

## **FIGURES TO FACILITATE CHECKING OF PROOFS**

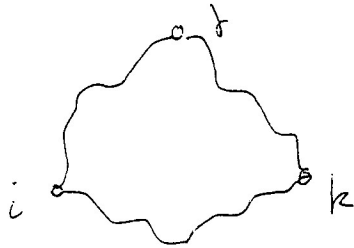
Note to Readers:

The attached figures facilitate the checking of the proofs. The pages are numbered N-1 to N-6 for the node connectivity case, and E-1 to E-13 for the edge connectivity case. Further, each of the figures is labeled with a number corresponding to the case number it corresponds (in the description of the cases). Finally, figures are attached for the 9 cases of the edge-decomposition on general graphs using 2-separators.

# Node Connectivity Case

## Cases for $S'_{ik}$

=

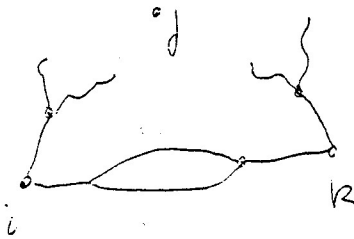


$$P_{ij} \cup P_{jk} \cup P_{ik}$$

(1)

(a)

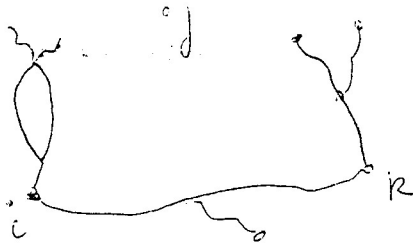
=



$$S_{ik} \cup T_{ij} \cup T_{kj}; a_{ij} = a_{kj} = 0$$

(b)

=



$$T_{ij} \cup P_{ik} \cup T_{kj}; a_{ik} = a_{jk} = 0$$

(c)

=

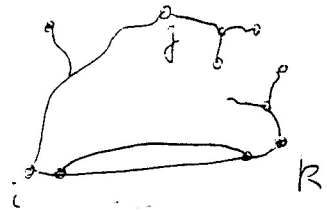


$$T_{ij} \cup P_{ik} \cup T_{kj}; a_{ki} = a_{ji} = 0$$

(2)

(a)

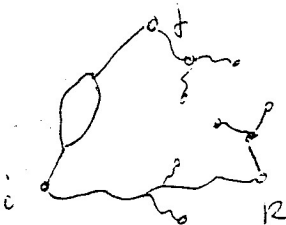
=



$$S_{ik} \cup P_{ij} \cup Q_{jk}; a_{ij} = a_{kj} = 0$$

(b)

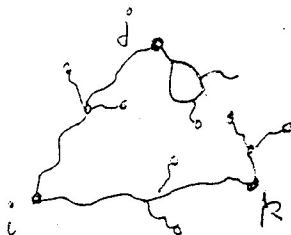
=



$$P_{ik} \cup S_{ij} \cup Q_{jk}; a_{ik} = a_{jk} = 0$$

(c)

=

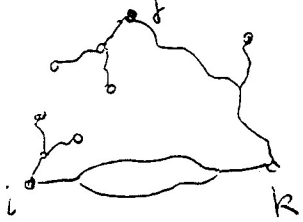


$$P_{ij} \cup P_{ik} \cup R_{jk}; a_{ji} = a_{ki} = 0$$

(3)

(a)

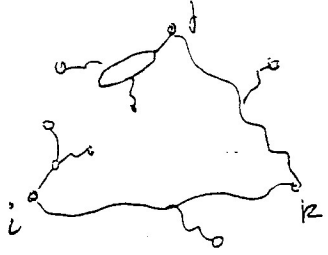
=



$$S_{ik} \cup P_{kj} \cup Q_{ij}; a_{ij} = a_{kj} = 0$$

(b)

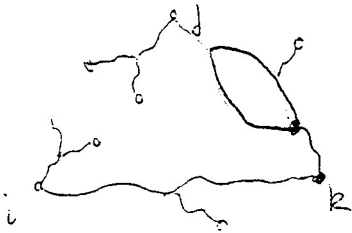
=



$$P_{ik} \cup P_{kj} \cup R_{ij}; a_{ik} = a_{jk} = 0$$

(c)

=

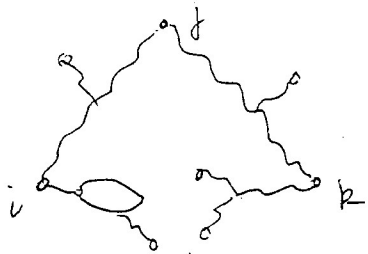


$$P_{ik} \cup S_{kj} \cup Q_{ij}; a_{ji} = a_{ki} = 0$$

(4)

(a)

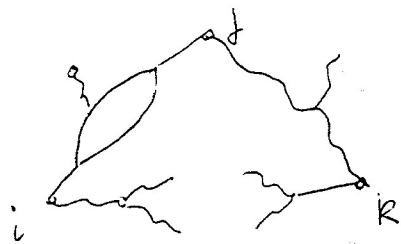
=



$$R_{ik} \cup P_{kj} \cup P_{ij}; a_{ij} = a_{kj} = 0$$

(b)

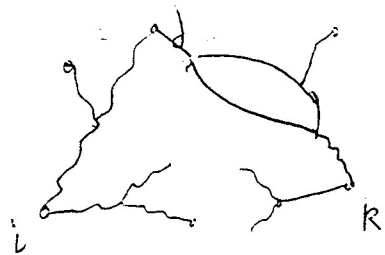
=



$$Q_{ik} \cup S_{ij} \cup P_{jk}; a_{ik} = a_{jk} = 0$$

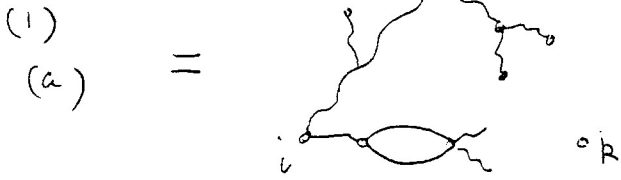
(c)

=



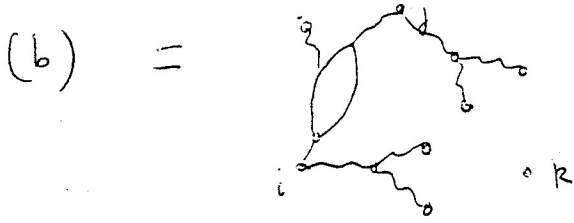
$$Q_{ik} \cup P_{ij} \cup S_{jk}; a_{ji} = a_{ki} = 0$$

$T'_{ik}$  (Cases)



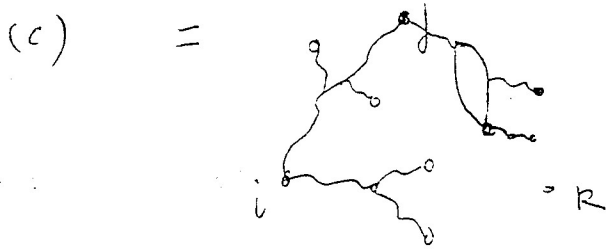
$$T_{ik} \cup P_{ij} \cup T_{jk}$$

$$a_{ij} = a_{kj} = 0$$



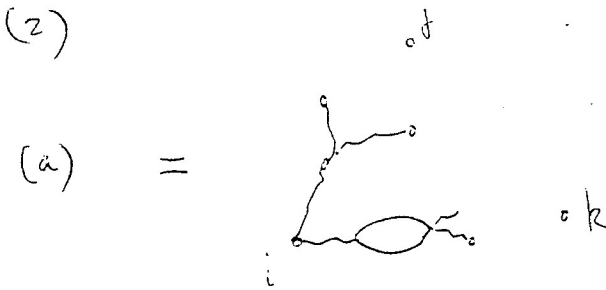
$$T_{ik} \cup S_{ij} \cup T_{jk}$$

$$a_{ik} = a_{jk} = 0$$



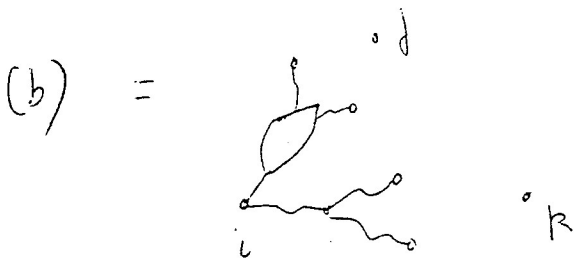
$$T_{ik} \cup P_{ij} \cup T_{jk}$$

$$a_{ji} = a_{ki} = 0$$



$$T_{ik} \cup T_{ij}$$

$$m_{jk} = 0, a_{ij} = 0$$



$$T_{ik} \cup T_{ij}$$

$$m_{jk} = 0, a_{ik} = 0$$

Cases for  $U_{ik}$

(1)

