda et al., 1998]. Closer to the Earth, the tail current sheet is tearing-stable and changes of the magnetic topology are not favorable energetically. As a result, the perturbations due either to the immediate solar wind trigger or to quasi-static changes in the process of magnetospheric convection should result in the formation of MHD discontinuities rather than the change of magnetic topology [Syrovatskii, 1971; Kulsrud and Hahn, 1982; Schindler and Birn, 1993]. The more detailed MHD [Birn and Schindler, 2002] and kinetic [Pritchett and Coroniti, 1994, 1995; Hesse et al., 1996; Lottermozer et al., 1998] modeling shows that these discontinuities are represented by thin current sheets (TCS) with thickness comparable to the thermal ion gyroradius based on the field outside the sheet.

[4] At the early stage of its formation the TCS may be dominated by the electron current because the difference between the electron and ion response to the convection fields leads to a negative charging of the central plasma sheet and the corresponding additional drift of both electrons and ions in the direction of the original electron drift in the Harris-type sheet [Pritchett and Coroniti, 1994]. Later, owing to the penetration of the convection field inside the sheet, the unmagnetized ions can be accelerated and can dominate the current [Burkhart et al., 1992; Pritchett and Coroniti, 1992; Holland and Chen, 1993; Kropotkin et al., 1997; Sitnov et al., 2000a, 2000b]. The current is generated by the quasi-adiabatic motion of ions that form the counter-streaming flows outside the TCS [Speiser, 1965]. After Burkhart et al. [1992], these TCSs are called forced current sheets. They may serve as the outflow regions of the X-line reconnection pattern [Hill, 1975; Lottermozer et al., 1998]. Note however that the penetration of the convection electric field inside the TCS is not necessarily related to the formation of the X-line. More correctly, these two processes should be considered as different manifestations of the same global process of the energy transformation in the tail-like systems.

[5] Concentration of the current density in the TCS forms a free energy source for current-driven instabilities, which do not change the initial magnetic field topology as they propagate in the dawn-dusk direction. The most extensively studied instability of this class is the lower-hybrid drift instability (LHDI) [Krall and Liewer, 1971; Davidson et al., 1977; Huba et al., 1977, 1980]. However, the classical LHDI is most unstable at the edges of the current sheet and it does not affect significantly the central region. Later, Lui et al. [1991, 1995] and Yoon and Lui [1996] found another class of instabilities, cross-field current instabilities (CFCI), driven by different dynamics of unmagnetized ions and magnetized electrons at the center of the current sheet. At the same time, simulations [Zhu et al., 1992; Ozaki et al., 1996; Pritchett and Coroniti, 1996; Zhu and Wingler, 1996] revealed the drift-kink instability (DKI), which strongly distorted the main equilibrium current like the CFCI modes. Lapenta and Brackbill [1997], Yoon et al. [1998], and Buchner and Kasu [1999] also found the drift-sausage instability (DSI), which had the opposite parity compared to the DKI (even profile of the dawn-dusk component of the electromagnetic potential in the case of the Coulomb gauge). However, Daughton [1998, 1999] performed the nonlocal kinetic linear stability analysis of DKI, DSI, and LHDI modes and reported no evidence of the DSI. More importantly, this analysis and later particle simulations [Hesse and Birn, 2000; Pritchett and Coroniti, 2001] revealed a strong decrease of the DKI growth rate with the increase of the mass ratio $m_e/m_i$, making that most promising mode irrelevant for the case with realistic $m_e/m_i = 1836$. We should note that recently Yoon et al. [2002] performed a nonlocal stability analysis of the Harris current sheet for current-driven instability based on the two-fluid formalism and found an entire class of unstable eigenmodes of either parity. The ground-state solutions of this class resemble the DKI and DSI modes, while the higher-order solutions increasingly behave as the LHDI as the order increases. The latter finding has been partly confirmed by the kinetic stability analysis of Daughton [2003]. He showed in particular that in a very thin Harris TCS with the thickness $L < \rho_{0e}$ ($\rho_{0e}$ is the thermal ion gyroradius in the field $B_0$ outside the sheet), the modes of both parities similar to the classical LHDI but having larger wavelength ($k_r \sqrt{\rho_{0e} \rho_{0i}} \sim 1$) may still have the significant growth rates and larger electromagnetic components, which are localized in the central region of the sheet.

[6] Meanwhile, Yoon and Lui [1996] and Yoon et al. [1996] have shown that the current-driven instabilities may survive in the case of the realistic mass ratio due to the bulk flow velocity shear in the initial TCS equilibrium. Furthermore, recent simulations [Hesse et al., 1998; Horiechi and Sato, 1999; Shinohara et al., 2001; Lapenta and Brackbill, 2002; Daughton, 2002] revealed the formation of the TCS profiles with the bulk flow velocity shear as a nonlinear effect of the LHDI with further excitation of the DKI and Kelvin-Helmholtz instabilities (KHI) as a consequence of that shear flow effect [Yoon et al., 1996]. The problem however is that the LHDI should be strongly suppressed by the normal magnetic field $B_n$. According to Pritchett and Coroniti [2001], in the case of the LHDI the $B_n$ field results in an effective component of the wave vector parallel to the local magnetic field, which strongly stabilizes the LHDI [Gladd, 1976]. Indeed, Pritchett and Coroniti [2001] and Pritchett [2002] found no signatures of the LHDI even for moderate mass ratio in the presence of a very weak normal component of the equilibrium magnetic field. As a result, both linear and nonlinear stages of the TCS evolution reveal no signatures of DKI, DSI, or KHI. Therefore it still remains unclear whether the large-scale instabilities such as the DKI or KHI can actually grow in the Harris TCS with $B_n \neq 0$ for realistic mass and temperature ratios. Note here that the effective shear of the bulk flow speed can be provided in the current sheet by the background plasma, and it may be a cause of the high DKI growth rate in the case of realistic mass ratio [Daughton, 1999; Karimabadi et al., 2003a,
However, the models with a uniform background plasma are known to have an additional free energy source for the ion-ion instability because the ion distribution is two-humped in the velocity space in the direction of the current. As a result, the assumption of a uniform background density is sometimes considered as artificial, the one to be eliminated [e.g., Shinohara et al., 2001].

7] Thus the absence of robustly growing modes in thin Harris sheet equilibria is in contradiction to the growing observational evidence of wave activity during substorms, earthward of the forming X-line. In addition to the original AMPTE/CEC observations of the current disruption perturbations at radial distance $r \sim 8R_E$ [Takahashi et al., 1987; Lui et al., 1988, 1992; Lopez et al., 1989; Ohtani et al., 1995, 1998], Bauer et al. [1995] statistically investigated low-frequency waves in the region $-19R_E < X_{GSM} < -9R_E$ using AMPTE/IRM data. They found in particular that the magnetic field fluctuations increase with increasing flow velocity in the plasma sheet. With the use of recent Geotail observations Fairfield et al. [1998, 1999], Sigsbee et al. [2002] and Shiokawa et al. [2002] reported strong wave activity between $-10$ and $-15R_E$ during substorms. Also, recent Cluster measurements [Sepevec et al., 2003; Volwerk et al., 2003] revealed modes corresponding to the DKI and possibly even the DSI [Runov et al., 2003] at $-19R_E$ during substorms. For the theoretical interpretation of these observations it is necessary to explore a mechanism of the excitation of the instabilities similar to the DKI and KHI, which do not involve the preliminary excitation of the LHDI.

