A model of the bifurcated current sheet

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Recent Geotail and Cluster observations revealed that thin current sheets in the near-Earth tail may have a bifurcated structure. In some cases the electrons are found to dominate the current. We present a generalization of the Harris current sheet equilibrium, which reproduces these features. The model ion distribution contains an additional element, viz. the quasi-adiabatic invariant of the ion motion across the sheet, and assumes ion temperature anisotropy outside the sheet. Bifurcated current sheets appear in the case of small ion anisotropy with \( T_{\perp i} > T_{\parallel i} \) (pancake distributions). In the opposite case (cigar distributions) the model describes a single-peaked ion-dominated current sheet embedded in a thicker Harris sheet, which is relevant to the outflow region of the collisionless reconnection pattern in the deHoffman-Teller frame.


1. Introduction

[2] Studies of the geomagnetotail current sheet (CS) using two closely located spacecraft [Fairfield, 1984; McPherron et al., 1987; Sergeev et al., 1993] revealed that its thickness \( L \) may be as small as the thermal ion gyroradius \( r_0 \) in the field outside. In such sheets the ions are nonadiabatic and electron and ion dynamics become decoupled, resulting in many new effects. In particular, the thin current sheet (TCS) may be embedded in a thicker plasma sheet [e.g., McComas et al., 1986]. Also, the current density in the TCS may be dominated by electrons [Mukai et al., 1998].

[3] One of the most recent findings is the explicit demonstration that the CS may have a bifurcated structure with two current density peaks separated by a current depression region at the sheet center [Nakamura et al., 2002; Runov et al., 2003; Sergeev et al., 2003]. These results have been obtained using four spacecraft Cluster observations, which allowed, for the first time, the unambiguous separation of spatial and temporal variability. They confirmed previous single event observations of bifurcated current sheets made using the ISEE 1 and 2 spacecraft [Sergeev et al., 1993] and the single-spacecraft Geotail observations [Hoshino et al., 1996; Asano, 2001]. An important additional feature of the new TCS observations was a very small (less than a few nT) dawn-dusk component of the magnetic field making it hard to explain the bifurcated structure in terms of electron-dominated currents near an X-line [Arzner and Scholer, 2001]. The bifurcated structure was stable in spite of the fast flapping motions of the CS as a whole [Sergeev et al., 2003].

[4] These new features demand a considerable modification and extension of the standard CS models based on the Harris equilibrium [Harris, 1962]. One such modification [Sitnov et al., 2000a, 2000b] has been proposed in the form of a new class of ion-dominated TCS equilibria, where the ion current is formed mainly by the quasi-adiabatic serpentine motion of ions [Speiser, 1965]. A distinctive feature of the model was the use of the new quasi-adiabatic invariant of the ion motion across the sheet [Sonnerup, 1971] in place of the dawn-dusk component of the canonical momentum used in Harris-type models. Another class of TCS models proposed recently by Schindler and Birn [2002] is based on a generalization of the Harris approach, which assumes non-Maxwellian distributions, but retains the original set of integrals of motion.

[5] In this letter we propose a generalization of the Harris approach by combining it with that of Sitnov et al. [2000a, 2000b]. It is shown that the generalized model describes the bifurcated current sheets (hereafter BCS). It also describes both ion- and electron-dominated TCS.

2. Basic Equations

[6] The model is based on a set of Ampère’s and Poisson’s equations, of which the latter is reduced to the quasi-neutrality condition

\[
\frac{dB_i}{dz} = \sum_{\alpha=e,i} \frac{4\pi n_\alpha}{c} \int v_z f_{0\alpha}(z,v) d^3v
\]

\[
\int f_{0\alpha}(z,v) d^3v = \int f_{0\alpha}(z,v) d^3v
\]

