Challenges to Creating a Multiple Solution Norm – Classroom Vignettes:

Creating a multiple solution norm is demanding and difficult work for a practitioner. This section will describe a series of annotated classroom vignettes in an attempt to describe a subset of the wide-array of challenges a teacher may confront in his/her efforts to instantiate a multiple solution norm.

The vignettes, though fictional, are based on an actual lesson I taught to two sections of an eighth grade Algebra class. The idea of using vignettes to highlight issues of creating a multiple solution norm came from the vignettes in *Professional Standards for Teaching Mathematics* (NCTM, 1991) used to illustrate the roles of teachers and students in a student-centered classroom. In addition, the vignettes represent a conglomeration of ideas from other research documents. Research by Kazemi and Stipek (2001), Pang (2000), and Yackel and Cobb (1996) examined issues regarding creating certain social and sociomathematical norms. Chazan and Ball (2000) and Lampert (2001) discuss the problems of teaching in ways predicated on student ideas.

Each vignette is meant to illustrate a potential challenge a practitioner may deal with in either constituting a given norm or a challenge that arises as a consequence of a norm being constituted. The vignettes are not meant to infer what teacher action or process led to a challenge or how the challenge could be addressed.

*Background Information:*
The vignettes describe an episode from the classrooms of four teachers: Mr. Carl, Ms. Joyner, Ms. Robinson, and Mr. Lyttle. Each teacher reports a commitment to allow students the opportunity to be engaged in mathematical discourse in which they invent, explain, and justify, their own mathematical ideas, and critique the ideas of others.

The classroom participation structure in each of the four classrooms was similar: (a) students were seated in groups of four or five; (b) the teacher initiated an activity or gave students a mathematical problem; (c) students independently solved the given problems; (d) the teacher asked students to report their solution methods to the whole class; (e)
students presented their solution methods; and (f) the teacher facilitated the classroom discussion.

Each teacher made a conscious effort not to adjudicate the correctness of ideas. All four teachers regularly asked students to explain their reasoning regardless of a correct or incorrect solution. Classes were dynamic and the students eagerly responded to requests made by the teacher. Each teacher had established a caring and permissive environment in which students’ mistakes were welcomed and accepted without ridicule.

The issue of task selection was critical for each teacher. They took care to select problems and activities that would be accessible to each student. Each of the four teachers used tasks that would elicit alternative representations and solution strategies. The teachers purposefully posed problems before a formal algorithm had been reached.

Each teacher posed the following task for their students:

*A person invests $1,000 into a savings account that earns 10% interest each year. How much money will the person have (a) After 1 year? (b) After 6 years? (c) After 60 years?*

**Vignette #1:**
Students in Mr. Carl’s class predominantly worked quietly and independently on the problem. Although students were placed in groups of four, there was little collaboration between classmates. In two of the groups where students were working together, the distribution of work across a group was not equal; members of the groups often accepted another student’s answer with little or no debate.

After several minutes, the majority of the class was no longer thinking of the problem and was engaged in conversations outside of mathematics. As Mr. Carl walked to different groups, he noticed most students had recorded a solution to part (a); however, Mr. Carl recognized that many students had made either procedural or conceptual errors in their solution. More than one student calculated the amount of money after one year as less than the original investment. Further, Mr. Carl observed many students made no visible attempt to solve the other two parts of the problem. One student who had not gone off task, asked Mr. Carl “I know how to get the first part, but how do I find how much there will be after six years?”

A challenge for Mr. Carl is to create an environment so that students reflect on mathematics and communicate ideas. In a classroom where a multiple solution norm is present, students will regularly ask questions of each other and evaluate solutions during student-student interactions.

In a classroom in which a multiple solution norm is evident, students will try to make sense of difficult problems and display a degree of persistence. Students may look to the teacher to clarify a task or point them in the right direction, but would not routinely ask the teacher how to solve a problem.
As Mr. Carl transitioned from student work to whole-class discussion, he managed to regain the attention of the class. Greg was the first student who spoke regarding part (a) of the task.

