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Branch and Price for WDM Optical Networks with No Bifurcation of Flow

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The second generation of optical networks with wavelength division multiplexing (WDM) is based on the notion of two layer networks, where the first layer represents a logical topology defined over the physical topology of optical fibers and the second layer represents multiple traffic requests combined (multiplexed) over the paths established in the logical topology. Because the design of both of these layers is challenging by itself, researchers have mainly focused on solving these problems either independently or in a sequential fashion. In this paper, we look at the WDM optical network design problem with nonbifurcated traffic flows and propose an exact branch-and-price procedure that simultaneously solves logical topology design and traffic routing over the established logical topology. The unique feature of the proposed algorithm is that it works with a row-incomplete mathematical formulation and two types of variables that exponentially grow in number with the problem size. We discuss computational issues related to the use of this procedure and propose two approximate branch-and-price procedures that can be used to obtain lower and upper bounds for this problem. Finally, we present the results of our computational experiments for two design objectives and alternative optical network settings.

Key words: optical networks; multilayer network design; network flows; mixed-integer programming models; branch-and-price procedure
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1. Introduction

The wavelength division multiplexing (WDM) optical network design problem with no bifurcation of flow is a two-layer version of a well-known origin–destination integer multicommodity flow (ODIMCF) problem. In this problem, we are given a graph $G = (V, A, \Omega)$, with a set of vertices V with a limited in-degree and out-degree, a set of arcs A with a limited capacity, and a set of pairwise demands Ω. We need to construct a virtual network and route the set of demands Ω over the virtual network so that the individual demands are not bifurcated and one or more network design objectives are optimized. The vertex set of the virtual network is identical to the set V, and its arc set (which we refer to as virtual paths) is denoted by Λ^*. Each arc in the virtual network (i.e., each virtual path) represents a directed path over the set of arcs A. The virtual network must be constructed so that the number of virtual paths originating and terminating at any given vertex does not exceed the out- and in-degree (respectively) of that vertex, and the number of virtual paths using any given arc $a \in A$ does not exceed the total number of virtual paths that can be supported by that arc.

This problem arises in WDM optical networks, where the graph G represents the physical layer with arcs A corresponding to optical fibers, and virtual paths Λ^* corresponding to lightpaths generated over the optical fibers. The arc capacity constraints correspond to the number of lightpaths that can be supported on each optical fiber. The degree constraints represent the number of transmitters and receivers available at each node, where a single transmitter and receiver is needed for each lightpath established over the physical layer. Finally, the entire collection of lightpaths established in the physical layer represents the logical topology Λ^* and is used to route the traffic (commodities) in the network.

The network design objectives considered for WDM optical networks vary and include minimization of the network-wide average packet delay (Mukherjee et al. 1996), maximization of the capacity reserved for network expansion (Mukherjee et al. 1996), minimization of congestion (Ramaswami and Sivarajan 1996), minimization of the number of transponders (Hu and Leida 2004), minimization of the total lost traffic (Zhu and Mukherjee 2002), and minimization of the total weighted lost traffic (Zhu and Mukherjee 2002). In this paper, we focus on the minimization of lost traffic in a network. This objective applies to WDM optical networks that allow the loss of traffic (for example, when it is possible to route traffic via alternate means if it cannot be routed over the WDM network) but require a design that provides service for as much traffic as possible. We also discuss how to apply our procedures to alternative network design objectives and present results of a computational study that focuses on minimization of the number of transmitters and receivers in a network. Our study indicates that the proposed procedures are more efficient when applied to networks where the objective is minimization of lost traffic.

In this paper, we assume that all transmitters and receivers are tunable to all wavelengths and that all nodes in the network are equipped with the wavelength changers. From the design perspective, wavelength changers are of great help because they eliminate the need to maintain the same wavelength (wavelength continuity constraint) from the origin to the destination of each commodity (instead, different wavelengths can be used for different lightpaths used by a single commodity). The assumption of wavelength conversion is driven by the results of previous studies, which indicate that wavelength conversion may be desirable in WDM networks as a way of resolving equipment compatibility issues (Banerjee and Mukherjee 2000). More details on the trade-offs between optical cross-connects with and without wavelength conversion capability can be found in Ramaswami and Sivarajan (2002). For more information of WDM optical networks without wavelength conversion, see Koster (2005), Koster and Zymolka (2005), Hu and Leida (2004), Lee et al. (2003), and Mukherjee et al. (1996).