The electron and ion distributions have the form

\[
f_{0\alpha} \propto \exp \left( \frac{-2W_{\alpha} - \omega_{\alpha} P_{\alpha}}{2T_{\alpha,0}} + \frac{v_{D\alpha} P_{\alpha}}{T_{\alpha,0}} - \frac{\omega_{\alpha} I_{\alpha}}{2T_{\alpha,0}} \right),
\]

where \( T_{\alpha,0} \), \( T_{\perp i,0} \), \( \omega_{i,0} = eB_0/m_i e \) \( \left( e = e_n \right) \), and \( v_{D\alpha} \) are the parallel and perpendicular temperatures, the cyclotron frequency, and constant speed parameter for the species \( \alpha \), respectively; \( \alpha = e, i \), and \( B_0 \) is the magnetic field outside the sheet. They are constructed from three integrals of motion. The first two, namely the total particle energy \( W_{\alpha} = m_i v^2 + q_i \phi \) (\( \phi \) is the electrostatic potential) and the out-of-plane (dawn-dusk) component of the canonical momentum

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\[ P_{\nu} = m_{\nu}v_{\nu} + (q_{\nu}/c)A_{\nu} \] are those used in the Harris model. We extend this set of invariants by including the quasi-adiabatic invariant of the particle motion across the sheet [Sonnier, 1971]

\[ I_{z}^{(q)} = \frac{1}{2\pi} \int_{z_{0}}^{z} \sqrt{W_{1}^{2} - W_{2}^{2}} \, dz \]

This is an exact invariant of the particle motion in the case of zero normal component of the magnetic field \( B_{n} \). Moreover, even for \( B_{n} \neq 0 \) it remains approximately constant for ions in a TCS with \( L \sim \rho_{0} \) as long as \( B_{n} \ll B_{0} \) (for details see [Sitnov et al., 2000a] and refs. therein). In that case the electron invariant \( I_{z}^{(q)} \) can be modified to form the conventional magnetic moment \( I_{z}^{(q)} \) becomes essential, in particular, for the case of the temperature anisotropy, \( T_{\perp} \neq T_{\parallel} \), outside the sheet.

[7] In the region \( z \geq 0 \) the expression for \( I_{z}^{(q)} \) can be re-written explicitly as

\[ I_{z}^{(q)} = \frac{2m_{\nu}}{\pi} \int_{z_{0}}^{z} (v_{\nu} + q_{\nu}/m_{\nu}) [\phi(z) - \phi(z')] \, dz \]

where \( W_{1} = \sqrt{v_{\nu}^{2} + \frac{2q_{\nu}}{m_{\nu}}[\phi(z) - \phi(z')]}, \quad W_{2} = v_{\nu} + (q_{\nu}/m_{\nu}) \int_{z_{0}}^{z} B(z') \, dz' \), and the limits of the integration are given by

\[ W_{2}(v_{\nu}, z, z_{01}) \pm W_{1}(v_{\nu}, z, z_{01}) = 0 \]

with \( z_{0} < z < z_{1} \) and the additional restriction that \( z_{0} = 0 \) if the formal solution of (6) becomes negative. For electrons, these expressions can be simplified in the main part of the sheet because \( \rho_{0e} \ll L \sim \rho_{0i} \). Expanding the electrostatic potential and magnetic field around the point \( z \) we obtain, instead of (5), the modified form

\[ I_{z}^{(q)} = \frac{2m_{\nu}}{\pi} \int_{z_{0}}^{z} (v_{\nu} + (q_{\nu}/m_{\nu}) \int_{z_{0}}^{z} B(z') \, dz') \, dz' \]

The quasi-adiabatic invariant \( I_{z}^{(q)} \) is given by the formula

\[ I_{z}^{(q)}(\nu_{x}, \nu_{y}, \nu_{z}) = \frac{2}{\pi} \int_{\nu_{z0}}^{\nu_{z}} \frac{d\nu_{z}}{\nu_{z0}^{2}} \int_{\nu_{z0}}^{\nu_{z}} \frac{d\nu_{z}}{\nu_{z0}^{2}} \]

where \( \nu_{z0} = \nu_{z0} + (\nu_{x} - \nu_{y})^{2} + \nu_{z}^{2} \), and the limits of the integration are given by

\[ \nu_{x}, \nu_{y}, \nu_{z} \]

with \( \nu_{x}, \nu_{y}, \nu_{z} \).

