Structure and dynamics of thin anisotropic current sheets: Challenges of Cluster and MRX observations

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Outline

- Summary of space and laboratory thin current sheet observations
- Generalization of the Harris equilibrium theory for thin current sheets
- TCS simulations using the P3D code and the new equilibrium theory
- Conclusion
Summary of space thin current sheet observations

**Embedding**

*McComas et al.* [1986] using ISEE 1 and 2 data on April 5, 1979 revealed that the relatively thin CS may be embedded into the much thicker plasma sheet with the much lower current density.

**Current carriers**

*Mitchell et al.* [1990], *Mukai et al.* [1998], *Asano et al.* [2003]: in contrast to the classical equilibrium theory [Harris, 1962], electrons may dominate the current even though $T_i >> T_e$.

**Current bifurcation**

*Sergeev et al.* [1993] found that the TCS current density may have two peaks off the central plane suggesting that $T_{\perp i} > T_{|| i}$ [Cowley, 1978, 1979].
Summary of space thin current sheet observations (continued)

Wave activity

Lui et al. [1992] (AMPTE CCE), Fairfield et al. [1998, 1999] (Geotail)

**current disruption phenomenon:**
strong magnetic fluctuations during the substorm onset that preserve the original tail magnetic field topology

Plasma anisotropy

Lui et al. [1992] (AMPTE CCE)

Pancake ion anisotropy prior to all current disruption events

\[ \frac{T_{\perp}}{T_{\parallel}} = 1.04 - 1.34 \]

June 1, 1985, 8 \( R_E \)
Summary of space thin current sheet observations:
Cluster observations: Bifurcated current sheets

*Nakamura et al. [2002], Runov et al. [2003], Sergeev et al. [2003]*

- Off-center peaks of the current density
- Plateau in the plasma density profile
- Negligible $B_y$ magnetic field

(a) Relationship between $V_z$ and $dB_x/dt$ in the flapping current sheet, (b,c) $B_x$ profiles vs. the $Z^*$ scale obtained by $V_z$ integration for individual crossings, across the neutral sheet (where $Z^* = 0$). Letters A, B, C denote the individual oscillations whereas u,d correspond to downward and upward halves.

Distributions of plasma sheet parameters and magnetic field difference ($dB_{x13}$) vs. $B_x$ during 2221:00–2240:37 from s/c #1 and #3, and occurrence rate distribution of $B_x$. 
Summary of space thin current sheet observations: Cluster observations (continued)

*Nakamura et al. [2002], Runov et al. [2003], Sergeev et al. [2003]*

Observations during complete flapping oscillations A–C. (a) Comparison of \( y_{\text{GSE}} \)-components of \( E \) (from EFW instrument) and \([V_p \times B]\) (from CODIF and MGF) at the s/c \#3; (b) time integral of GSM ion bulk flow z-component \( V_{pz} \); (c) variations of \( B_x \) components at four s/c (every 3rd data point is marked); (d) Y-Z projections of the MVA the current sheet normals determined during upward and downward phases of flapping. The thick dashed line is a rough sketch of the wavy neutral sheet geometry based on the computed normal directions.

Schematic picture of the flapping motions and the current sheet structure

The whole flapping and bifurcated pattern moves in the dusk direction
Summary of laboratory thin current sheet observations: Magnetic Reconnection Experiment

Current sheet thickness

[Yamada et al., 2000]

Embedding

[Carter, 2001]
Summary of laboratory thin current sheet observations
Magnetic Reconnection Experiment (continued)

Wave activity

Electromagnetic fluctuations in the central current region
[Ji et al., 2003]

Plasma anisotropy

Plasma (ion) anisotropy (field-aligned flows)
[Hsu et al., 2000] $V_{iz} \sim 0.2V_A$
Harris [1962] current sheet theory

\[
\frac{dB_x}{dz} = \sum_{\alpha=e,i} \frac{4\pi q_\alpha}{c} \int v_y f_{0\alpha}(z,v) \, d^3v
\]

\[
\int f_{0i}(z,v) \, d^3v = \int f_{0e}(z,v) \, d^3v
\]

\[
f_{0\alpha} \propto \exp\left(- \frac{W_\alpha - v_{Da} P_{ya}}{T_\alpha}\right), \quad W_\alpha = \frac{m_\alpha v^2}{2} + q_\alpha \phi, \quad P_{ya} = m_\alpha v_y + \frac{q_\alpha}{c} A_y
\]