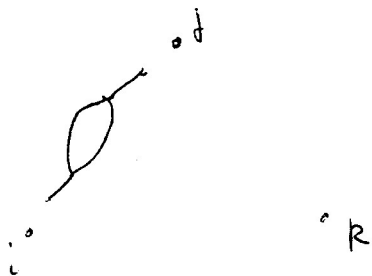
(a) =



$U_{ik}$

$m_{ij} = m_{jk} = 0$

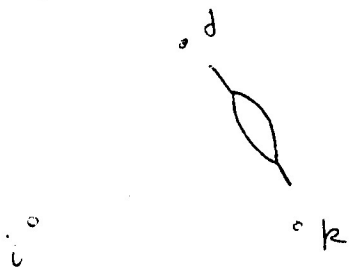
(b) =



$U_{ij}$

$m_{ik} = m_{jk} = 0$

(c) =

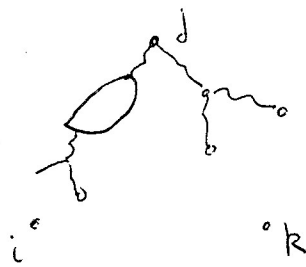


$U_{jk}$

$m_{ij} = m_{ik} = 0$

(2)

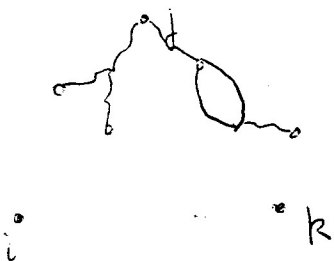
(a) =



$T_{ji} \cup T_{jk}$

$m_{ik} = 0, a_{jk} = 0$

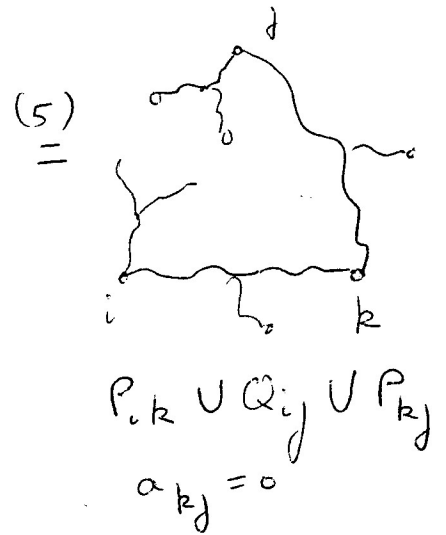
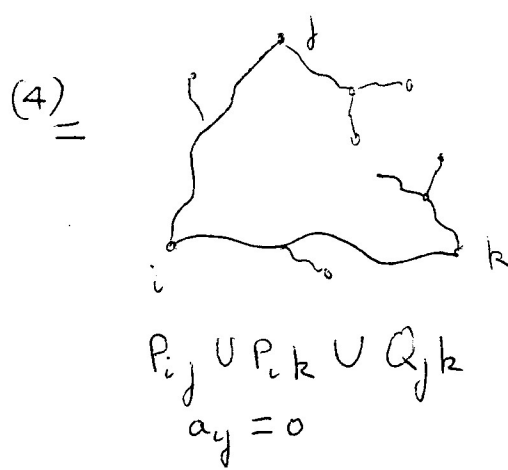
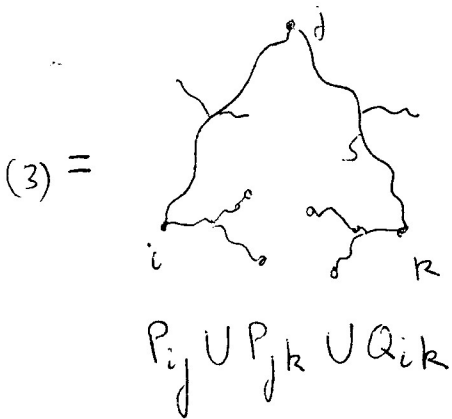
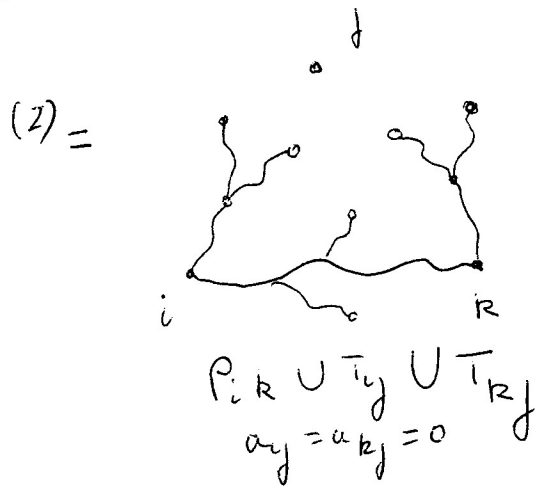
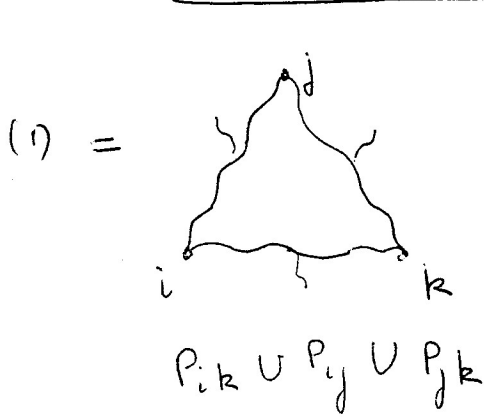
(b) =



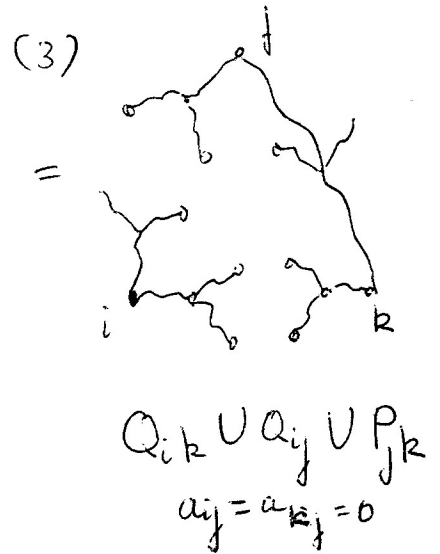
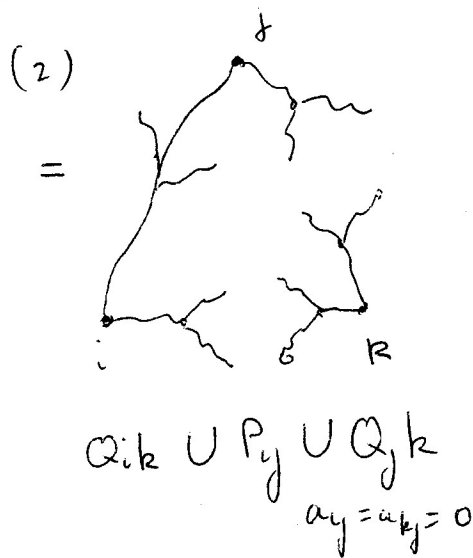
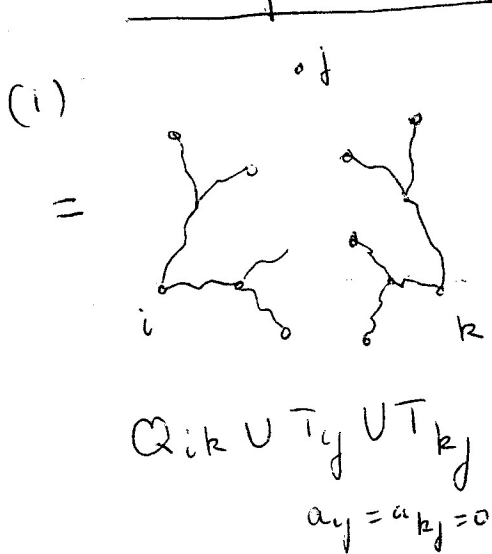
$T_{ji} \cup T_{jk}$