[9] Now we introduce a number of dimensionless parameters and variables. The group of fixed parameters includes the temperature ratio \( \tau = T_{\perp}/T_{\parallel} \), the anisotropy ratio \( \eta_{\alpha} = T_{\perp,\alpha}/T_{\parallel,\alpha} \), the mass ratio \( \mu = m_{\nu}/m_{i} \), and the dimensionless drift speed \( v_{D_{\nu}} = v_{D_{\nu}}/v_{T_{\perp,\alpha}} \). The parameter to be determined is \( \beta_{0} = 8\pi\nu_{0}/T_{\perp,\alpha} \), which characterizes the compression of the sheet. We also normalize the magnetic field \( B = B_{D}/B_{0} \), the electromagnetic potential \( A_{\nu} = -aB_{D,\nu,0} \), where \( a = \sqrt{\frac{e}{\mu_{0}c}B_{n}} \), the electrostatic potential \( \phi = T_{\parallel,\nu}/\phi_{e} \), the spatial coordinate \( z = z_{0} \), the particle velocity \( v_{\nu} = v_{\nu}/v_{T_{\perp,0}} \), and the invariant \( I_{z}^{(q)} = I_{z}^{(q)}(m_{\nu}v_{\nu}T_{\perp,\alpha,0}^{1/2}) \). Moreover, we transform Ampère’s equation (1) into an integral equation for the function \( b(a) = b(\zeta(a)) \) by changing variables \( \zeta \rightarrow a \) and using the identity \( db/d\zeta = (db/d\alpha)(da/d\zeta) \); \( (1/2)(db/d\alpha) \). Finally, we limit the consideration below to the case of isotropic electron species. Then Ampère’s equation can be written as

\[ b(a) = \sqrt{3\eta_{\nu}}(P_{\nu}(a) + P_{e}(a)), \]

with

\[ P_{\nu} = -\frac{1}{\pi} \int_{a_{0}}^{a_{1}} \frac{d\alpha}{b(a)} \]

where

\[ a_{0} = a_{0} = a_{0} - a_{0} \]

\[ a_{0} = a_{0} - a_{0} \]

[8] In the limit \( T_{\perp} = T_{\parallel} \) the distribution (3) takes the classical isotropic form [Harris, 1962]. In another limiting case, \( v_{D_{\nu}} = 0 \) and \( T_{\parallel} > T_{\perp} \), it becomes cigar-like and resembles the counterstreaming ion distribution of the forced current sheet model [Sitnov et al., 2000a, 2000b]. Below we consider a more general case \( v_{D_{\nu}} \neq 0 \) and \( T_{\parallel} \neq T_{\perp} \). The distribution (3), which is used below

\[ f_{\nu} = \frac{\delta_{0}}{T_{\perp}} \frac{v_{T_{\perp}}^{2}}{v_{T_{\perp}}^{2}} \sqrt{T_{\perp}} \int_{T_{\perp}}^{T_{\perp}} \frac{d\nu_{z}}{T_{\perp}} \]

The quasi-neutrality equation takes the form

\[ \varphi(\tau^{-1} + \eta_{\nu}) = \log(\frac{1}{\pi} \int_{a_{0}}^{a_{0}} \frac{d\alpha}{b(a)} \]

\[ -w_{D_{\nu}}^{2} \eta_{\nu}(\nu_{z} - 1) - 2\nu_{w_{D_{\nu}}}/\eta_{\nu} - 1/2 \eta_{\nu}w_{D_{\nu}} \]

[10] The quasi-neutrality equation takes the form

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\[ -w_{D_{\nu}}^{2} \eta_{\nu}(\nu_{z} - 1) - 2\nu_{w_{D_{\nu}}}/\eta_{\nu} - 1/2 \eta_{\nu}w_{D_{\nu}} \]

The system (8)–(12) is complemented by the condition \( w_{D_{\nu}} = 1/2 + \eta_{\nu}w_{D_{\nu}} = 0 \), which provides neutrality outside the sheet and transforms into the well-known relation \( w_{D_{\nu}} = -T_{\parallel}T_{\perp} \) in the case of the Harris CS.