Solution:

\[
\frac{\partial^2 A_y}{\partial z^2} + ae^{-bA_y} = 0
\]

\[
\frac{v_{Di}}{v_{De}} = - \frac{T_i}{T_e}, \quad n_0 \left( T_i + T_e \right) = \frac{B_0^2}{8\pi}, \quad L = \frac{v_{Ti}}{v_{Di}} \rho_{0i}
\]

\[
\phi = 0, \quad B = B_0 \tanh\left( \frac{z}{L} \right) \quad n = \frac{n_0}{\cosh^2 (z/L)}
\]
Summary of the Harris equilibrium theory

- Thickness: $L = \left( \frac{\nu_T}{\nu_D} \right) \rho_{0i}$
- Embedding: No
- Main current carriers: Ions ($T_i > T_e$)
- Plasma anisotropy: No
- Bifurcated thin current sheets: No

Summary of thin current sheet observations

- Thickness: $L \sim \rho_{0i} \sim c/\omega_{pi}$
- Embedding: TCS is often embedded in a thicker plasma sheet
- Main current carriers: May be electrons even though $T_i > T_e$
- Plasma anisotropy: Plasma is often anisotropic outside the TCS
- Bifurcated thin current sheets
- Flapping motions
Ion motion in thin current sheets

Speiser [1965], Sonnerup [1971], Chen and Palmadesso [1986], Chen [1993]

Adiabatic parameter

\[ \kappa = \frac{B_n}{B_0} \sqrt{\frac{L}{\rho_0}} \]

\( \kappa \gg 1 \)

Adiabatic motion

\[ \mu = \frac{m_\alpha v^2}{2B} \]

\( \kappa \sim 1 \)

Dynamical chaos

\( \kappa \ll 1 \)

Quasi-adiabatic motion

\[ I_z^{(\alpha)} = \frac{1}{2\pi} \oint m_\alpha v_z \, dz \]
Anisotropic thin current sheet models: Earlier results


Kropotkin et al., [1997]; Sitnov et al., [2000]:

\[ f_{0i}(W_i, P_{yi}) \rightarrow f_{0i}(W_i, I_{zi}) \]

Proper scales \( L \sim \rho_{oi} \) 
Embedding effect

No bifurcation 
No reduction to the Harris model

Current bifurcation prior to forced current sheet catastrophe [Zelenyi et al., 2002]

Bifurcation appears for very steep distributions over \( I_a \) presumably as a transient effect during the evolution from cigar to effectively pancake distribution (trapped ions), which is not self-consistent in the absence of the Harris constituent.
Generalization of the Harris current sheet theory

[Sitnov, Guzdar and Swisdak, 2003]

\[
\frac{dB_x}{dz} = \sum_{\alpha=e,i} \frac{4\pi q_{\alpha}}{c} \int v_y f_{0\alpha}(z,v) d^3v = \int f_{0i}(z,v) d^3v = \int f_{0e}(z,v) d^3v
\]

\[
f_{0\alpha} \propto \exp \left( -\frac{2W_\alpha - \omega_{0\alpha} I_z^{(\alpha)}}{2T_{\parallel\alpha}} + \frac{v_Da P_y}{T_{\parallel\alpha}} - \frac{\omega_{0\alpha} I_z^{(\alpha)}}{2T_{\perp\alpha}} \right)
\]

Adiabatic motion

Quasi-adiabatic motion

\[
\mu = \frac{m_\alpha v^2}{2B}
\]

\[
I_z^{(\alpha)} = \frac{1}{2\pi} \int m_\alpha v_z dz
\]
Generalized Harris theory: Results