$m_{ik} = 0, a_{ji} = 0$

Cases for  $P'_{ik}$



Cases for  $Q'_{ik}$



# Cases for $R_{ik}$

(1)

(a)

$R_{ik} \cup T_{iy} \cup T_{kj}$   
 $a_{iy} = a_{kj} = 0$

(b)

$Q_{ik} \cup T_{iy} \cup T_{kj}$   
 $a_{ik} = a_{jk} = 0$

(c)

$Q_{ik} \cup T_{iy} \cup T_{kj}$   
 $a_{ji} = a_{ki} = 0$

(2)

(a)

$R_{ik} \cup P_{iy} \cup Q_{jk}$   
 $a_{iy} = a_{kj} = 0$

(b)

$Q_{ik} \cup S_{iy} \cup Q_{jk}$   
 $a_{ik} = a_{jk} = 0$

(c)

$Q_{ik} \cup Q_{iy} \cup S_{jk}$   
 $a_{ji} = a_{ki} = 0$

(3)

(a)

$Q_{ik} \cup P_{iy} \cup R_{jk}$   
 $a_{ji} = a_{ki} = 0$

(b)

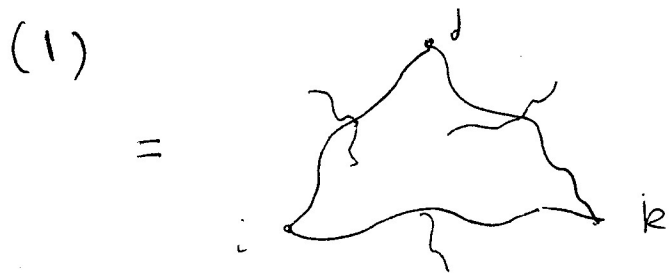
$R_{ik} \cup Q_{iy} \cup P_{jk}$   
 $a_{iy} = a_{kj} = 0$

(c)

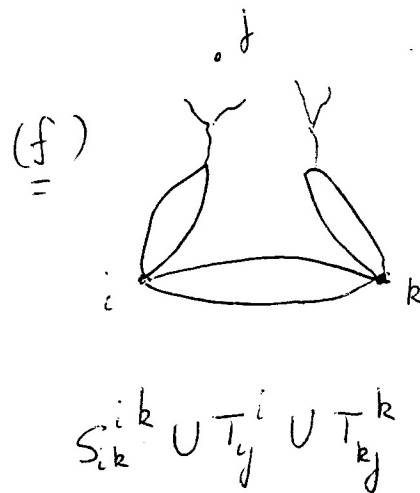
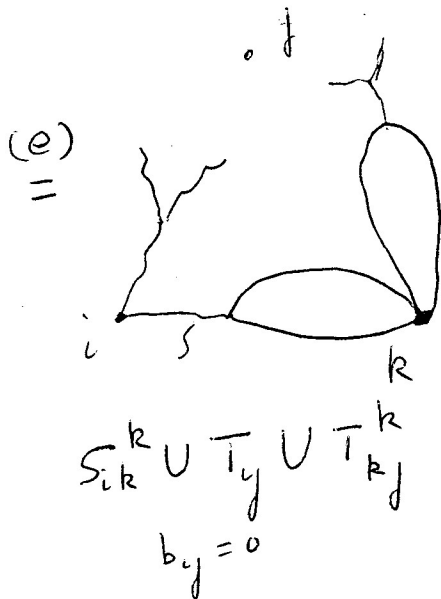
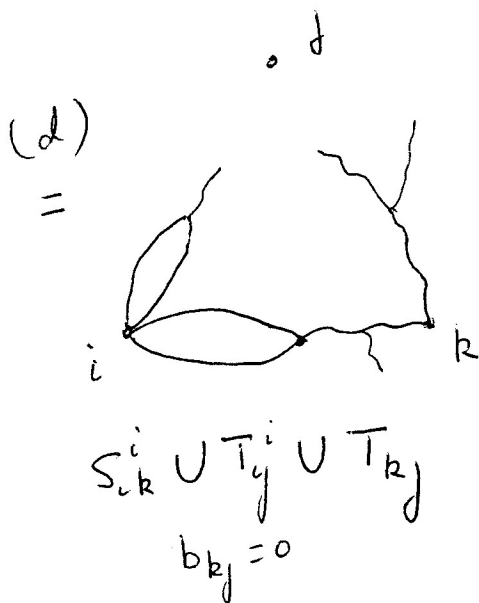
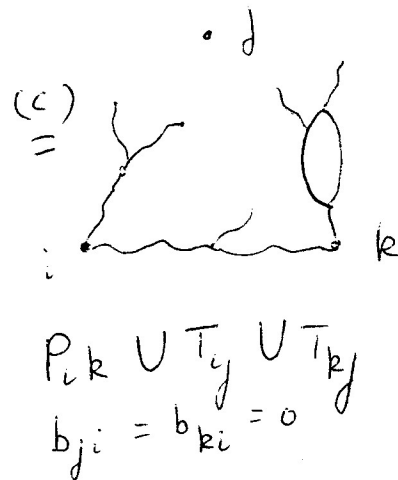
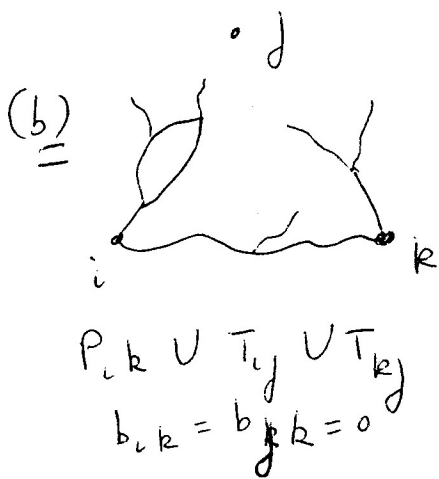
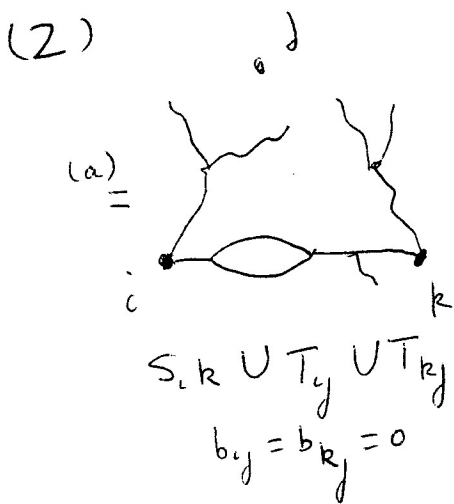
$Q_{ik} \cup R_{iy} \cup P_{jk}$   
 $a_{ik} = a_{jk} = 0$

# EDGE CONNECTIVITY CASE

## Cases for $S_{ik}$

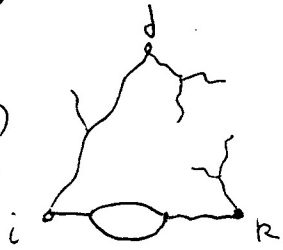


$$P_{ik} \cup P_{yd} \cup P_{jk}$$



(3)