Greg: I think it’s about one thousand and ten dollars.
Mr. C.: Why do you think that?
Greg: Because ten percent is like ten more dollars.
Mr. C.: Say more, why is that?
Greg: Because ten percent of something is about ten or eleven more dollars.

Mr. Carl asked the class what they thought of Greg’s response. The students at first were largely unresponsive. Mr. Carl again urged students to comment on Greg’s work. Erica spoke out.

Erica: Is he [Greg] right or wrong?
Mr. C.: What do you think?
Erica: I don’t know.
Mr. C. [To the class]: Does anybody have a different answer?
Erica: I think it is just one hundred.
Mr. C.: Why?
Erica: I remembered doing problems like this last year. Ms. Cooper taught us to write $n$ over one thousand equals ten over one hundred. Then you criss-cross and get one hundred times $n$ and one thousand times one hundred which is ten thousand, and then divide by one hundred and you get one hundred.

Mr. C.: Why do you write it out like that?
Erica: That’s just how Ms. Cooper taught us to do them.

Although Mr. Carl twice attempted to get Greg to explain his answer, Greg did not offer a mathematical argument for his solution. Similarly, Erica’s explanation involved a summary of the steps taken to solve the problem, but lacked mathematical justification or verification. A challenge for a teacher when pressing for mathematical explanations is that students may not understand what is being asked, may not know what type of rationale is acceptable, or lack the language needed to explain.

A class predicated on student ideas requires a degree of participation on the students. A challenge for a teacher attempting to implement a multiple solution norm is to get students to respond to their classmates’ solution strategies. Unresponsive students may lack sufficient knowledge of content or reasoning strategies to evaluate explanations; they may lack interest in the problem; and/or they may be unwilling to critically evaluate their classmates.

Erica’s comment further suggests that she believes Mr. Carl is responsible for judging the correctness of student solutions. Erica may not be comfortable sharing her solution if she thinks she has the wrong answer. These are both challenges Mr. Carl needs to address to create a multiple solution norm.
Vignette #2:
In Ms. Joyner’s class, students enthusiastically began working on the problem. At first, most of the students worked individually; then, as progress was made on the task, the students communicated their ideas with their group members. In the different groups, the distribution of labor was fairly equal and all students were engaged in solving the problem. Students shared their thoughts, asked questions of one another, and compared their solution strategies.

As groups calculated solutions, they were anxious to show Ms. Joyner what they had accomplished. Students waved their hands to get Ms. Joyner’s attention and wanted Ms. Joyner to check their work. Some groups saw the task as a competition and were pleased when they solved the problem before other groups. Many of the groups suspended their work on parts (b) and (c) until they asked Ms. Joyner for confirmation for their answers on part (a).

A challenge for Ms. Joyner, in her attempts to constitute a multiple solution norm, is to create an environment in which her students regularly rely on mathematical logic and evidence to evaluate the validity of an answer. Students in a classroom in which a multiple solution norm is present would rarely seek the teacher’s confirmation regarding the correctness of solutions. Additionally, the criteria for doing well would rest in making sense of mathematical ideas, not in the speed or pace of student work.

In the whole-class discussion for part (a), students expressed their ideas by freely speaking out. Ms. Joyner recorded the students’ ideas on the board. Rob was the first to begin the conversation.

Rob: I divided one thousand by ten and got one hundred, then I added that to one thousand and got one thousand one hundred dollars.

Megan: I got the same answer, but I did it a different way. I multiplied one thousand by point ten because ten percent is point ten, and added what I got to one thousand.

Rob: That’s really the same thing.

Megan: [to Rob] I don’t understand why you divided by ten.

Kristen: Megan, multiplying by point ten is the same as dividing by ten because it’s a tenth

Ms. J.: Any other questions or comments or different solutions or solution strategies?

John: I solved it using a proportion – is over of equals percent over one hundred; so I did n over one thousand equals ten over one hundred. I cross multiplied and divided to find n is one thousand one hundred.