3. Solution of the Equilibrium Equations

[11] The solution of the equilibrium equations (8)–(12) represents a nonlinear eigenvalue problem for the functions \( b(a), \varphi(a) \), and the unknown parameter \( \beta_{0} \). It can be iteratively solved starting from the Harris equilibrium for
the parameters $\tau = 1/4$ and $\mu = 1/1836$ (for details of the numerical technique see [Sitnov et al., 2000a, 2000b] and refs. therein). The eigenvalues $\beta_0$ are shown in Figure 1 as functions of the parameter $\eta_i$ for different values of the drift velocity $w_{Di}$. All the curves converge at the point $\eta_i = 1$ (isotropic plasma) with $\beta_0(1 + \tau) = 1$, which corresponds to the well known relation of the Harris model $B_0^2 = 8\pi n_0(T_i + T_e)$. The eigenfunctions in the form of the profiles of the magnetic field $b(\zeta)$, electrostatic potential $\varphi(\zeta)$, plasma density $n(\zeta)$ and current density $J(\zeta)$ are shown in Figures 2–4. In particular, Figure 2 shows these profiles for the cigar ion distribution with the parameters $\eta_i = 0.8$ and $w_{Di} = 0.125$. They reproduce the positive charging and embedding within the thicker plasma sheet, that is the distinctive features of the TCS formed by counter-streaming field-aligned ion flows and known as the forced CS ([Sitnov et al., 2000a, 2000b] and refs. therein). However, in contrast to the earlier models, the thicker plasma sheet is now the conventional Harris CS rather than a background plasma with a constant density and zero current.

[12] The formation of BCS becomes possible for pancake ion distributions ($T_{\perp i} > T_{\parallel i}$) outside the sheet. Note that this type of the ion anisotropy is a natural consequence of the CS compression by the convection dawn-dusk electric field $E_y$, although this field itself cannot be included in the present equilibrium model. Since the ions spread out across the whole plasma sheet instead of following the electron convection flow toward the CS [Pritchett and Coroniti, 1995], the energy going into this species will dominantly heat it. However, this heating will first affect the perpendicular component of the ion velocity because of the small normal component $B_n/C_28 B_0$ of the CS magnetic field (in the limit $B_n = 0$ the parallel component of the velocity is an integral of motion even for the case $E_y \neq 0$). It is interesting to note that Lui et al. [1992] found pancake proton distributions with $\eta_i = 1.04–1.34$ prior to all current disruption events studied using the AMPTE CCE spacecraft. Figure 3 shows that even a small (10%) pancake anisotropy of the ions outside the CS drastically modifies the Harris sheet profile, transforming it into a BCS (bottom panel). It also shows a considerable increase of the electron contribution to the total current density at the center of the sheet, which is provided however mainly because of the reduction of the corresponding ion contribution. Further increase of the

![Figure 1](image1.png)

**Figure 1.** Values of the parameter $\beta_0$ found as solutions of the equilibrium equations (8)–(12) for different values of anisotropy $\eta_i$ and the Harris part of the ion bulk flow velocity $w_{Di} = 0.125 (\tau), 0.25 (\triangle)$, and $0.5 (\Box)$.