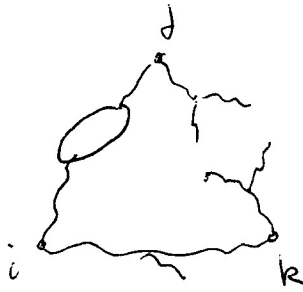
(a)  
=



$$S_{ik} \cup P_{ij} \cup Q_{kj}$$

$$b_{ij} = b_{kj} = 0$$

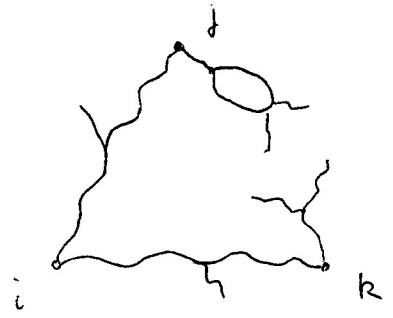
(b)  
=



$$P_{ik} \cup S_{ij} \cup Q_{kj}$$

$$b_{ik} = b_{jk} = 0$$

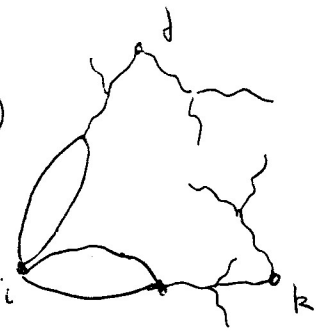
(c)  
=



$$P_{ik} \cup P_{ij} \cup R_{jk}$$

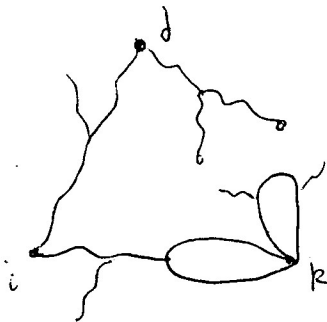
$$b_{ji} = b_{ki} = 0$$

(d)  
=



$$S_{ik}^i \cup S_{ij}^i \cup Q_{kj}$$

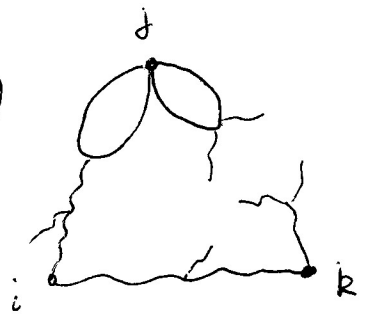
(e)  
=



$$S_{ik}^k \cup P_{ij} \cup Q_{kj}^k$$

$$b_{ij} = 0$$

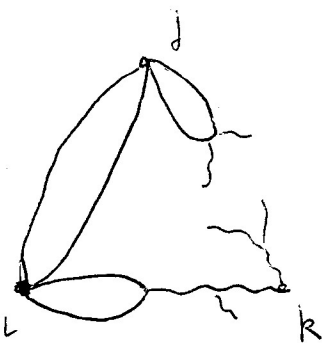
(f)  
=



$$P_{ik} \cup S_{ij}^d \cup Q_{jk}^d$$

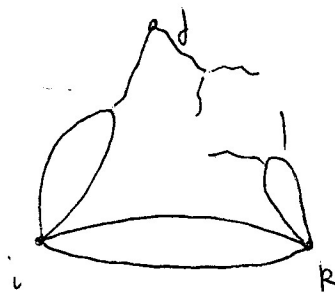
$$b_{ik} = 0$$

(g)  
=



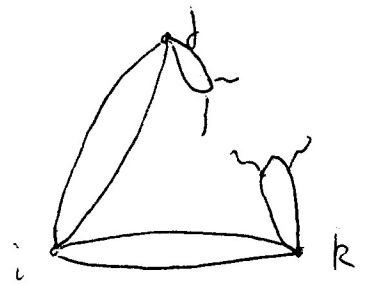
$$S_{ik}^i \cup S_{ij}^{id} \cup Q_{jk}^d$$

(h)  
=



$$S_{ik}^{ik} \cup S_{ij}^i \cup Q_{kj}^k$$

(i)  
=



$$S_{ik}^{ik} \cup S_{ij}^{id} \cup Q_{jk}^{ik}$$

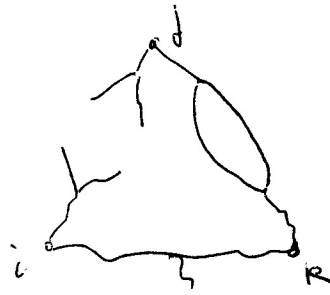
(4)

(a)  
=



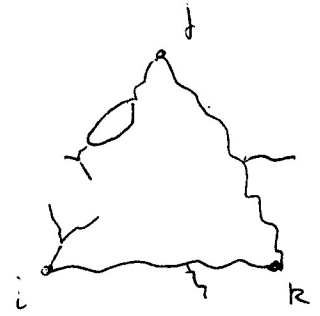
$$S_{ik} \cup Q_{ij} \cup P_{kj} \\ b_{ij} = b_{kj} = 0$$

(b)  
=



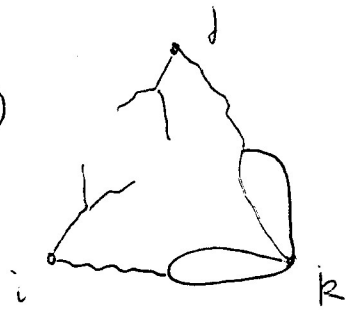
$$P_{ik} \cup Q_{ij} \cup S_{kj} \\ b_{ki} = b_{ji} = 0$$

(c)  
=



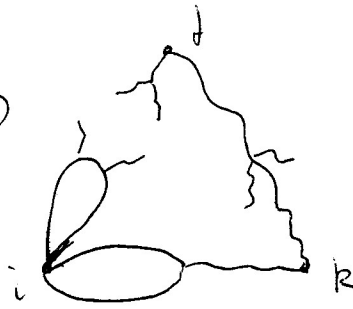
$$P_{ik} \cup P_{jk} \cup R_{ij} \\ b_{ik} = b_{jk} = 0$$

(d)  
=



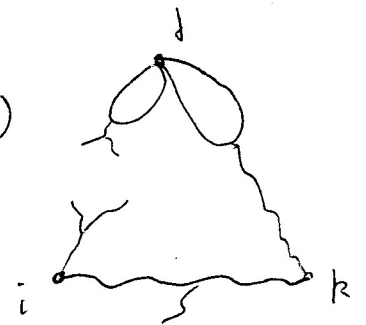
$$S_{ik}^k \cup S_{kj}^k \cup Q_{ij}$$

(e)  
=



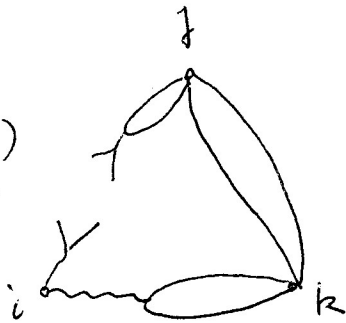
$$S_{ik}^i \cup Q_{ij}^i \cup P_{kj}$$
$$b_{kj} = 0$$

(f)  
=



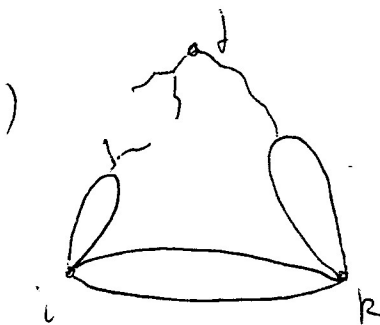
$$P_{ik} \cup Q_{ij}^d \cup S_{kj}^d \\ b_{ki} = 0$$

(g)  
=



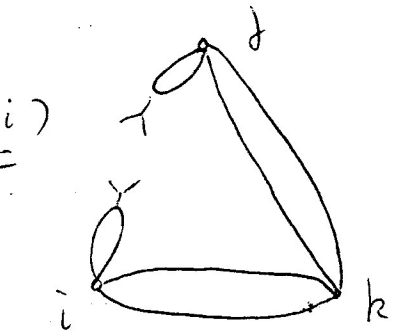
$$S_{ik}^k \cup S_{jk}^d \cup Q_{ij}^d$$

(h)  
=



$$S_{ik}^i \cup Q_{ij}^i \cup S_{kj}^k$$

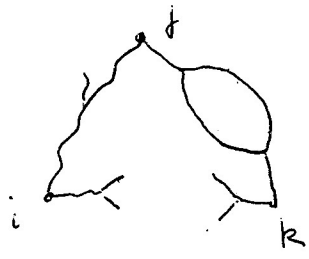
(i)  
=



$$S_{ik}^i \cup S_{jk}^k \cup Q_{ij}^d$$

(5)