Dylan: We just multiplied one thousand by one point one.

Students: Where is the one point one coming from?

Dylan: I’m not sure. Mac had explained it to me but I forgot why he did that. Mac, why did we use one point one?
Mac: Multiplying by one point one is the same as multiplying by point one and adding back what you started with because one point one is point one plus the one whole.

This episode demonstrates that Ms. Joyner’s class has constituted norms in which students freely offer their solutions, and students listen and respond to one another.

In a class where a multiple solution norm exists, not all student solutions would be accepted as reasonable and valid. Solutions, like John’s, which consisted of a procedural explanation, would likely be probed further by students as they evaluate his solution strategy to understand how it compares or contrasts to other solution methods.

In monitoring whole-class student discourse, Ms. Joyner is faced with the challenge of making decisions that could potentially affect classroom norms. As students take charge in sharing solutions, Ms. Joyner needs to decide how to monitor participation so everyone has an equal chance to share their solutions or express their concerns. Ms. Joyner must decide how to attach notation and language to student ideas as she records them on the board. Ms. Joyner must use her knowledge of students, mathematics, and the curriculum to determine what ideas to pursue in depth among a potential wide-array of student strategies.

A bit later in the period, the class discussed part (b) of the task, finding how much money there would be after 6 years. The following conversation took place.

Olivia: After one year there was one hundred more dollars, so after six years there will be six hundred more dollars for a total of one thousand six hundred.

John: I multiplied one thousand by point ten and got one hundred, then multiplied one hundred six times to get one thousand six hundred.

Tressa: I agree with John and Olivia, but I did it more like Mac’s way. Ten percent each year for six years is sixty percent, so one point six times one thousand is one thousand six hundred.

Ms. J.: Does anybody have a question? Or does anybody have a different solution or solution strategy?

After Tressa’s comments, no other questions or solution strategies were put forth. Several students expressed they understood. Ms. Joyner had anticipated students would not compound the interest each year and arrive at the solution obtained above. While students worked on the task earlier in the class, Ms. Joyner observed that Alison’s group had compounded the interest and recursively found an answer for the amount of money at the end of six years. Knowing that a different solution was found, the following exchange took place.

Ms. J: Alison, I saw your group had found a different solution.

Alison: We had something different, but I see what we did wrong now.

Ms. J: You are satisfied that there is one thousand six hundred dollars after six years?
Alison: Yes.

A challenge for Ms. Joyner in creating a classroom environment predicated on student ideas is to have students explicitly appreciate similar and different solution strategies. In a classroom with a multiple solution norm, the meaning of mathematical difference is negotiated by a teacher and his/her students. Solution methods equivalent to each other are either not put forth by students, or identified by the class as being mathematically equivalent. Here, John offered a solution method isomorphic to Olivia’s explanation. Whether or not John attempted to process Olivia’s response is unknown; however no objection was raised by the class.

Ms. Joyner is faced with the dilemma of having the entire class reach a consensus on a mathematically objectionable solution. Although Alison and her group had calculated the desired answer, she seemed unwilling to share her perceived mistake with the class. In a class with a multiple solution norm, students would routinely share their (perceived) mistakes as an important component of learning.

**Vignette #3:**
The social and sociomathematical norms evident in Ms. Joyner’s classroom also appeared in Ms. Robinson’s class. Students collaboratively persevered in attempting to solve the task, students enthusiastically shared their different solution strategies, and there was evidence suggesting that students appropriated the responses of their classmates.

In the whole-class discussion for part (b), the following discussion occurred.

Kalie: In one year ten percent of one thousand is one hundred, so if you wait six years, you will get six hundred more dollars.

Alex: I agree with Kalie. If you do ten percent a year for six years you have sixty percent. Sixty percent of one thousand dollars is six hundred. So you will have one thousand six hundred dollars.