8] In this paper we show that the modes similar to the DKI and KHI, which have larger wavelength and electromagnetic component than the lower-hybrid modes and significantly perturb the central current sheet region, may have large enough growth rates even for the realistic mass ratio in a non-Harris thin current sheet embedded in a thicker anisotropic plasma sheet [Sitnov et al., 2000a, 2000b], also known as the forced current sheet (hereinafter referred to as FCS) [Burkhart et al., 1992]. The FCS has bulk flow velocity shear, which is a natural consequence of the quasi-adiabatic ion motion and plasma anisotropy outside the sheet. Moreover, this shear is not related to any additional background plasma, and therefore the system is free from the ion-ion instabilities. According to the current disruption model of substorms [Lui, 1996], the current disruption starts when the near-Earth current sheet thins enough to make ions un magnetized with respect to the field $B_n$ and accelerated by the dawn-dusk convection electric field which penetrates the sheet. This is exactly how the FCS is formed [e.g., Hill, 1975; Lottermoser et al., 1998; Nakamura et al., 1998]. The FCS represents therefore the natural model for a considerable region of the tail current sheet during active periods. It is shown to be unstable with respect to current-driven instabilities with the frequency around the ion gyrofrequency outside the sheet for a wide range of wave numbers.

9] The FCS equilibrium is fully self-consistent for the case of zero normal magnetic field just like the popular Harris equilibrium [Harris, 1962]. As shown by Sitnov et al. [2003a], these two types of current sheet equilibria are different limiting cases of a more general class of one-dimensional (1-D) equilibrium models. In contrast to the Harris model, the FCS equilibrium becomes possible due to the plasma anisotropy and characteristics of the ion orbits in thin current sheets that cannot be described in terms of the conventional magnetic moment (although the orbits remain adiabatic and fully integrable). This is why the stability analysis of the FCS equilibrium for the case $B_n = 0$ is as valid and self-consistent as a similar analysis of the Harris sheet [e.g., Daughton, 1998, 1999, 2003].

10] Like practically all other linear kinetic stability analyses of the current-driven instabilities [Lapenta and Brackbill, 1997; Daughton, 1998, 1999, 2003; Silin et al., 2002], the present study is limited to the case of zero normal magnetic field $B_n = 0$. It is believed nevertheless to clarify the important aspects of the stability picture in the case of a small nonzero $B_n$, for the following reasons. First, like the Harris equilibrium, which locally keeps its original form in the presence of a small finite normal component of the magnetic field $B_n$ [e.g., Schindler, 1972], the FCS equilibrium is not changed significantly in the latter case. As argued below, in the case of a small finite $B_n$ the FCS model describes the sheets where the magnetic tension is balanced by the ion inertia of the quasi-adiabatic ions. However, this condition for small $B_n$ can be reduced to the form, which is independent of $B_n$. Similarly, the main new ingredient of the FCS model, namely the invariant of the quasi-adiabatic motion [e.g., Sonnerup, 1971] remains approximately constant on the time scales considered.

11] The question, whether one can neglect the influence of the $B_n$ component in the stability analysis of the current-driven instabilities, is still an open question. The 3-D particle simulations [Pritchett and Coroniti, 2001] suggest that the influence of the $B_n$ field is drastically different for the LHDI-like and DKI-like modes. While the lower-hybrid drift modes are stabilized due to the aforementioned Gladd [1976] mechanism, this is not the case for the DKI modes, as they are much less structured along the normal to the sheet plane. We show below that though the unstable FCS modes share some properties with the LHDI, they are much less structured and thus may avoid the stabilization.

12] The structure of the paper is as follows. In section 2 we describe the analog of the Harris [1962] self-consistent model for FCS class of TCSs as well as some reduced descriptions of the model useful for the subsequent numerical stability analysis. The basic system of the linearized Vlasov-Maxwell equations for the considered class of current-driven instabilities is derived in section 3. The solution of the linearized Vlasov equation resulting in the ideal-differential system of equations for perturbed electrostatic and electromagnetic potentials is given in section 4. The results of the numerical analysis of this system using the finite element approach are presented in section 5. These results are discussed and summarized in section 6.

2. Self-Consistent Model of the Forced Current Sheet

13] For many years the Harris model of current sheets [Harris, 1962] and its modification for the case of a nonzero normal component $B_n$ of the equilibrium magnetic field [Schindler, 1972] represented the only class of the self-consistent models used in applications to the magnetospheric current sheets. In Harris-type models with finite $B_n$ the
plasma is isotropic and the magnetic field line tension is balanced by the pressure gradient along the tail axis. There is, however, another way to balance the magnetic field line tension, namely, by the inertia of the counter-streaming ion flows penetrating the current sheet [Hill, 1975] and providing the current due to their quasi-adiabatic motion [Schindler, 1965; Sonnerup, 1971]. The corresponding force balance condition [e.g., Burkhart et al., 1992] can be obtained by integrating the force balance equation over the current sheet thickness

$$B_0 B_n / 4\pi = m_i \int v_x v_y f(v) dv,$$

where $B_0$ is the x-component of the magnetic field outside the sheet in the GSM coordinate system. Integration over the velocity space is made outside the sheet, and for simplicity we neglect the effects of the stochastic scattering investigated by Burkhart et al. [1992]. In spite of the fact that the finite value of the normal magnetic field $B_n$ is crucial for the force balance (equation (1)), as this field provides both the magnetic tension and the main dynamical features of the Speiser ions balancing that tension, the parameter $B_n$ formally disappears from the subsequent approximate equilibrium theory. In particular, assuming the ion gyrotropy outside the sheet, one can convert integration in equation (1) to the pitch angle and gyrophase coordinates $(v_x, v_y, v_z) \rightarrow (\phi, \psi, v_i)$ with $f(v_x, v_y, v_z) \rightarrow (2\pi)^{-1} f(v_x, v_y, \psi) d^2 v = v_x d v_x d v_y d \phi, v_i = v_y, \cos 0_\phi - v_z \cos \phi \sin 0_\phi, v_z = v_y \sin 0_\phi + v_x \cos \phi \cos 0_\phi$, and tan $0_\phi = B_n/B_0$, and after integrating over the gyrophase $\phi$, equation (1) can be written as

$$B_0 B_n / 4\pi = m_i \sin 0_\phi \cos 0_\phi \int_0^{\infty} v_i d v_i \int_0^{2\pi} f(v_i, \psi) dv_i.$$

In the limit $B_n/B_0 \ll 1$ it takes the form

$$B_0^2 / 4\pi = 2m_i \int_0^{\infty} v_i d v_i \int_0^{2\pi} f(v_i, \psi) dv_i,$$

where $B_0$ is absent. It is also known as the so-called marginal firehose condition [Rich et al., 1972; Hill, 1975]. In the case of the shifted Maxwellian distribution $f \sim \exp \left[-v_y^2//2v_r^2 - (v_\parallel - v_D)^2/2v_D^2\right]$, where $v_r$ and $v_D$ are the thermal velocity of ions and the bulk speed of their counter-streaming flows, respectively, this balance equation takes the form [Burkhart et al., 1992]

$$v_y / v_D = \sqrt{1 + \exp \left(-\delta^2 / \delta^2 \right) / [1 + \text{erf}(\delta)]},$$

where $\delta = v_D/v_r$ is the measure of the ion anisotropy outside the sheet and erf is the “error function” [Abramowitz and Stegun, 1972]. In the limit of strong anisotropy $\delta \gg 1$, which can also be considered as the limit of cold ions, equation (4) further transforms into the relation $v_y = v_D$, known as the Wahlen relation [Wahlen, 1944].