We refer to the problem of routing the lightpaths over the physical topology as the *logical topology design* (LTD) problem, and we use the term *traffic routing* for the problem of routing of commodities over the logical topology.

The LTD problem and the traffic routing problem have been extensively studied in the literature, but only as independent problems or as a part of iterative sequential solution procedures. For example, Ramaswami and Sivarajan (1996) proposed several heuristic procedures for the LTD problem. The logical topology determined by a heuristic was then used as input for a mixed-integer program (MIP) that solves the traffic routing problem over the fixed logical topology.

Haque et al. (2002) used a column generation-based approach for the traffic routing problem over a fixed logical topology. Prathombutr et al. (2005) used a heuristic approach for solving the WDM optical network design problem by prespecifying a small number of lightpaths that can be used for LTD. Similarly, Banerjee and Mukherjee (2000) proposed an MIP for WDM optical network design problems where only a small subset of all possible lightpaths can be used.

Recently, Belotti and Malucelli (2005) proposed a column generation algorithm for a two-layer telecommunications network design problem, where the traffic can be routed over a combination of individual physical edges and virtual paths (referred to as semipaths in Belotti and Malucelli 2005). The only constraints considered in this problem are the capacities of the semipaths and the individual physical edges (if used instead of semipaths). The proposed column generation starts with a subset of lower and higher layer paths and progressively solves the problem. If the solution found by the column generation is fractional, a heuristic rounding algorithm is applied in search for a feasible integer solution. The column generation is then repeated using the modified reduced costs that favor the use of physical edges not close to their capacity.

Current literature suggests that there has not been much work in terms of exact procedures for simultaneous logical topology design and traffic routing. One such procedure was proposed by Sung and Song (2003). However, in their setting there are no restrictions at the network nodes in terms of equipment available, and the capacity of lightpaths is not considered to be a constraint. It is also assumed that at most one logical path can be established between any two nodes in the network. With these assumptions, Sung and Song develop a simple branching procedure that they report had no impact on the pricing problem solved with column generation. However, the branch-and-price procedure developed in Sung and Song is based on an assertion that the unit integer multicommodity flow problem is polynomially solvable. This assertion is not true, as it can be shown that this problem is NP-hard (see Garg et al. 1997 for a special case of the unit integer multicommodity flow problem in tree networks).

Raghavan and Stanojević (2006) proposed an exact branch-and-price procedure for the design of more general WDM optical networks where bifurcation of flow is allowed and where the number of lightpaths that can be established between any two nodes in the network is not prespecified. In this paper, we address WDM optical networks that require the traffic flows to be routed without bifurcation and propose one exact and two approximate branch-and-price procedures for this problem. We show that the requirement for no bifurcation of flow significantly changes the complexity of the mathematical formulation used in Raghavan and Stanojević (2006) and further complicates the development of the branch-and-price techniques for WDM optical network design.

The rest of this paper is organized as follows. In the next section, we present a mathematical formulation for WDM optical networks with no bifurcation of flow. In §3, we propose an exact branch-and-price algorithm for this problem and discuss computational issues related to this procedure. We also propose a branch-and-price algorithm that can be used to obtain upper bounds for this problem and discuss how the proposed procedures can be applied to optical network design problems with alternative network design objectives. In §4, we propose a modification of the branch-and-price algorithm proposed in Raghavan and Stanojević (2006) that can be used to obtain lower bounds for the design of WDM optical networks without bifurcation of flow. The results of our computational experiments are provided in §5. They indicate that the combined use of our lower bound and upper bound approximate branch-and-price procedures provides better results than our exact branch-and-price procedure alone. Section 6 provides concluding remarks.

