Singular Current Layers and the Sweet-Parker/Petschek models of Reconnection in MHD

B.D. Jemella
M.A. Shay
J.F. Drake
B.N. Rogers

University of Maryland, College Park, MD
Dartmouth College, Hanover, NH
Resistive MHD Description

- Formation of macroscopic Sweet-Parker layer

\[ V \sim (\Delta / L) C_A \sim (\tau_A / \tau_r)^{1/2} C_A \ll C_A \]

  - Slow reconnection
  - Sensitive to resistivity
  - Macroscopic nozzle

- Petschek-like open outflow configuration does not appear in resistive MHD models with constant resistivity (Biskamp ‘86). Why?
Equilibrium and stability

\[ \vec{B} = \hat{z} \times \nabla \psi + B_z \hat{z} \]

\[ \psi = \lambda \cos \left( \frac{y}{\lambda} \right) \]

\[ \lambda = \frac{L_y}{2\pi} \]

\[ B_z = \sqrt{B_{z0}^2 - B_x^2} \]

- Total Pressure Balance

\[ \Delta' \lambda = 2 \sqrt{1 - \varepsilon^2} \tan \left( \frac{\pi}{2} \sqrt{1 - \varepsilon^2} \right) \]

- Tearing mode stability parameter for wavevector \( k_x \)

\[ \varepsilon = k_x \lambda \]

\[ k_x = \frac{2\pi}{L_x} \] – Going to look at different box lengths.
Typical Magnetic Island

- **Elongated current layer.**
  - **Sweet-Parker scaling dominates.**
Island Growth

- Largest value of $\Delta'$ has fastest island growth.
\[ \Delta' \lambda = 0.92 \]

Islands are grown from the full resistive MHD Equations with

\[ \eta = 2.8 \times 10^{-4} \]

1.32

3.13

4.92

8.11

20.93

– Sweet-Parker Current layers become stronger with increasing \( \Delta' \).
Clear linear relationship between $V_{out}$ and $C_{A\nu r}$ is evident for finite values of $\Delta'$.

Scaling with resistivity:

$\eta = 2.8 \times 10^{-4}, 1.12 \times 10^{-3}$

$\Delta' = 20.93 \Rightarrow E_r \sim \eta^{0.51}$

$\Delta' = 8.11 \Rightarrow E_r \sim \eta^{0.53}$

⇒ Sweet-Parker Scaling
More Scaling with Resistivity

$\Delta' = 8.108$

-Sweet-Parker scaling dominates at earlier time at lower resistivity.

Why do Sweet-Parker Layers Develop?
Equilibrium

- Allow reconnection to produce a finite magnetic island ($\eta \neq 0$)
- Zero ion flows
- Shut off reconnection ($\eta = 0$) and evolve to relaxed state
  - Formation of singular current sheet
• Noticeable decrease in width of the current layer

• Collapse to point of loss of resolution in simulations – re-grid at twice resolution.

• Transition from open x-line to a y-point configuration

• Layer length appears nearly constant as system relaxes to equilibrium

• The post-reconnection MHD equilibrium contains an intrinsic singularity

• This singular structure is responsible for the Sweet-Parker current layer.
All cases with Sweet-Parker Current layers have singular layers in relaxed state.
Island Width Dependence of Singular Current Layer

\[ \Delta' = 20.93 \]

Note the difference in scale in y and the narrowing of the layer.
Equilibrium State

- In a perfect Equilibria $B_y$ should be zero between the two $y$-points.

- Cut through current layer in outflow direction.

- All but smallest value of tearing mode stability parameter yield a stable singular current layer in their equilibrium state.
Formation of layer

- Formation of layer is consequence of the conservation of magnetic flux and requirement that magnetic energy be reduced. (Waelbroeck 89)
  - Will show with a simple cartoon model
Model for Current Layer Formation


\[ L_j = L \left(1 - \sqrt{\frac{U_f}{U_i}}\right) \]
Growth of Island

\[ \dot{w} = \frac{L}{(\tau_r \tau_{A_i})^{1/2}} \frac{\Delta' w / 4}{1 + \Delta' w / 4} \]

Alfven Time Evaluate with \( B_i \sim \dot{w} \)

- Based on current layer length calculation we generalize the Sweet-Parker Model.
- For \( \Delta' w / 4 > 1 \) this reduces to the Sweet-Parker scaling for reconnection.
- Remains Slower than Petschek reconnection.
Recap

- Current layers develop in systems only slightly deviated from marginal stability.
  - Layers increase in length and strength with increasing $\Delta'$
  
  - Macroscopic layers are Sweet-Parker in nature.

- Formation is a byproduct of conservation laws.

- Singular layers created in the Magnetic Island Equilibria govern the system. All dynamical properties build on top of them.
Applications

- Astrophysical plasmas in general, correspond to systems with large values of the tearing mode stability parameter.

- Dorelli et al., completed global MHD simulations with a fully resolved dissipation region on the sun side – results indicate Sweet-Parker layer.
Magnetosphere: Global MHD

- Global MHD Simulation
  - Very high resolution sun-side.

- Magnetic field pileup:
  - Not physical for $B_z < 0$ and moderate solar wind speeds

Satellite Data

(Phan et al., 1994)
Conclusions

- From our research, resistive MHD produces Sweet-Parker reconnection in virtually all systems.
  - These Layers are the result of the singular equilibria.

- Well resolved Global Models develop similar Sweet-Parker layers
  - Reconnection rates far too slow
  - Build up of Magnetic Field flux is inconsistent with satellite data

- Pure MHD insufficient for modeling reconnection in Magnetosphere.