[14] The self-consistent kinetic theory of the FCS equilibria [Sitnov et al., 2000a, 2000b] has been built on the earlier contributions in this direction [Eastwood, 1972, 1974; Rich et al., 1972; Hill, 1975; Francfort and Pellat, 1976; Chen et al., 1990; Pritchett and Coroniti, 1992; Burkhart et al., 1992; Holland and Chen, 1993; Ashour-Abdalla et al., 1994; Kropotkin et al., 1997]. It is based on a new set of integrals of motion, namely, the total particle energy $W = m_v v^2 / 2 + \phi$, which is also used in the Harris-type models, and the sheet invariant $I_z$ of the quasi-adiabatic motion [Schindler, 1965; Sonnerup, 1971; Whipple et al., 1990]

$$I_z = \frac{1}{2\pi} \int m v_y dz,$$

which replaces the canonical momentum $p_y = m v_y + (q/c) a_y$ used in the Harris model. Then the ion distribution, representing two counterstreaming Maxwellian beams outside the sheet, can be described in the form

$$f_0(I < w^2 + \varphi) = \frac{n_0}{\pi^{3/2} v_T^{3/2}} \exp \left\{ - \left[ \left( \sqrt{\varphi + w - \delta^2/3} \right)^2 + I^2 \right] \delta^2/3 \right\} / [1 + \text{erf}(\delta)],$$

where $w = \delta^2/3 v_T v_D, \varphi = 2\delta^{4/3} \phi_0 m v_T^2, I = \delta^{-2/3} \omega_B m v_T^2$. It must be complemented by the distribution of ions that are trapped inside the sheet. Its specific form

$$f_0(I > w^2 + \varphi) = \frac{n_0}{\pi^{3/2} v_T^{3/2}} \exp \left\{ - \left[ \delta^{4/3} + w^2 + \varphi \right] \delta^{4/3} \right\} / [1 + \text{erf}(\delta)],$$

satisfies in particular the condition of continuous transition from trapped to transient distribution, which is necessary for self-consistency [Sitnov et al., 2000a]. The distinctive features of the ion distribution (equations (6) and (7)) can be grasped from Figure 1. It shows in particular how the distribution of two field-aligned counterstreaming ion beams outside the sheet transforms into a bean-shaped distribution inside the sheet, which is asymmetric along the $y$ direction. It is this asymmetry in phase space that becomes the mechanism of the current generation in forced current sheets.

[15] The electron distribution function can be taken as a Maxwellian distribution over the particle energy with zero bulk flow speed [Sitnov et al., 2000b]

$$f_{0e} = \frac{n_0}{\pi^{1/2} v_T} \exp \left\{ - \frac{v_e^2}{v_T} + \frac{e \varphi_0}{T_e} \right\}.$$
anisotropy outside the sheet the magnetic field profile can be presented in the form

\[ B(z) = B_0 b(z/L), \]  

where in the case of weak anisotropy the thickness \( L \) is of the order of the thermal ion gyroradius in the field outside the sheet \( L \sim \rho_i \), while in the case of strong anisotropy \( L \sim \delta^{-1/3} \rho_i \), and the specific profiles \( b(\zeta) \) are universal in that same sense as the profile \( \tanh(z/L) \) in the case of the Harris [1962] equilibrium (for details see [Sitnov et al., 2000a, 2000b]).

[16] The use of the FCS equilibrium model in the stability analysis requires an additional interpolation procedure to speed up the process of the orbit integration. This is done in Appendix A using the scaling relation (equation (9)) and the similar relation for the electrostatic potential found in the work of Sitnov et al. [2000b] for the region of strong anisotropy, which is considered in detail in the following stability analysis. In particular, the magnetic field is approximated by the formula

\[ b(\zeta) = \tanh^\nu \left( \mu \zeta^{1/\mu} \right), \]  

while for the electrostatic potential we use the approximation

\[ \phi_0(\zeta) = T_e e^{-\sigma \phi_0} \exp \left[ -\phi_1 \left( \zeta \sigma^{-1/2} \right)^{\nu_0} \right], \]  

where \( \mu, \nu, \) and \( \phi_1 \) are constants and \( \sigma = 1 - (4/3 \, \phi_0) \log(10/\delta) \). The results of the interpolation (equations (10) and (11)) are given in Figure 3 for different values of the parameter \( \delta \).

[17] The disappearance of the normal component of the magnetic field from the force balance condition (equation (2)) suggests that the FCS equilibrium will exist in the case of zero normal magnetic field just like the popular Harris equilibrium. This suggestion was recently confirmed by the theory of 1-D current sheets [Sitnov et al., 2003a], in which these two types of current sheet equilibria appear as different limiting cases of a more general class of 1-D equilibrium models. In contrast to the Harris model, the FCS equilibrium becomes possible due to the plasma anisotropy outside the sheet and characteristics of the ion

Figure 1. Ion distributions given by equations (6) and (7) outside the sheet (left panel) and at its center (right panel) for \( \delta = 3 \). The distributions are color coded on the plane \( \left( v_x/v_D, v_y/v_D \right) \) and normalized by their maximum values.

Figure 2. Universal magnetic field profile \( b(\zeta) \) with \( \zeta = z/L \) (equation (9)) for the case of strong anisotropy \( \delta \gg 1 \) (solid line) and its interpolation (equation (10)) with the parameters \( \mu = 0.517 \) and \( \nu = 0.897 \) (dashed line). The inset shows the difference between the profiles.
noted that the magnitude of nonadiabatic changes of $I_z$ nonzero $B_n$ during the transitions between crossing and noncrossing transitions from one type of the fast motion, namely the orbits decreases with the decrease of the parameter $\delta$. This is why the stability analysis of the FCS equilibrium for the latter case is as valid and self-consistent as a similar analysis of the Harris sheet. The FCS equilibrium is an exact equilibrium in the case of zero normal magnetic field in particular because the invariant (equation (5)) is exact in the latter case (for details see, for instance, Kropotkin et al., [1997] and references therein). In the case of nonzero $B_n$ the invariant is only approximate. It is conserved nevertheless much better than many other parameters, including the conventional magnetic moment [e.g., Whipple et al., 1990]. Nonadiabatic changes of $I_z$ occur during the transition from one type of the fast motion, namely the conventional Larmor rotation with the particle orbit not crossing the $z = 0$ plane, to a figure-eight orbit crossing the $z = 0$ plane. Changes of $I_z$ during such transitions can be described as a weak diffusion and they were studied in many papers [Chen and Palmadesso, 1986; Büchner and Zelenyi, 1989; Chen, 1992; Kropotkin et al., 1997]. Here it should be noted that the magnitude of nonadiabatic changes of $I_z$ during the transitions between crossing and noncrossing orbits decreases with the decrease of the parameter $B_n/B_0$. Also the characteristic period between these nonadiabatic jumps is of the order of the ion gyroperiod $\Omega_i$ in the field $B_n$. This is why the above changes of $I_z$ can be neglected in the stability analysis as long as the characteristic frequencies and growth rates are larger than $\Omega_i$.