Dimensionless parameters:
\[ \tau = \frac{T_{le}}{T_{li}}, \quad \eta_\alpha = \frac{T_{\perp\alpha}}{T_{||\alpha}}, \quad \mu = \frac{m_e}{m_i}, \]

\[ w_{D\alpha} = \frac{v_{D\alpha}}{v_{T\perp\alpha}}, \quad \beta_0 = \frac{8\pi n_0 T_{\perp i}}{B_0^2} \]

Dimensionless variables:
\[ b = \frac{B}{B_0}, \quad \zeta = \frac{z}{\rho_{0i}}, \quad n = \frac{n}{n_0}, \quad \phi = \frac{T_{\perp i}\varphi}{e} \]

\[ \frac{w_{Di}}{w_{De}} = -\frac{1}{\eta_i\sqrt{\tau\mu}} \]

Embedded TCS
- \( \mu = 1/1836 \)
- \( \tau = 1/4 \)
- \( \eta_i = 0.8 \)
- \( w_{Di} = 0.125 \)

Bifurcated TCS
- \( \mu = 1/1836 \)
- \( \tau = 1/4 \)
- \( \eta_i = 1.2 \)
- \( w_{Di} = 0.125 \)
Generalized Harris theory: Effect of electron anisotropy

Key parameters:

- solid line
  - $\mu = 1/16$
  - $\tau = 1/4$
  - $\eta_i = 2.0$
  - $\eta_e = 1.0$
  - $w_{Di} = 0.3$

- dotted line
  - $\mu = 1/16$
  - $\tau = 1/4$
  - $\eta_i = 2.0$
  - $\eta_e = 1.065$ <
  - $w_{Di} = 0.3$
Generalized Harris theory: Comparison to other models

Generalization of isotropic 2D models [Birn, Schindler and Hesse, 2004]

\[ f_{0\alpha} \propto \exp\left[ -\left( W_\alpha - v_{Da} P_{y\alpha} \right) / T_\alpha \right] \rightarrow \exp(-W_\alpha / T_\alpha) g_\alpha \left( P_{y\alpha} \right), \quad P_{y\alpha} = m_\alpha y + (q_\alpha / c) A_{y\partial} \]

Sitnov, Guzdar and Swisdak [2003]

\[ f_{0\alpha} \propto \exp\left( -\frac{2W_\alpha - \omega_0 I^{(\alpha)}_z}{2T_{||\alpha}} + \frac{v_{Da} P_{y\alpha}}{T_{||\alpha}} - \frac{\omega_0 I^{(\alpha)}_z}{2T_{\perp\alpha}} \right) \]

\[ I^{(\alpha)}_z = \frac{1}{2\pi} \int m_\alpha v_z dz = I^{(\alpha)}_z (\tilde{A}_y) \quad \tilde{A}_y(z) = -\int_0^z B_x(z')dz' = A_y - \int_{x_0}^x B_z dx \neq A_y \]

\[ BSH \]

\[ \alpha \sim B_n / B_0 \]

\[ SGS \]

\[ \alpha \rightarrow B_n / B_0 \]
Generalized Harris theory: Particle simulations
Case 1: Stable bifurcated current sheet

Dimensionless parameters: \( \tau = 1, \quad \eta_i = 1.2, \quad \eta_e = 1., \quad \mu = 1/16, \quad w_{Di} = 0.14 \)

PIC simulations confirm the new equilibrium theory
Case 2: LHDI and transition to another equilibrium

Dimensionless parameters: \( \tau = 1/4, \quad \eta_i = 2, \quad \eta_e = 1, \quad \mu = 1/16, \quad w_{Di} = 0.3 \)

Magnetic field evolution
Case 2: LHDI and transition to another equilibrium (continued)

Dimensionless parameters: \( \tau = 1/4, \eta_i = 2., \eta_e = 1., \mu = 1/16, \ w_{Di} = 0.3 \)

Current density evolution

![Current density evolution](image)
Case 2: LHDI and transition to another equilibrium: current evolution

Key parameters:

<table>
<thead>
<tr>
<th>Solid Line</th>
<th>Dotted Line</th>
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<tbody>
<tr>
<td>$\mu = 1/16$</td>
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<td>$\tau = 1/4$</td>
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Case 2: Flapping motions of the bifurcated current sheet (even parity)
Case 2: Flapping motions of the bifurcated current sheet (odd parity)
Case 3: Flapping motions of the bifurcated current sheet II: Long box