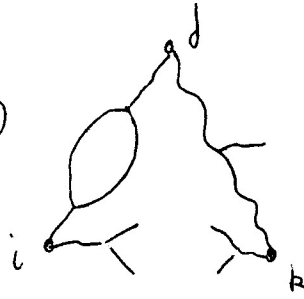
(a)



$$P_{ij} \cup S_{jk} \cup Q_{ik}$$

$$b_{ji} = b_{ki} = 0$$

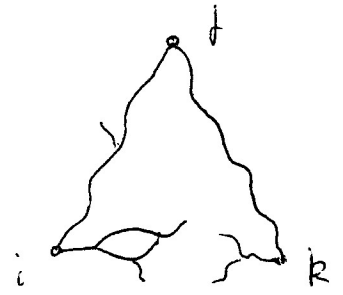
(b)



$$S_{ij} \cup P_{jk} \cup Q_{ik}$$

$$b_{ik} = b_{jk} = 0$$

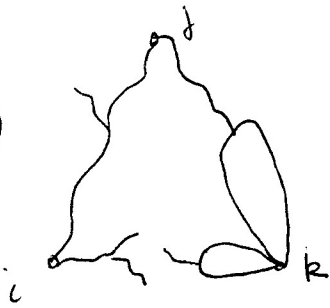
(c)



$$P_{ij} \cup P_{jk} \cup R_{ik}$$

$$b_{ij} = b_{kj} = 0$$

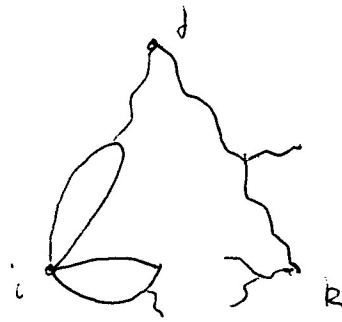
(d)



$$Q_{ki}^k \cup S_{kj}^k \cup P_{ij}^y$$

$$b_{ji} = 0$$

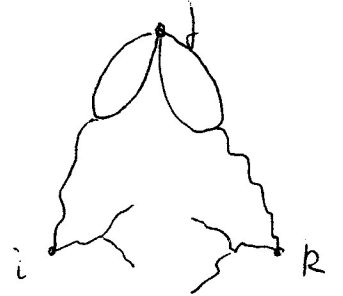
(e)



$$Q_{ik}^i \cup S_{ij}^i \cup P_{jk}^x$$

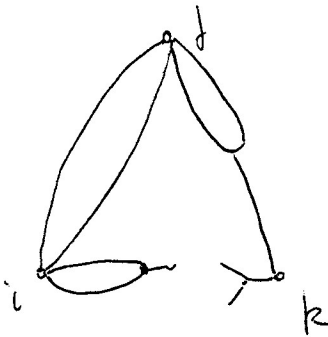
$$b_{jk} = 0$$

(f)



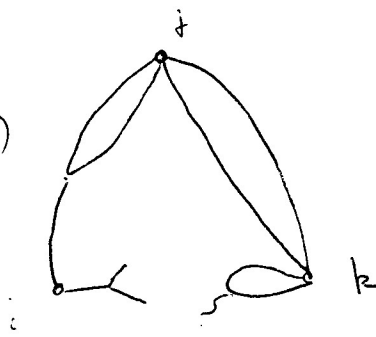
$$S_{ij}^t \cup S_{jk}^t \cup Q_{ik}$$

(g)



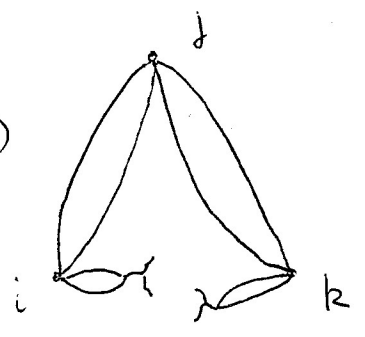
$$S_{ij}^{id} \cup S_{jk}^t \cup Q_{ik}^i$$

(h)



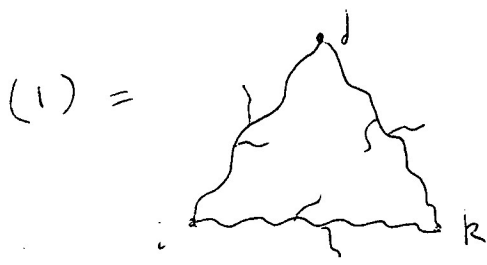
$$S_{ij}^t \cup S_{jk}^{dk} \cup Q_{ik}^k$$

(i)

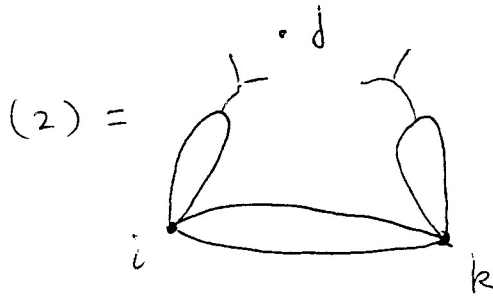


$$S_{ij}^{id} \cup S_{jk}^{dk} \cup Q_{ik}^k$$

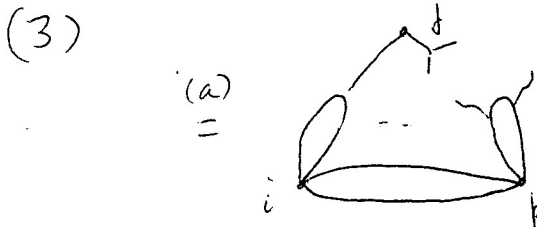
Cases for  $S_{ik}^{ik}$



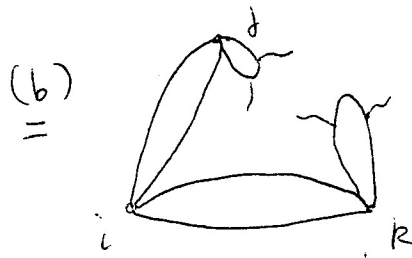
$$P_{ik} \cup P_{jd} \cup P_{jk}$$



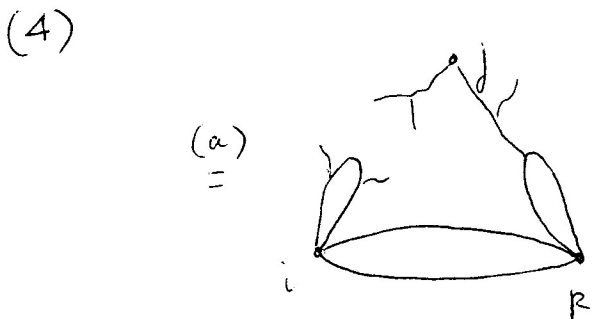
$$S_{ik}^{ik} \cup T_{ij}^i \cup T_{kj}^k$$



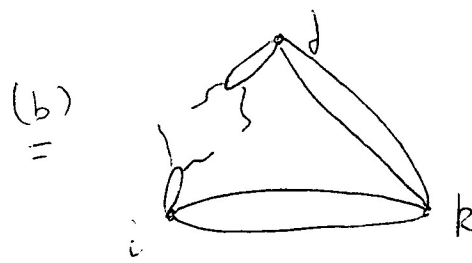
$$S_{ik}^{ik} \cup S_{ij}^i \cup Q_{kj}^k$$



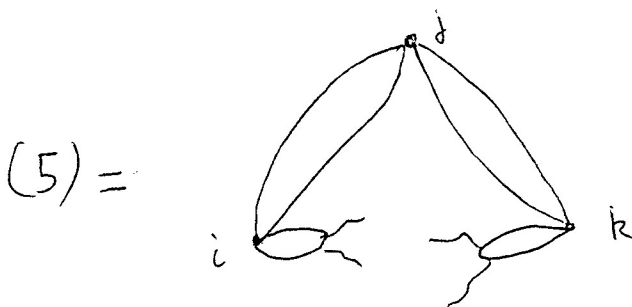
$$S_{ik}^{ik} \cup S_{ij}^{id} \cup Q_{jk}^{jk}$$



$$S_{ik}^{ik} \cup Q_{ij}^i \cup S_{jk}^{jk}$$





$$S_{ik}^{ik} \cup Q_{ij}^{id} \cup S_{jk}^{jk}$$





$$S_{ij}^{jd} \cup S_{jk}^{jk} \cup Q_{ik}^{ik}$$


Cases for  $S_{ik}^i$

(1) =   
 $P_{ik} \cup P_{yj} \cup P_{jk}$


(2) (a) =   
 $S_{ik}^i \cup T_{yj}^i \cup T_{kj}^i$   
 $b_{kj} = 0$


(b) =   
 $S_{ik}^{ik} \cup T_{yj}^i \cup T_{kj}^k$


(3) (a) =   
 $S_{ik}^i \cup S_{yj}^i \cup Q_{jk}^i$


(b) =   
 $S_{ik}^i \cup S_{yj}^{ij} \cup Q_{jk}^d$


(c) =   
 $S_{ik}^{ik} \cup S_{yj}^i \cup Q_{kj}^k$


(d) =   
 $S_{ik}^{ik} \cup S_{yj}^{ij} \cup Q_{jk}^{jk}$


(4) (a) =   
 $S_{ik}^i \cup Q_{yj}^i \cup P_{kj}^i$   
 $b_{kj} = 0$

(b) =   
 $S_{ik}^{ik} \cup Q_{yj}^i \cup S_{kj}^k$

(c) =   
 $S_{ik}^{ik} \cup Q_{yj}^{ij} \cup S_{kj}^{kj}$

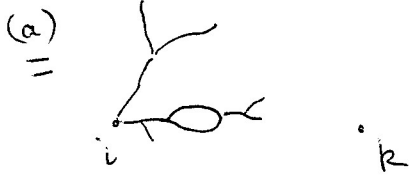
(5) (a) =   
 $S_{yj}^i \cup Q_{ik}^i \cup P_{jk}^i$   
 $b_{jk} = 0$