3. Basic Equations for the Nonlocal Kinetic Stability Analysis

The analysis of small perturbations in the FCS is based on the linearized Vlasov equation, which can be written in the form

$$\tilde{L}_{(0)} = \left( \frac{\partial}{\partial t} + v \nabla + \frac{q_0}{m_0} \left( E_0 + \frac{1}{c} (v \times B_0) \right) \nabla \right) \psi_{f_0},$$

where

$$\tilde{L}_{(0)} = \left( \frac{\partial}{\partial t} + v \nabla + \frac{q_0}{m_0} \left( E_0 + \frac{1}{c} (v \times B_0) \right) \nabla \right) \psi_{f_0},$$

(13)

and an arbitrary (tearing, kink, etc.) perturbation is described by the potentials $A_1$ and $\phi_1$ with $B_1 = \nabla \times A_1$ and $E_1 = -\nabla \phi_1 - (1/c) (\partial/\partial t) A_1$.

We simplify the subsequent stability analysis by neglecting the normal component of the magnetic field $B_n$. Then the component of the particle velocity $v_z$ becomes an integral of motion. We also consider the particular case of the current-driven instabilities with $A_1 = A(z) \exp (\gamma t + iky)$, and then the stability problem can be reduced to that for the two-component vector-potential $A_1 = (0, A_x, A_y)$, in which the components are additionally connected by the Coulomb gauge

$$\partial A_{1z} + i k A_{1y} = 0.$$

(14)

Using this simplification, we exclude one of the Maxwell equations for the potential $A_{1y}$, and write other equations in the form

$$\nabla^2 A_{1z} = -\frac{4\pi}{c} \sum_\alpha q_0 \int v_z g_{1\alpha} d^3v,$$

(15)

$$\nabla^2 \phi_1 = \phi_1 \chi(z) - 4\pi \sum_\alpha q_0 \int g_{1\alpha} d^3v,$$

(16)

where $g_{1\alpha} = f_{1\alpha} - \phi_1 \langle \partial_\alpha \phi_1 / \partial u \rangle$ and

$$\chi(z) = -4\pi \sum_\alpha q_0^2 m_0 \int \left( \frac{\partial f_{0\alpha}}{\partial u} \right) d^3v.$$

(17)

is the fixed function of the TCS profile. Taking into account the specific set of variables of the FCS distribution function $f_{0\alpha} = f_{0\alpha}(u + (e/m_c) \phi, L)$, where $u = v_Z^2/2$, and the specific form of the perturbed vector potential, the linearized Vlasov equation (13) for the perturbed distribution function $g_{1\alpha}$ can be rewritten as

$$\tilde{L}_{(0)} g_{1\alpha} = \frac{q_0}{m_0} \frac{\partial f_{0\alpha}}{\partial u} \left\{ \left( \frac{1}{c} \left[ v_z A_{1z} + \frac{v_z}{c} A_{1Z} - \phi_1 \right] + \frac{q_0}{m_0} \frac{\partial f_{0\alpha}}{\partial L_x} \left[ ik \phi_1 + \frac{\gamma}{c} A_{1y} + \frac{v_z}{c} \partial_z A_{1y} - \frac{v_z}{c} i k A_{1z} \right] + \frac{q_0}{m_0} \frac{\partial f_{0\alpha}}{\partial L_y} \left[ \partial_z \phi_1 + \frac{\gamma}{c} A_{1z} + \frac{v_x}{c} i k A_{1z} - \frac{v_x}{c} \partial_x A_{1z} \right] \right) \right\} + v_x \frac{\partial A_{1z}}{\partial u} \chi_0$$

(18)

4. Orbit Integration

The solution of equation (18) can be presented in the form of the integrals over the unperturbed orbits

$$g_{1\alpha} = \frac{q_0}{m_0} \frac{\partial f_{0\alpha}}{\partial u} \left\{ \left[ \frac{1}{c} \left( v_z A_{1z} + \frac{v_z}{c} A_{1Z} - \phi_1 \right) \right] + \frac{q_0}{m_0} \frac{\partial f_{0\alpha}}{\partial L_x} \left[ ik \phi_1 + \frac{\gamma}{c} A_{1y} + \frac{v_z}{c} \partial_z A_{1y} - \frac{v_z}{c} i k A_{1z} \right] + \frac{q_0}{m_0} \frac{\partial f_{0\alpha}}{\partial L_y} \left[ \partial_z \phi_1 + \frac{\gamma}{c} A_{1z} + \frac{v_x}{c} i k A_{1z} - \frac{v_x}{c} \partial_x A_{1z} \right] \right\} + v_x \frac{\partial A_{1z}}{\partial u} \chi_0$$

(19)
where \( \phi_i = \phi(z) \exp (\gamma t + iky) \), \( A_{1y,z} = A_{2y,z}(z) \exp (\gamma t + iky) \), 
\( y' (\tau = 0) = y, y' (\tau = 0) = z, v_y (\tau = 0) = v_y, \) and \( v_y' (\tau = 0) = v_z. \) As a result, the equations (15)–(16) take the form

\[
\nabla^2 A_z + \frac{4\pi}{c} \sum_{m} n_m \left[ \nabla \left( \gamma \frac{\partial \phi_0}{\partial \rho} \frac{\phi_0}{\partial x} + \frac{\partial \phi_0}{\partial \rho} \frac{\phi_0}{\partial y} \right) \right] = 0 \tag{20}
\]

\[
\nabla^2 \phi - \phi \chi(z) + 4\pi \sum_{m} n_m \left[ \nabla \left( \gamma \frac{\partial \phi_0}{\partial \rho} \frac{\phi_0}{\partial x} + \frac{\partial \phi_0}{\partial \rho} \frac{\phi_0}{\partial y} \right) \right] = 0, \tag{21}
\]

where

\[
\begin{align*}
\bar{S}_1 &= \int_{-\infty}^{0} \left\{ \frac{1}{c} \left[ v_{y} A_y (z') + v_{y} A_y (z') \right] - \phi (z') \right\} \\
&\quad \times \exp[\gamma t + ik(y' - y)]d\tau
\end{align*}
\]

\[
\bar{S}_2 = \int_{-\infty}^{0} \left[ \frac{1}{c} \left[ v_{y} A_y (z') + v_{y} A_y (z') \right] + \frac{\gamma}{c} A_z (z') \right] \\
&\times \exp[\gamma t + ik(y' - y)]d\tau + \int_{-\infty}^{0} \left[ \frac{\gamma}{c} A_z (z') \right] d\tau
\]