$\omega_{oi}t=100$
Case 3: FFT analysis of instabilities

- **Lower-hybrid drift instability**
  
  - (a) $\omega t=10$
  
  - (b) $m=24$
  
  - (c) $m=24$

- **Even parity (sausage?) flapping instability**
  
  - (d) $\omega t=30$
  
  - (e) $m=3$

- **Odd parity (kink) flapping instability**
  
  - (g) $\omega t=90$
  
  - (h) $m=4$

  - (i) $m=4$
Modeling MRX current sheet: Two specific problems

**Current sheet thickness**

\[ L \sim c / \omega_{pi} \]

**Wave activity**

![Graph showing current sheet thickness](image)

![Graph showing wave activity](image)
Modeling MRX current sheet: Equilibrium theory

Harris model:

\[ L = \frac{c}{\omega_{pi}} \frac{v_{ti}}{v_D} \sqrt{1 + \tau} \]

\[ \frac{c}{\omega_{pi}} = \frac{\rho_{0i}}{\sqrt{\beta_0}} \]

\[ \beta_0 = \frac{8\pi n_0 T_{li}}{B^2} \]

\[ \mu = 1/64 \]

\[ \tau = 2/3 \]

\[ \eta_i = 1.2 \]

\[ w_{Di} = 2. \]
Modeling MRX current sheet: PIC simulations $\omega_0t=1.2$

- Magnetic field $B_x$ ($\omega_0t=1.2$)
- Current density $J_y$ ($\omega_0t=1.2$)
- Plasma density $n_i$ ($\omega_0t=1.2$)
- Electric field $E_y$ ($\omega_0t=1.2$)
Modeling MRX current sheet: PIC simulations $\omega_0 t=2.4$

- Magnetic field $B_x (\omega_0 t=2.4)$
- Current density $J_y (\omega_0 t=2.4)$
- Plasma density $n_i (\omega_0 t=2.4)$
- Electric field $E_y (\omega_0 t=2.4)$
Modeling MRX current sheet: PIC simulations $\omega_0 t = 3.6$

- Magnetic field $B_x (\omega_0 t = 3.6)$
- Current density $J_y (\omega_0 t = 3.6)$
- Plasma density $n_i (\omega_0 t = 3.6)$
- Electric field $E_y (\omega_0 t = 3.6)$
Modeling MRX current sheet : PIC simulations $\omega_0 t = 4.8$

- Magnetic field $B_x (\omega_0 t = 4.8)$
- Current density $J_y (\omega_0 t = 4.8)$
- Plasma density $n_i (\omega_0 t = 4.8)$
- Electric field $E_y (\omega_0 t = 4.8)$
PIC simulations of MRX current sheet: FFT analysis

\[ \omega_0 t \]

\[ m = 3 \]

\[ m = 5 \]

\[ m = 9 \]

\[ \langle B_m^2 \rangle \]

\[ \langle B_m \rangle \]

\[ \omega t = 1.2 \]

\[ \omega t = 2.4 \]

\[ \omega t = 3.6 \]

\[ \omega t = 4.8 \]

\[ z \]
Conclusion

- The new current sheet equilibrium model generalizes the well-known Harris equilibrium assuming particle anisotropy and taking into account the features of the non-guiding-center ion motion in thin current sheets. The model explains current sheet embedding and bifurcation and the characteristic scale of laboratory current sheets.

- The model is combined with the massively parallel particle code [Zeiler et al., 2002] to explore the evolution of thin current sheets. PIC simulations confirm the new equilibrium theory.

- Simulations reveal the build-up of the bifurcated electron current indicating the transition to a new TCS equilibrium. The new equilibrium can be described if one assumes small anisotropy of the electron species.

- Simulations reveal cascades of instabilities:
  (I) LHDI, sausage and kink in case of geotail-type TCS
  (II) sausage and kink in case of MRX-type TCS
Pancake anisotropy: How and why does it form?

*Shay et al. [1998]*

*Swisdak et al. [2004]*