![Figure 2](image2.png)

**Figure 2.** Profiles of the magnetic field $b(\zeta)$, electrostatic potential $\varphi(\zeta)$, plasma density $n(\zeta)$, and current density $J(\zeta)$ found for the parameters $\eta_i = 0.8$ (cigar distribution) and $w_{Di} = 0.125$. Dashed lines show the corresponding profiles of the Harris sheet. Dash-dotted and dash-triple dotted lines show the contributions to the current density of ion and electron species, respectively.

![Figure 3](image3.png)

**Figure 3.** Profiles described in Figure 2 shown for the case of the pancake ion distribution outside the sheet with the parameters $\eta_i = 1.1$ and $w_{Di} = 0.125$.

![Figure 4](image4.png)

**Figure 4.** Profiles described in Figure 2 for the case of electron current domination at the center of the BCS ($\eta_i = 1.2$ and $w_{Di} = 0.125$).
aniotropy results in the domination of the electron current in the central CS region as is seen in Figure 4. Note that while the electron current density exceeds the ion density at the center of the sheet, the two off-center peaks are still due to the ions. Thus, the drift of electrons in the crossed fields $E_z = -\nabla\phi/\partial z$ and $B$ is not the main mechanism for maintaining the equilibrium BCS, but that may be the case during its formation by the convection field $E_z$. [Pritchett and Coroniti, 1995; Hesse et al., 1998]. Although the central part of the equilibrium BCS is negatively charged as compared to the region of the maximum current density, the latter region is closer to the peak of the potential rather than its $z$-derivative as would be the case for currents dominated by the $E \times B$ drift.

13 Signatures of the current bifurcation were found earlier in a number of numerical TCS models with various ion source distributions [Holland and Chen, 1995; Harold and Chen, 1996] including those of trapped ions [Zelenyi et al., 2002]. Interestingly, the abundance of trapped ions and pancake anisotropy may have similar effects on the CS structure as both phenomena imply the domination of ions with large values of the invariant $I_z^0$. And yet, the BCS solution found in our model cannot be explained by the dynamics of the quasi-adiabatic ions and in particular its trapped component alone. It forms as a result of the balance between the features of the quasi-adiabatic ion motion (due to the new invariant $I_z^0$) and those of the Harris part of the CS. We find, in particular, no BCS solutions in the limit $w_{Di} \to 0$, where the Harris constituent is sufficiently small.

14 Another distinctive feature of the present BCS model, which is consistent with observations [Sergeev et al., 2003], is the formation of a plateau in the profile of the plasma density in the central CS region between the current peaks (Figure 4). This effect is particularly well seen when compared to the pure Harris profile (dashed line in Figure 4). One distinctive feature of the model, namely the ion current density domination in the peak regions, seems to be at variance with observations, which display no such features yet. It should be noted however, that a similar characteristic profile of the ion current (in the form of an inverted “W”) was reported by Pritchett and Coroniti [1995] whereas the simulated electron current did not show any such feature.

15 With further increase of the anisotropy $\eta_*$ for the drift speeds $w_{Di} = 0.125$ and 0.25 the total current density at the center of the sheet becomes negative and a self-consistent equilibrium vanishes. On the other hand, with the increase of the dimensionless ion drift velocity $w_{Di}$ the total thickness of the CS approaches the ion gyroradius $\rho_i$ but the BCS structure disappears. An interesting and yet unexplained feature of this case is the qualitatively different behavior of the parameter $\beta_0$ as a function of anisotropy $\eta_*$ (Figure 1).

4. Conclusion

16 We have shown that the generalization of the Harris CS model assuming ion anisotropy outside the CS and taking into account the quasi-adiabatic properties of the ion serpentine motion, gives rise to a large family of TCS equilibria, which describe such distinctive features as their characteristic scales, domination of either ion or electron species in the current production, embedding, and bifurcated structures. In particular, the model reproduces the double-peak structure of the current density and the plateau of the plasma density in the current depression region. Thus it can be used for comparison with the observed BCS along with the slow shock [Hoshino et al., 1996] and electron-dominated CS [Hesse et al., 1998; Arzner and Scholer, 2001] models.

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