\[
\bar{S}_3 = \int_{-\infty}^{0} \left[ \frac{1}{c} \left[ v_{y} A_y (z') + v_{y} A_y (z') \right] - \phi (z') \right] \\
&\times \exp[\gamma t + ik(y' - y)]d\tau
\]

\[
\bar{S}_4 = \int_{-\infty}^{0} \left[ \frac{1}{c} \left[ v_{y} A_y (z') + v_{y} A_y (z') \right] + \frac{\gamma}{c} A_z (z') \right] \\
&\times \exp[\gamma t + ik(y' - y)]d\tau + \int_{-\infty}^{0} \left[ \frac{\gamma}{c} A_z (z') \right] d\tau
\]

where the displacement in the y-direction has the form 
\( y' (\tau) = n \Delta \), \( y' (\tau) = y' (\tau) = 0 \), \( \Delta_t \) is the net drift in the y-direction during the gyropulse period \( \tau_b \), and \( y' (\tau) \) is the periodic function.

[22] It is convenient for the purpose of the subsequent numerical solution to present the basic set of equations (14),
(20), (21), (24), and (25) in the dimensionless block form

\[
L_{11} (A_z) + L_{12} (A_z) + L_{13} (\bar{S}) = 0 \tag{26}
\]

\[
L_{21} (A_z) + L_{22} (A_z) + L_{23} (\bar{S}) = 0 \tag{27}
\]

\[
L_{31} (A_z) + L_{32} (A_z) + L_{33} (\bar{S}) = 0 \tag{28}
\]

where the electrostatic potential is renormalized as \( \phi = \hat{\phi} \psi_e/c. \) The dimensionless parameters used to describe the blocks \( L_{ij} \) include \( \tilde{y} = \gamma / \omega_b, \eta = k \zeta, \tilde{y} = \psi / \omega_b, \tilde{z} = z / \rho_b, \tilde{v}_{y,z} = y' / \gamma, \) and \( \tau = \tau / \omega_b, \) where \( \omega_b = e B_0 m_e c. \) We also used the quasi-neutrality approximation \( \nabla^2 \phi = 0 \) to simplify the Poisson’s equation (21). Then the blocks can be written in the form

\[
L_{11} = i A_z \tag{29}
\]

\[
L_{12} = \partial A_z / \partial 
\]

\[
L_{13} = 0 \tag{31}
\]

\[
L_{21} = -2 \tilde{\gamma} \frac{\omega_p}{k^2 c^2} \exp \left( \frac{\psi_0}{T_e} \right) \int \tilde{v}_{y} \tilde{f}\phi \tilde{S}_y (z, \tilde{v}, \left\{ \tilde{v}_y A_y (z') \right\}) d\tilde{v}_y d\tilde{v}_z
\]

\[
+ 2 \tilde{\gamma} \frac{\omega_p}{k^2 c^2} \int \tilde{v}_z \tilde{f}_\phi \tilde{S}_z (z, \tilde{v}, \left\{ \tilde{v}_y A_y (z') \right\}) d\tilde{v}_y d\tilde{v}_z
\]

\[
+ \frac{\omega_p}{k^2 c^2} \int \tilde{v}_z \tilde{f}_\phi \tilde{S}_z (z, \tilde{v}, \left\{ \tilde{v}_y A_y (z') \right\}) d\tilde{v}_y d\tilde{v}_z
\]

\[
L_{22} = \frac{dA_z}{dT_e} - A_z - 2 \tilde{\gamma} \frac{\omega_p}{k^2 c^2} \exp \left( \frac{\psi_0}{T_e} \right) \int \tilde{v}_z \tilde{f}_\phi \tilde{S}_z (z, \tilde{v}, \left\{ \tilde{v}_y A_y (z') \right\}) d\tilde{v}_y d\tilde{v}_z
\]

\[
L_{23} = 2 \tilde{\gamma} \frac{\omega_p}{k^2 c^2} \exp \left( \frac{\psi_0}{T_e} \right) \int \tilde{v}_z \tilde{f}_\phi \tilde{S}_z (z, \tilde{v}, \left\{ \tilde{v}_y A_y (z') \right\}) d\tilde{v}_y d\tilde{v}_z
\]

\[
L_{31} = - \tilde{\gamma} \frac{\psi_0}{T_e} \exp \left( \frac{\psi_0}{T_e} \right) \int \tilde{f}_\phi \tilde{S}_z (z, \tilde{v}, \left\{ \tilde{v}_y A_y (z') \right\}) d\tilde{v}_y d\tilde{v}_z
\]

\[
+ \tilde{\gamma} \int \tilde{f}_\phi \tilde{S}_z (z, \tilde{v}, \left\{ \tilde{v}_y A_y (z') \right\}) d\tilde{v}_y d\tilde{v}_z
\]

\[
+ \frac{1}{2} \int \tilde{f}_\phi \tilde{S}_z (z, \tilde{v}, \left\{ \tilde{v}_y A_y (z') \right\}) d\tilde{v}_y d\tilde{v}_z
\]

\[
- \frac{\partial}{\partial \tilde{v}_z} \tilde{f}\phi (A_y (z')) d\tilde{v}_y d\tilde{v}_z
\]

\[
L_{32} = \frac{dA_z}{dT_e} - A_z - 2 \tilde{\gamma} \frac{\omega_p}{k^2 c^2} \exp \left( \frac{\psi_0}{T_e} \right) \int \tilde{v}_z \tilde{f}_\phi \tilde{S}_z (z, \tilde{v}, \left\{ \tilde{v}_y A_y (z') \right\}) d\tilde{v}_y d\tilde{v}_z
\]

\[
+ \frac{\omega_p}{k^2 c^2} \int \tilde{v}_z \tilde{f}_\phi \tilde{S}_z (z, \tilde{v}, \left\{ \tilde{v}_y A_y (z') \right\}) d\tilde{v}_y d\tilde{v}_z
\]

\[
L_{33} = 0 \tag{35}
\]
Given the turning points, one can find the corresponding potentials for Larmor and figure-eight orbits (note that the presence of the integral $S_b$ from the Harris CS case by the different form of the magnetic field profile $b(\zeta)$ equation (10) and the presence of the electrostatic potential $\phi_0(\zeta)$ equation (11)). The calculation of the integral $S_\alpha$, starts from finding the turning points $z_{0,1}$ for Larmor and figure-eight orbits (note that the presence of the potential $\phi_0$ does not affect the orbit classification). Given the turning points, one can find the corresponding period $p_\alpha$. Then $S_\alpha$ can be calculated with the orbit parameters $z', v'$ and $v'_y$ being updated at each time step using the standard Runge-Kutta procedure [e.g., Press et al., 1999].