(b) =   
 $S_{yj}^{ij} \cup Q_{ik}^i \cup S_{jk}^d$

(c) =   
 $S_{yj}^{ij} \cup S_{jk}^{jk} \cup Q_{ik}^{ik}$

# Cases for $T'_{ik}$

(1)  $\bullet d$



$$T_{ik} \cup T_{ij}$$

$$m_{jk} = 0, b_{ij} = 0.$$



$$T_{ik} \cup T_{ij}$$

$$m_{jk} = 0, b_{ik} = 0.$$



$$T_{ij}^i \cup T_{ik}^i$$

$$m_{jk} = 0$$

(2)  $\bullet d$



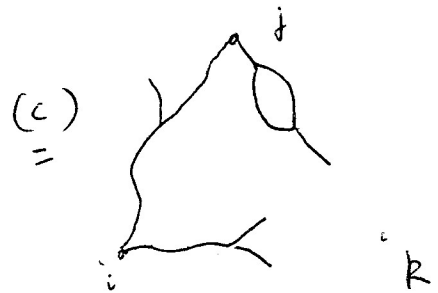
$$T_{ik} \cup P_{ij} \cup T_{jk}$$

$$b_{ij} = b_{kj} = 0.$$



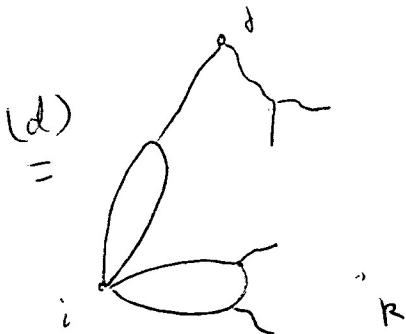
$$T_{ik} \cup S_{ij} \cup T_{jk}$$

$$b_{ik} = b_{jk} = 0.$$



$$T_{ik} \cup P_{ij} \cup T_{jk}$$

$$b_{ji} = b_{ki} = 0.$$



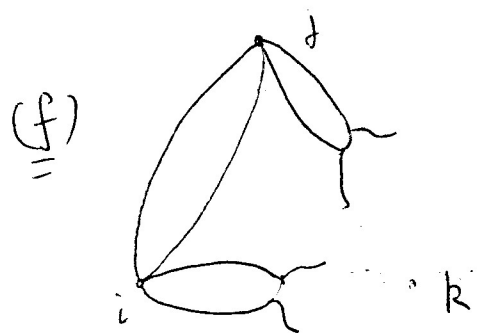
$$T_{ik}^i \cup S_{ij}^i \cup T_{jk}$$

$$b_{jk} = 0.$$



$$T_{ik} \cup S_{ij}^d \cup T_{jk}^d$$

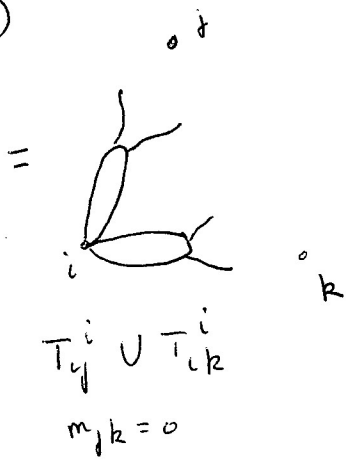
$$b_{ik} = 0.$$



$$T_{ik}^i \cup S_{ij}^d \cup T_{jk}^d$$

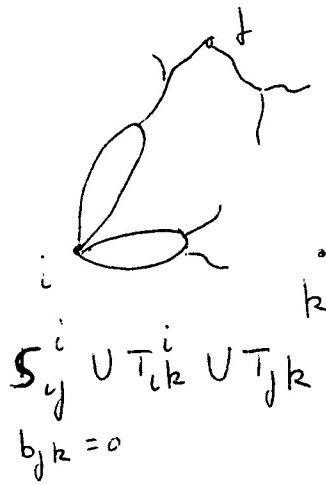
Cases for  $T_{ik}^i$

(1)

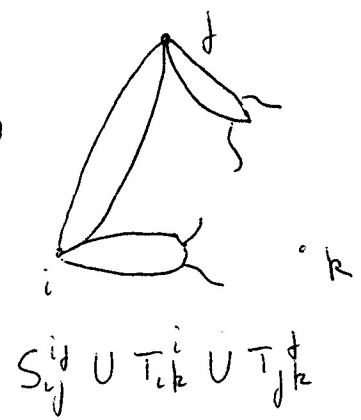


(2)

(a)



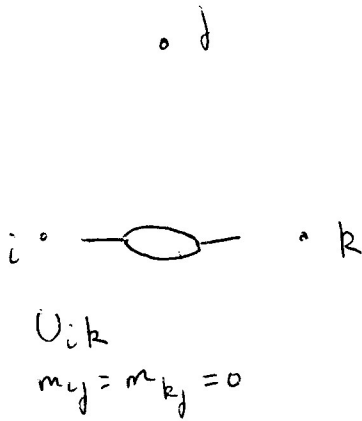
(b)



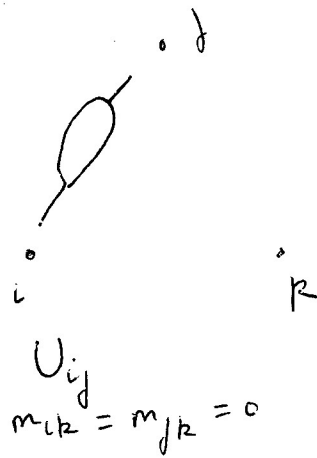
Cases for  $U_{ik}^i$

(1)

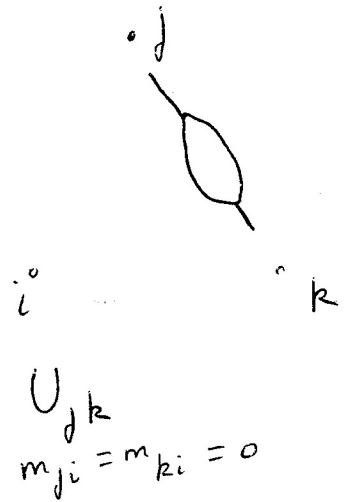
(a)



(b)

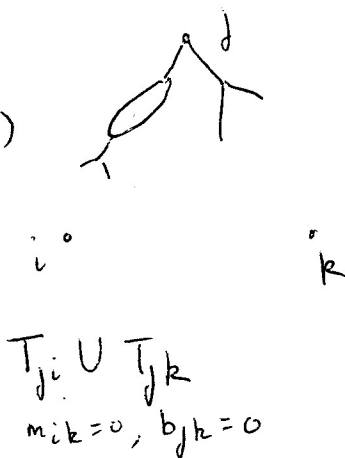


(c)



(2)

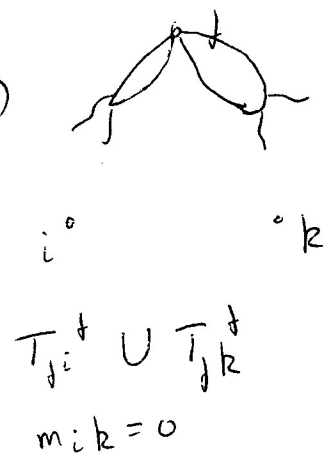
(a)



(b)