Cristal: I might be wrong, but I did something else and got a different answer. I started out like Kalie and found ten percent of one thousand is one hundred, so after one year there was one thousand one hundred dollars. Then I took ten percent of one thousand one hundred which was one hundred ten and added it to get one thousand two hundred ten dollars. That is how much there was after two years. I kept going until I got to six.

Ms. R: You got a different answer than Kalie?

Cristal: I ended up getting one thousand seven hundred seventy one dollars.

Ms. R: Can anyone make an argument either for or against Cristal or Kalie?

At this request, a number of students expressed their allegiance to both sides. Students argued for their choice by essentially revoicing what Kalie and Cristal offered. One student suggested the class vote on which answer was right. Another student offered a compromise and suggested that “maybe they are both right.”
Ms. Robinson, attempting to get her students to compare the discrepancies in the two solutions, asked “under what conditions would Kalie be correct, and what conditions or assumptions would Cristal’s argument make more sense?” Again several students responded to Ms. Robinson’s question by re-summarizing the procedures Kalie and Cristal used to get their respective answers. No new idea was put forth. At this point, students seemed frustrated regarding the stalemate and pressed Ms. Robinson to tell them which answer is correct. One student exclaimed “I understand both ways, so if I know which one is correct, I can explain why it works.”

Here, Ms. Robinson faces a challenge different than Ms. Joyner’s. Whereas Ms. Joyner was challenged when the entire class arrived at an undesirable mathematical consensus, Ms. Robinson’s dilemma centers on a class divided over the legitimacy of an answer. In a classroom with a multiple solution norm, students will routinely use mathematical arguments to support or refute a given solution strategy. The bases for student actions would be mathematical, not status-based. Students would unlikely suggest voting as a means for determining a correct answer. Students would not rely on the teacher’s authority for adjudicating the correctness of a solution.

In the face of student frustration, Ms. Robinson needs to make difficult decisions regarding her next move. She must decide if and what information to give to her students versus letting her students struggle.

**Vignette #4:**
By all accounts, Mr. Lyttle and his students have jointly negotiated a multiple solution norm. During small group discussions, students described and defended their mathematical interpretations and solutions for the problem. When Mr. Lyttle approached a group, their mathematical work did not alter. Group discussions led to a consensus in which each member was accountable for understanding the accepted solution strategy. During whole-class discussion, students offered detailed analysis of their solution methods. Students accepted explanations only if the explaining students included a mathematical justification for their answers. Students questioned and compared various solution methods. The basis on which each solution strategy was evaluated rested on the strength and logic of its mathematical argument.

Although multiple methods for part (a) were put forth, the class agreed that multiplying the original investment by one and one tenth was the most sophisticated and efficient way to arrive at solution. For part (b), the discussion, at first, resembled that of Ms. Robinson’s class. However, instead of reaching a gridlock, the students’ turned to the different ways of interpreting the problem. The students reached a consensus that the most logical interpretation was to compound the original investment each year. With that concord, the students agreed on the strategy of recursively calculating a solution by multiplying the previous year’s amount by one and one tenth six times.

During the discussion to part (c), the students realized they could continue to recursively solve the problem to determine the amount of money after sixty years. This was not the
preferred strategy of the class. Ruthie exclaimed “there has to be a formula we can use.” Jonah, referring to the table on the board from part (b), conjectured that the amount each year corresponded to a row of numbers in Pascal’s triangle.

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<tr>
<th>This episode illustrates how a teacher will inevitably be asked to deal with important challenges as a consequence of negotiating a multiple solution with his/her students. In this case, Mr. Lyttle is confronted with an unanticipated student response. Mr. Lyttle needs to be able to assess the mathematics in Jonah’s idea, its level of sophistication, and student interest for it in order to decide if it is worthwhile to invest a substantial amount of class time discussing it.</th>
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<td>Additionally, Mr. Lyttle must decide what type of questions to ask and what information to give to his class in facilitating a productive classroom discussion.</td>
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<td>These decisions and the subsequent interaction with his students could positively or negatively impact the constitution of a multiple solution norm.</td>
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