5. Finite Element Analysis

To solve the set of equations (26)–(38), we use the finite element method [Chen and Lee, 1985; Burkhart and Brittnacher et al., 1995, 1998; Daughton, 1998, 1999, 2003; Sitnov et al., 2002]. In this method the expansion of potentials $A_{\alpha\beta}$ and $\phi$ in a series of basis functions $\Psi_{\alpha}$, $\alpha = 1, \ldots, N$, transforms the nonlocal eigenvalue problem (equations (26)–(28)) into an algebraic one

$$M_0 C_0 = 0,$$

where the elements of the matrix $M_0$ are the following inner products:

$$\langle \Psi_{\alpha} L_{\alpha\beta}(\Psi_{\beta}) \rangle, \alpha, \beta = 1, 2, 3$$

and the potentials entering the blocks $L_{\alpha\beta}$ are expanded as follows:

$$A_\beta(z) = \sum_{n=1}^{N} C_n \Psi_n(z), \qquad A_\alpha(z) = \sum_{n=1}^{N} C_{N+n} \Psi_n(z)$$

The specific set of basis functions used in our analysis are Hermite functions [e.g., Daughton, 1999]

$$\Psi_n(z) = (\sqrt{\pi} n! 2^n)^{-1/2} H_n(\zeta) e^{-z^2/2},$$

with the inner product $(f) = \int_{-\infty}^{\infty} f(z) \, dz$.

5.1. Benchmark Case: Harris Equilibrium

The finite element analysis in itself is a complex numerical procedure. This is why we first performed a set of benchmark runs to check the performance of this procedure for the known case of the Harris equilibrium described by Daughton [1998, 1999, 2003]. Figures 4 and 5 show the results of these test runs for the case $m_i/m_e = 16$, $T_i = T_e$, and $L = \rho_0$. 

![Figure 4](image-url)
In particular, Figure 4 resembles the corresponding plots for the frequency and growth rate obtained by Daughton with the same set of parameters but $r_0/\lambda = 0.7$ [Daughton, 1998, Figure 8; Daughton, 1999, Figure 10]. The eigenfunctions at $k\lambda = 0.7$ (Figure 5), corresponding to the drift kink mode, are also consistent with Daughton [1998, Figure 6] and Daughton [1999, Figure 9], although they reveal more diversity. This can be explained by the fact that the eigenfunctions can be determined to within an arbitrary complex phase. We additionally checked the convergence of eigenfunctions by plotting in the upper panel the component $A_y$ found from the gauge condition (equation (14)) $i k A_y = - \partial_z A_{1z}$. The number of basis functions used in this benchmark runs was $N = 6$. Another solution corresponding to the region of the relatively long wavelength LHDI, which was recently considered by Daughton [2003], is obtained using a larger number of basis functions $N = 20$ and is shown in Figure 6. As one can see from this figure, the solution is basically consistent with the results shown in Daughton [2003, Figure 5] except for a bit smaller growth rate. Note also that in contrast to the case of the DKI (Figure 5) and similar to the classical LHDI case, the electromagnetic component of the solution is rather small compared to its electrostatic part. This is why even though the electromagnetic component $A_z$ is peaked at the center of the sheet, it will hardly perturb this region significantly.

5.2. Forced Current Sheet Stability Results

The unstable modes of forced current sheets have been investigated for the parameters $m_i/m_e = 1836$, $\delta = 3$, $L = 0.69\rho_0$, two different temperature ratios $T_i/T_e = 1$, and $T_i/T_e = 4$ with the use of $N = 6$ basis functions. The basic set of linear equations (26)–(38) can be further simplified in this region with the details given in Appendix B. Figure 7 shows that even for realistic mass ratios the growth rate of the unstable modes is quite significant, and according to Figure 10, it depends weakly on the temperature ratio. We have found unstable modes of both parities (Figures 8, 9, 10, 11, and 12). It is important to note that these modes differ significantly from the well-known DKI solutions found in the cases of small $m_i/m_e$ [Daughton, 1998, 1999] and the hypothetical DSI modes [e.g., Lapenta and Brackbill, 1997]. In fact, they resemble more closely the long wavelength LHDI solutions found recently in the

![Figure 5](image_url)

Figure 5. Electrostatic potential $\phi$ and components $A_x$ and $A_z$ of the vector-potential found for the parameters of Figure 3 with $k\lambda = 0.7$. Dash-dotted and dash-triple dotted lines show the profiles of the real and imaginary part of the potential $A_y$ inferred from the profiles of $A_z$ using the gauge condition. To provide the proper resolution, the latter profiles are additionally shifted along the ordinate axis by the value $\delta A_y = 0.01$.

![Figure 6](image_url)

Figure 6. An example of the LHDI solution for the Harris TCS obtained using $N = 20$ basis functions. The solution with $-\omega/\Omega_{ni} = 4.7$ and $\gamma/\Omega_{ni} = 0.21$ is found for the case $m_i/m_e = 512$, $T_i = T_e$, $L = 0.5\rho_0$, and $k\lambda = 2$.

![Figure 7](image_url)

Figure 7. Growth rate (upper panel) and frequency (lower panel) of the unstable eigenmodes of odd (diamond symbol) and even (star symbol) parity in forced current sheets with $m_i/m_e = 1836$, $T_i = T_e$, $\delta = 3$, and $L = 0.69\rho_0$. 
Vlasov stability analysis of the Harris sheet [Daughton, 2003] and earlier two-fluid analysis [Yoon et al., 2002]. In particular, similar to the LHDI, which is driven by the plasma density gradient at the edges of the sheet, the FCS instability appears to be driven by the bulk flow velocity shear, which is also located off the sheet center. This explains the similar growth rates for even and odd parity modes that are also characteristic of the LHDI (see, for instance, [Daughton, 2003, Figure 5]). Since in both these cases the instabilities develop on either side away from the center of the sheet they are fairly independent of each other. Thus one can expect that the corresponding global perturbations of even and odd parity will be almost equally probable.

[27] The relatively large growth rate of even parity modes does not necessarily mean that these modes will dominate the FCS evolution beyond the linear growth stage. In contrast to the kink perturbations, the amplitude of the even parity modes is limited to the current sheet thickness, which...
is fairly small. Thus one can expect observing the latter modes as transient phenomena before the global current sheet kinking. Note that these transient phenomena have been actually observed in particle simulations of another TCS type, namely the bifurcated sheet, which implies even stronger separation of the instability driving sources off the sheet center [Sitnov et al., 2003b].

The bulk flow velocity shear may be one of the driving forces in another class of current sheet equilibria composed of the Harris sheet and a background plasma. In these equilibria the stability picture is drastically different from the case of the FCS. In particular, only the instability of the odd parity mode similar to the DKI develops, although it does have relatively large growth rate for realistic mass ratio [Daughton, 1999; Karimabadi et al., 2003a, 2003b]. However, the equilibria with a background plasma are drastically different from the FCS equilibria. In particular, the background population of ions results in a two-humped ion distribution in velocity space [Karimabadi et al., 2003b]. Such distributions have an additional free-energy source similar to that in the beam-plasma systems. To avoid the corresponding artificial ion-ion instabilities, some authors [e.g., Shinohara et al., 2001] attempted to reduce the background plasma density inside the current sheet. This results, however, in a non-self-consistent description of the current sheet equilibrium. In contrast, the current sheet equilibrium studied in our paper does not have a two-humped velocity distribution of ions in the direction of the current. As shown in Figure 1 [see also Burkhart et al., 1992, Figure 3h], the distribution inside the sheet has instead a bean-shaped structure in the sheet plane. Therefore it should be stable with respect to ion-ion instabilities inside the sheet, and our present study highlights the effect of the shear flow.