(c)



# Cases for $P_{ik}^1$

(1)

=

$$P_{ik} \cup P_{ij} \cup P_{jk}$$

(2)

=

$$P_{ik} \cup T_{ij}^i \cup T_{kj}^k$$

(3)

(a)

=

$$P_{ik} \cup S_{ij}^i \cup Q_{kj}^k$$

(b)

=

$$P_{ik} \cup S_{ij}^{id} \cup Q_{kj}^{kj}$$

(4)

(a)

=

$$P_{ik} \cup Q_{ij}^i \cup S_{kj}^k$$

(b)

=


$$P_{ik} \cup Q_{ij}^{id} \cup S_{kj}^{kj}$$

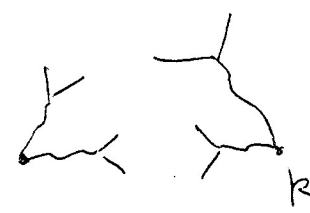
(5)

=


$$P_{ij} \cup P_{jk} \cup Q_{ik}^{ik}$$

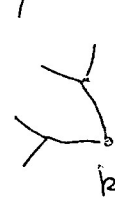
# Cases for $Q_{ik}^d$

(1) 


= 

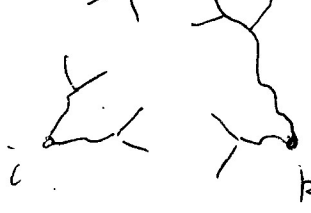
$Q_{ik} \cup T_y \cup T_{kj}$   
 $b_{ij} = b_{kj} = 0$

(2) 

= 


$Q_{ik} \cup P_y \cup Q_{jk}$   
 $b_{ij} = b_{kj} = 0$


(3) 

= 


$Q_{ik} \cup Q_y \cup P_{jk}$   
 $b_{ij} = b_{kj} = 0$

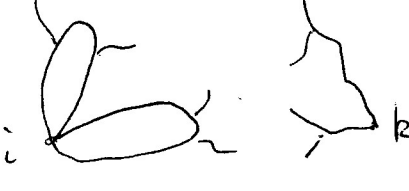
# Cases for $Q_{ik}^i$

(1) 


= 


$Q_{ik}^i \cup T_y^i \cup T_{kj}$   
 $b_{kj} = 0$

(2) 


= 


$Q_{ik}^i \cup Q_y^i \cup P_{jk}$   
 $b_{kj} = 0$

(3) (a) 

= 

$Q_{ik}^i \cup S_{ij}^i \cup Q_{jk}$

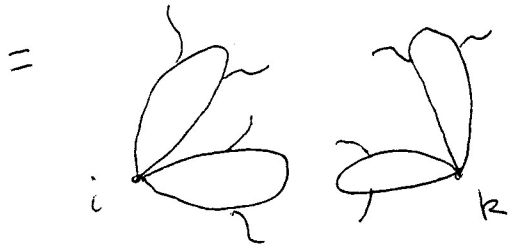
(3) (b) 

= 

$Q_{ik}^i \cup S_{ij}^{id} \cup Q_{jk}^d$

Cases for  $Q_{ik}^{jk}$

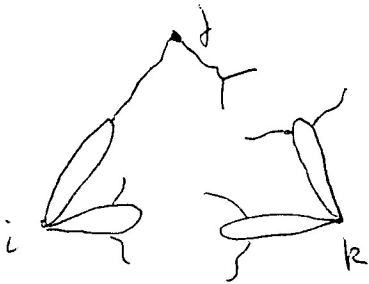
(1)  $\cdot d$



$$Q_{ik}^{jk} \cup T_{ij}^i \cup T_{kj}^k$$

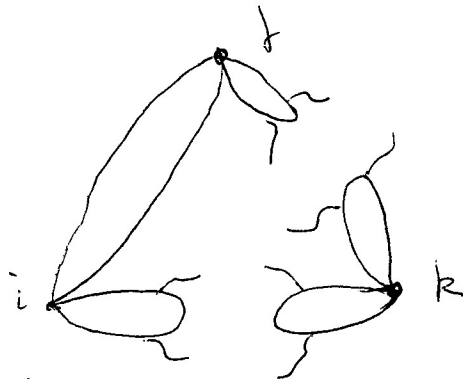
(2)

(a)



$$Q_{ik}^{jk} \cup S_{ij}^i \cup Q_{kj}^k$$

(b)



$$Q_{ik}^{jk} \cup S_{ij}^d \cup Q_{kj}^k$$

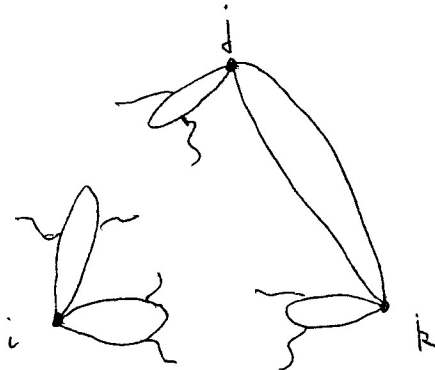
(3)

(a)



$$Q_{ik}^{jk} \cup Q_{ij}^i \cup S_{kj}^k$$

(b)



$$Q_{ik}^{jk} \cup Q_{ij}^d \cup S_{kj}^k$$

# Cases for $R_{ik}$

(1)

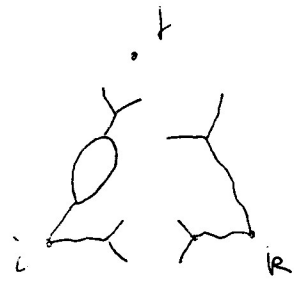
(a)



$$R_{ik} \cup T_{ij} \cup T_{kj}$$

$$b_{ij} = b_{kj} = 0$$

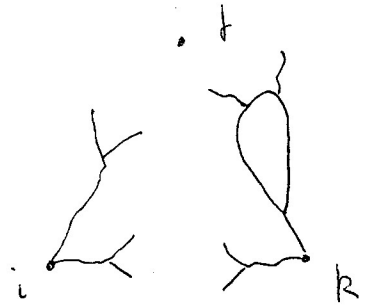
(b)



$$Q_{ik} \cup T_{ij} \cup T_{kj}$$

$$b_{ik} = b_{jk} = 0$$

(c)



$$Q_{ik} \cup T_{ij} \cup T_{kj}$$

$$b_{ki} = b_{ji} = 0$$

(d)



$$Q_{ik}^i \cup T_{ij}^i \cup T_{kj}$$

$$b_{jk} = 0$$

(e)



$$Q_{ik}^k \cup T_{ij} \cup T_{kj}^k$$

$$b_{ji} = 0$$

(2)

(a)



$$R_{ik} \cup Q_{ij} \cup P_{kj}$$

$$b_{ij} = b_{kj} = 0$$

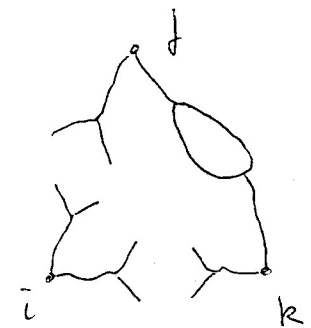
(b)



$$Q_{ik} \cup R_{ij} \cup P_{kj}$$

$$b_{ik} = b_{jk} = 0$$

(c)



$$Q_{ik} \cup Q_{ij} \cup S_{kj}$$

$$b_{ji} = b_{ki} = 0$$

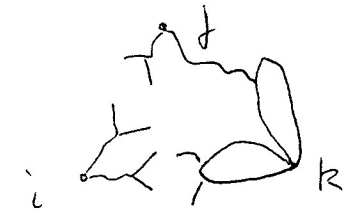
(d)



$$Q_{ik}^i \cup Q_{ij}^i \cup P_{kj}$$

$$b_{jk} = b_{kj} = 0$$

(e)



$$Q_{ik}^k \cup Q_{ij} \cup S_{kj}^k$$

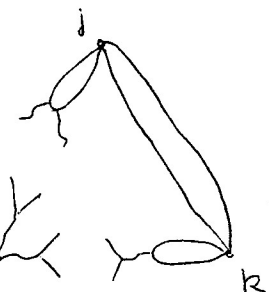
$$b_{ji} = 0$$

(f)



$$Q_{ik} \cup Q_{ij}^d \cup S_{kj}^d$$

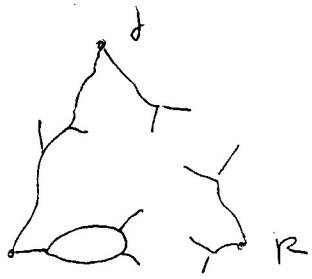
$$b_{ki} = 0$$

(g) 

=

$Q_{ik}^k \cup Q_{ij}^j \cup S_{kj}^k$

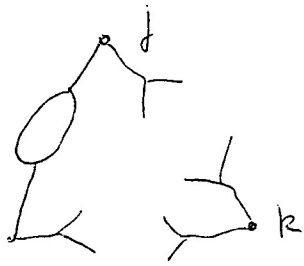
$\gamma_i = 0$

(3) (a) 

=

$R_{ik} \cup P_{ij} \cup Q_{jk}$


$b_{ij} = b_{kj} = 0$

(b) 

=

$Q_{ik} \cup S_{ij} \cup Q_{jk}$


$b_{ik} = b_{jk} = 0$

(c) 

=

$Q_{ik} \cup P_{ij} \cup R_{jk}$

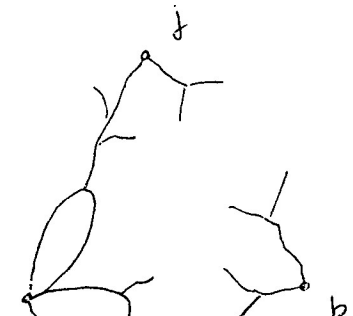
$b_{ji} = b_{ki} = 0$

(d) 

=

$Q_{ik}^k \cup P_{ij} \cup Q_{jk}^k$

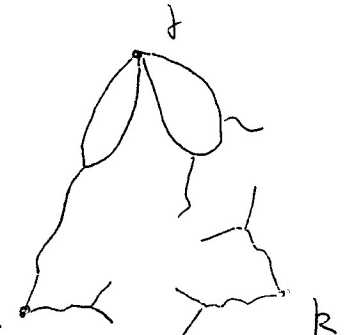
$b_{ij} = b_{ji} = 0$

(e) 

=

$Q_{ik}^i \cup S_{ij}^i \cup Q_{jk}$

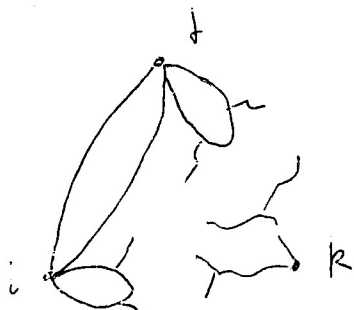
$b_{jk} = 0$

(f) 

=

$Q_{ik} \cup S_{ij}^j \cup Q_{jk}^j$

$b_{ik} = 0$

(g) 

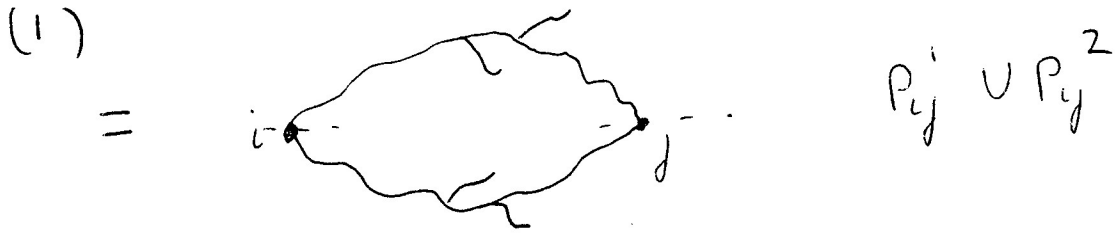
=

$Q_{ik}^i \cup S_{ij}^j \cup Q_{jk}^j$

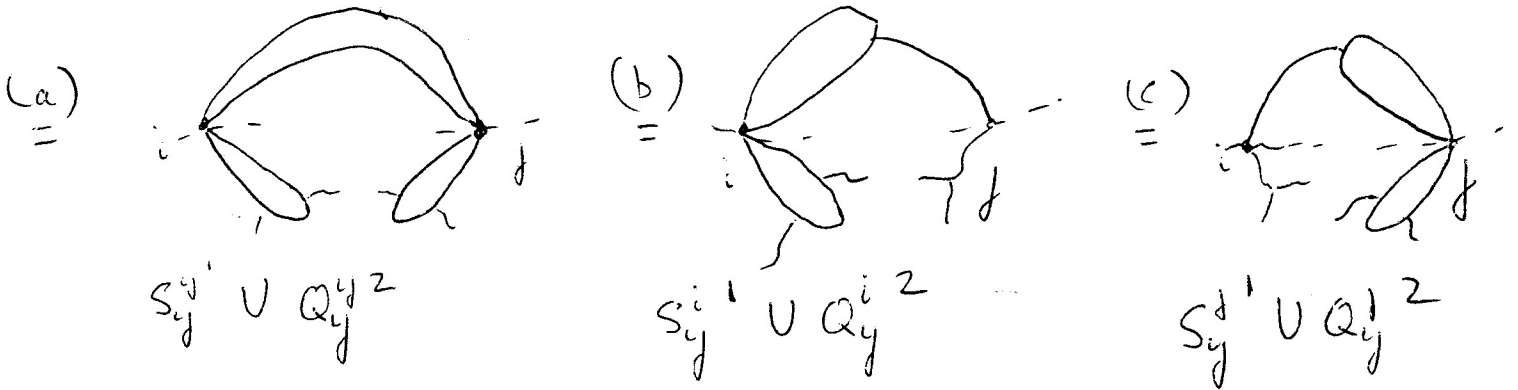
$\gamma_k = 0$

Decompositions for edge connectivity case.

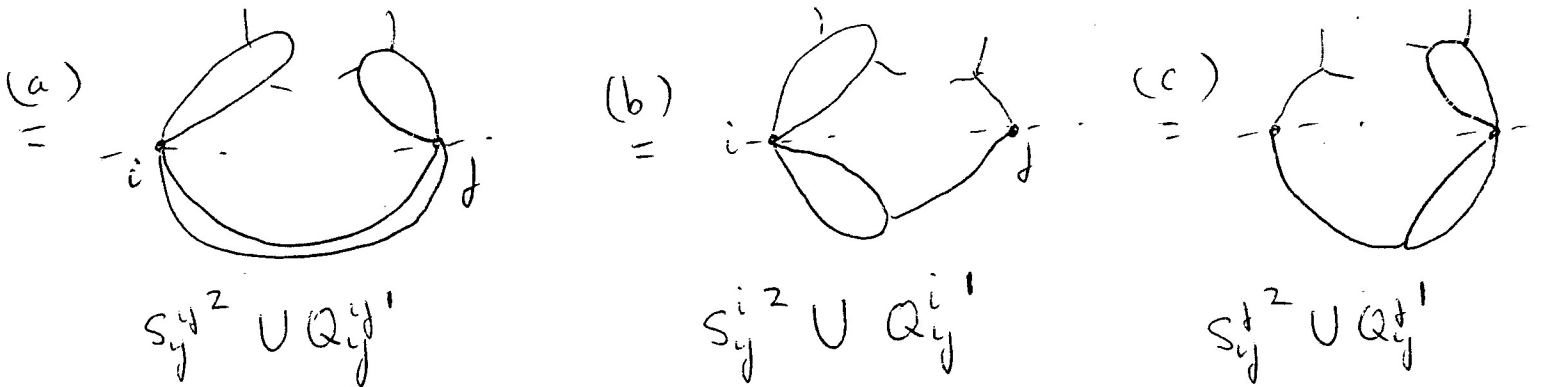
Upper graph =  $G_1$ , lower graph =  $G_2$ . Separated by ---



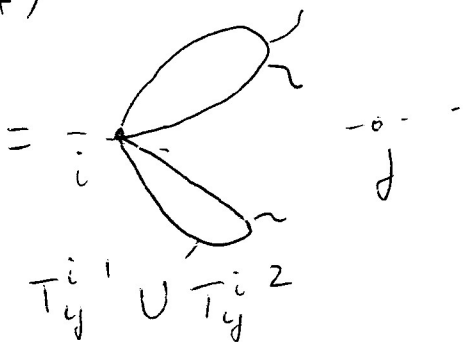
(2)



(3)



(4)



(5)