On the other hand, the unstable FCS modes have a number of properties, which distinguish them from those of the lower-hybrid drift modes and may be important for applications. They have in particular much lower frequency, which is of the order of the ion gyrofrequency $\omega_i$, outside the sheet, and much larger electromagnetic component, similar to the DKI. Most noteworthy is the profile of the FCS modes across the sheet, which is much less structured as compared to that of the LHDI (compare, for instance, Figure 6 and 8). As a result, based on the arguments of Gladd [1976] and Pritchett and Coroniti [2001], one cannot expect any significant stabilization of these modes by the normal magnetic field $B_n$, which is characteristic of the magnetotail.

The wavelength $\lambda$ of the most unstable waves with $kL \sim 0.2$ is quite large. However, taking into account that the current sheet itself may be very thin, the value of $\lambda$ is quite consistent, for instance, with recent Cluster observations [Runov et al., 2003; Sergeev et al., 2003]. In particular, with $\rho_0 = 600$ km, $kL \sim 0.2$, and $L = 0.69 \rho_0$, we find $\lambda \sim 2R_E$, which coincides with the estimate of $\lambda$ for the flapping motions of the magnetotail reported by Sergeev et al. [2003].

6. Discussion and Conclusion

In this paper we have reported on the nonlocal kinetic linear stability analysis, which is similar to the analysis made by Daughton [1998, 1999, 2003] for the Harris sheet. The new analysis is done for one of a few known self-consistent current sheet equilibria different from the Harris model, namely the so-called forced current sheet [Sitnov et al., 2000a, 2000b]. In contrast to the Harris model, the FCS equilibrium exists due to the plasma anisotropy outside the sheet and characteristics of the ion orbits in thin current sheets that cannot be described in terms of the conventional magnetic moment. The FCS models describe thin current sheets with the thickness comparable to the thermal ion gyroradius in the field outside the sheet and shear of the bulk flow velocity of the ion species. They can be readily generalized to the case of a small finite magnetic field $B_n$. Like the case of the modified Harris model with $B_n \neq 0$, the more general theory does not contain the parameter $B_n$ explicitly. Moreover, in contrast to the Harris case, the model remains 1-D as the magnetic tension is balanced by the ion inertia rather than the pressure gradient. Such generalized FCS models are often considered as a basis for the models of magnetic merging [Hill, 1975; Francfort and Pellat, 1976; Kropotkin et al., 1997; Sitnov et al., 2002], which may be an alternative to the X-line reconnection as a mechanism of the transformation of the magnetic field energy into particle kinetic energy [Biskamp, 1986, 2000]. In the physics of magnetospheric substorms this alternative mechanism is known as current disruption [Lui, 1996].

We have found that the stability picture in the FCS drastically differs from the case of the Harris sheet. The unstable FCS modes resemble the long wavelength $(k_y \rho_0 \omega_i \sim 1)$ LHDI modes, which were recently studied by Yoon et al. [2002] and Daughton [2003]. Their growth rates are quite significant even for realistic mass ratio $m_i/m_e = 1836$ and are comparable for the modes with odd
and even parity. In terms of the structure of the eigenmodes across the sheet, they differ from the DKI modes and resemble the higher-order LHDI-like analogs of the ground state solutions corresponding to the DKI and hypothetical DSL. On the other hand, the unstable FCS modes further strengthen some features of the long wavelength LHDI modes that distinguish themselves from the classical LHDI. They have in particular a lower frequency and a larger electromagnetic component, similar to the DKI. Finally, they are much less structured across the sheet as compared even to the long wavelength LHDI. This gives us hope that like the cases of the DKI at low mass ratios or with a background plasma, the newly found unstable FCS modes will survive in the presence of the finite normal magnetic field typical for the Earth’s magnetotail and the outflow regions of the reconnections patterns.

[33] Our results reveal the important role of the bulk flow velocity shear in destabilizing thin current sheets. They are consistent with recent PIC simulations of current-driven instabilities in the models of Harris sheets assuming a bulk flow velocity shear due to an additional non-self-consistent background plasma [Shinohara et al., 2001], the models where the bulk flow velocity shear arises as a consequence of the nonlinear saturation of the LHDI [Horiuchi and Sato, 1999; Daughton, 2002; Lapenta and Brackbill, 2002], and the Harris models with a uniform background plasma [Daughton, 1999; Karimabadi et al., 2003a, 2003b]. However, in the latter case the shear flow effects may be strongly masked by the more conventional two-stream instability. In contrast to the Harris sheet, the FCS equilibria appear to assume a wider spectrum of the unstable modes, including modes of even parity. This is consistent with recent observations of electromagnetic fluctuations in laboratory plasmas (Ji et al., submitted manuscript, 2004) and similar fluctuations in the geomagnetotail during current disruptions [Lui and Najmi, 1997; Sigsbee et al., 2002], which clearly reveal a wide variety of excited modes.

Appendix A: Approximation of the Equilibrium Forced Current Sheet Profiles

[34] As shown in Figure 2, the magnetic field profile (equation (9)) in the region of strong anisotropic case can be approximated as

\[ b(\zeta) = \tanh^{\nu} \left( \mu \zeta \right), \]  

(A1)

where \( \zeta = 2\pi^2 \omega_0 / v_D = 2\pi^2 / (\rho_{0w} / \rho_0) \), and the coefficients \( \mu = 0.517 \) and \( \nu = 0.897 \) are found by minimizing the standard deviation between equation (A1) and the exact solution of the Grad-Shafranov equation [Sitnov et al., 2000b]. For computation of the dimensionless sheet invariant one needs also the dimensionless function \( \tilde{b}(\tilde{a}) = b(\zeta(\tilde{a})) \) where

\[ \zeta(\tilde{a}) = \int_0^{\tilde{a}} \frac{b(\zeta)}{\sqrt{1 + b(\zeta)^2}} d\zeta = \frac{2}{3} \frac{\rho_{0w}}{\rho_0} a(\zeta) \]

(A2)

Therefore one can represent \( \tilde{b}(\tilde{a}) \) in the form \( \tilde{b}(\tilde{a}) = \mu \zeta(\tilde{a}) \), where \( \mu = 0.517 / (\rho_{0w} / \rho_0) \). This scaling is similar to equation (A1).

[35] Let us now consider the limit \( \tilde{a} \to 0 \). Then we have \( \zeta(\tilde{a}) = \mu \tilde{a}^2 / 2 \) and \( a(\zeta) = \zeta / \mu \tilde{a}^2 / 2 \), and therefore \( \tilde{b}(\tilde{a}) = \sqrt{2} \mu \tilde{a}^2 \). Thus the function \( \tilde{b}(\tilde{a}) \) should be approximated as

\[ \tilde{b}(\tilde{a}) = \tanh^{\nu} \left( \mu \tilde{a}^{1/2} a(\tilde{a}) \right). \]  

(A3)

Here however the coefficients \( \mu_1 \) and \( \nu_1 \) do not necessarily coincide with the pair \( \mu \) and \( \nu \) used in the approximation (A1) because the only relation, which provides consistency of equations (A1) and (A3) is \( \mu_1 = \mu \tilde{a}^2 \). Indeed, approximating equation (A3) yields \( \mu_1 = 0.449 \) and \( \nu_1 = 0.748 \) with \( \mu_1 = 0.549 \), whereas \( \mu_1 = 0.553 \).

[36] For completeness we need now to find the electrostatic potential. This can be done using the quasi-neutrality relation

\[ n_{0w} = \exp \left( \frac{e\phi_0}{T_e} \right) = \int \tilde{f}_0 d\tilde{v}, \]  

(A4)

which in the limit of strong anisotropy takes the form

\[ \exp \left( \frac{e\phi_0}{T_e} \right) = \frac{8}{2\pi} \int_0^{\pi} d\varphi \int_0^{\pi} \exp(-i(\tilde{v}_z, \varphi)) d\varphi. \]  

(A5)

One of our main tasks here is to find the scaling of \( \phi_0 \) as a function of \( \tilde{b} \). To reveal this scaling we rewrite the invariant \( \tilde{t} \) in the right-hand side of equation (A5) as follows

\[ \tilde{t} = \frac{2}{\pi} \int_{\mu(\tilde{t})}^{\pi} \sqrt{1 + b(a)^2} - [\tilde{t} + b(a)] d\tilde{a}/b(a) \]

\[ \approx \frac{2}{\pi} \sqrt{2} \sin \tilde{t} \int_{\mu(\tilde{t})}^{\pi} \left[ \sqrt{1 + b(a)^2} - b(a)^2 / b(a) \right] d\tilde{a}/b(a), \]

(A6)

where \( w_z = b^{1/2} \). Expanding the expression under the square root we have

\[ \sqrt{1 + b(a)^2} - b(a)^2 / b(a) = w_z^2 / 2b^{1/3} \sin \varphi. \]

(A7)

Now we introduce the new variable in the integral on the right-hand side of equation (A5) \( u'' = w_z^2 / (2b^{1/3} \sin \varphi) + a(\tilde{z}) \) to get

\[ \exp \left( \frac{e\phi_0}{T_e} \right) = \frac{8}{2\pi} \int_0^{2\pi} \sqrt{1 + b(a)^2} - a(\tilde{z}) d\tilde{a} / b(a). \]  

(A8)

\[ \tilde{t} = \frac{2}{\pi} \sqrt{2} \sin \tilde{t} \int_0^{\pi} \sqrt{a'' - a(\tilde{z})} d\tilde{a} / b(a). \]  

(A9)
This shows that the right-hand side of equation (A5) scales as $\delta^{4/3}$. However, this scaling cannot be merely introduced in the approximating formula for electrostatic potential, which has to disappear outside the current sheet $|z| \to \infty$. One can propose nevertheless the approximating formula, which reconciles the above two requirements. It has the form

$$
\phi_0(z) = T \epsilon^{-1} \sigma \psi_0 \exp \left[-\phi_0 \left(z \sigma^{-1/3} \right) \right],
$$

where $\phi_0 = 2.886$, $\sigma = 0.08$, $\psi_2 = 2.2$, and $\sigma = 1 - (4/3) \phi_0$ (log ($b/10$)). This approximation is based on the numerical solution of the Grad-Shafranov equation for thin current sheets [Sitnov et al., 2000b] for $\delta = 10$, and confirmed then for $b = 7$, $5$, and $3$. In particular, $\psi_0 = (e^{\phi_0}/T \epsilon)_{z=0}$, and $(e^{\phi_0}/T \epsilon)_{z=0} = \phi_0 = (4/3) \log (b/10)$.

Appendix B: Basic Set of Equations in the Case of Strong Ion Anisotropy

[37] Here we consider further simplifications of the matrix elements (equations (32)–(37)) in the region $b \gg 1$. In this limit one can neglect first of all the trapped ion population and represent the distribution function in the form

$$
f_0 \approx \frac{1}{2\pi^{3/2}} \exp \left[-\left(\sqrt{v^2 - \delta^2}\right)^2 \right],
$$

while $\partial f_0 / \partial W = (b/\sqrt{W} - 1) f_0$, and $\partial f_0 / \partial \delta = -f_0$. Moreover, in this limit $b \gg 1$ we have $f_0 \approx (1/2\pi) \exp (-\sqrt{W})$ $\delta (\sqrt{W} - \delta)$, and therefore

$$
\int f_0 d^3 v = \int f_0 dv_1 dv_2 d\varphi \approx \frac{b}{2\pi} \int_0^{2\pi} d\varphi \int_0^{2\pi} \exp (-\sqrt{v^2 - \delta^2}) d\varphi,
$$

where $v_x = \delta \cos \varphi$ and $v_y = \delta \sin \varphi$.

[38] Using these simplifications the matrix elements (equations (32)–(37)) can be rewritten in the form

$$
L_{21}(A_{ij}) = - 2b \frac{\omega_{pl}^2}{k^2 e^2} \exp \left(\frac{e^{\phi_0}}{T \epsilon} \right) \int z \overline{v} f_0 S \left(\overline{v}_z, \left\{ \overline{v}_y A_z \left(\overline{v}_z \right) \right\} \right) d\overline{v}_y d\overline{v}_z
- \frac{\delta}{2\pi} \frac{\omega_{pl}^2}{k^2 e^2} \int_{-\infty}^{\infty} \overline{v}_y d\overline{v}_z \int_0^{2\pi} \exp (-\sqrt{v^2 - \delta^2}) d\varphi
\cdot \left( z, \overline{v}_z, \left( \frac{\partial}{\partial \overline{v}_y} \overline{v}_y A_z \left(\overline{v}_z \right) \right) + \overline{v}_y \frac{\partial}{\partial \overline{v}_z} A_z \left(\overline{v}_z \right) \right)
$$

(A3)

$$
L_{22}(A_{ij}) = \frac{d^2 A_{ij}}{dC^2} - A_{ij} - 2b \frac{\omega_{pl}^2}{k^2 e^2} \exp \left(\frac{e^{\phi_0}}{T \epsilon} \right) \int z \overline{v} f_0
\cdot \left( \overline{v}_z, \left\{ \overline{v}_y A_z \left(\overline{v}_z \right) \right\} \right) d\overline{v}_y d\overline{v}_z
- \frac{\delta}{2\pi} \frac{\omega_{pl}^2}{k^2 e^2} \int_{-\infty}^{\infty} \overline{v}_z d\overline{v}_z \int_0^{2\pi} \exp (-\sqrt{v^2 - \delta^2}) d\varphi
\cdot \left( z, \overline{v}_z, \left( - k_{\parallel 0} \frac{\partial}{\partial \overline{v}_y} \overline{v}_y \right) \right) + \left( \frac{\partial}{\partial \overline{v}_y} \left( \overline{v}_y + i k_{\parallel 0} \overline{v}_y \right) \right) A_z \left(\overline{v}_z \right)
$$

(B4)

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