2. Path-Based Mathematical Formulation for the WDM Optical Network Design Problem

One way to formulate the WDM optical network design problem is to use arc-based variables both for the definition of the logical topology and for the routing of traffic demands. (The arc-based variables used for the definition of the logical topology represent a set of multicommodity flow variables that determine the routing of the logical topology variables, or lightpaths, on the physical network, whereas the arc-based variables used for the routing of traffic demands represent a set of multicommodity flow variables that determine routing of traffic over the logical topology variables.) One such formulation was proposed by Banerjee and Mukherjee (2000). However, to maintain the tractability of the formulation size, they simplified the formulation in the sense that only a small subset of all possible lightpaths was considered. Even with this simplification, they found that the formulation with nonbifurcated flows required very long solution times, and they only reported computational results for networks with bifurcated flows.

Given that the arc-based formulation for the WDM optical network design problem quickly becomes computationally intractable as the size of the network increases, we develop a column generation-based procedure using the path-based mathematical formulation that we describe next.

We will use the following notation.

Physical Topology

$G=(V,A)$—The physical topology defined over a set of nodes V, and connected by a set of arcs (optical fibers) A.

L_{ij}—Number of wavelengths available between nodes i and j that are directly connected by optical fibers (that is, $(i,j)\in A$). When multiple fibers are available, this number represents a product of the number of fibers and the number of wavelengths available on each fiber.

Δ_t^i—Number of transmitters available at node i.

Δ_r^j—Number of receivers available at node j.

Demand

Ω—Set of demands between nodes of the network.

$T^{(s,d)}$—Total demand between nodes s and d ($(s,d)\in\Omega$), expressed as a portion of fiber capacity. (We assume that demand between any two nodes is always less than the capacity of a single fiber. This assumption is not restrictive. If demand is greater than that of a single fiber it can be split up into multiple demands between s and d.)

Lightpaths

Λ—Set of all possible lightpath origin–destination pairs. (When it is possible to set up lightpaths between all pairs of nodes in the network, we have $\Lambda=V\times V$.)

Z—Set of all possible lightpaths.

Flow Paths

$P^{(s,d)}$—Set of all possible flow paths p for commodity (s,d). (The flow paths for a commodity (s,d) are the paths used for routing of the corresponding demand for that commodity over the network defined by the lightpaths.)

Variables

We define two types of path variables, one for each of the two layers of WDM optical networks. The *local lightpath* variables provide information about the lightpath propagation path in the physical topology (we refer to these variables as *local* because, for each lightpath, we know the set of arcs, or fibers, used by the lightpath). The second type of path variables that we use are the *flow path* variables defined for each demand origin and destination node pair. Each flow path variable represents a possible path for sending the demand from its origin to its destination over the established set of lightpaths in the network.

x_z—local lightpath indicator variable that indicates whether lightpath z is used in a given solution.

$f_p^{(s,d)}$—flow path indicator variable that indicates whether flow path p is used to carry traffic demand for commodity (s,d).

$h^{(s,d)}$—lost traffic indicator variable that indicates whether demand of commodity (s,d) is lost.

MIP-PATH-LOCAL Formulation

$$\text{Min} \sum_{(s,d)\in\Omega} T^{(s,d)} h^{(s,d)} \tag{1}$$

$$\text{subject to} \sum_{z:\,O(z)=i} x_z \le \Delta_t^i \quad \forall\, i\in V, \tag{2}$$

$$\sum_{z: D(z)=j} x_z \leq \Delta_r^j \quad \forall j \in V, \tag{3}$$

$$\sum_{z: (l,m)\in z} x_z \leq L_{lm} \quad \forall (l,m) \in A, \tag{4}$$

$$x_z - \sum_{p: z\in p} f_p^{(s,d)} \geq 0 \quad \forall z \in Z,\ (s,d) \in \Omega, \tag{5}$$

$$x_z - \sum_{(s,d)\in\Omega, p: z\in p} T^{(s,d)} f_p^{(s,d)} \geq 0 \quad \forall z \in Z, \tag{6}$$

$$\sum_{p\in P^{(s,d)}} f_p^{(s,d)} + h^{(s,d)} = 1 \quad \forall (s,d) \in \Omega, \tag{7}$$

$$f_p^{(s,d)} \in \{0,1\} \quad \forall p \in P^{(s,d)},\ (s,d) \in \Omega, \tag{8}$$

$$h^{(s,d)} \geq 0 \quad \forall (s,d) \in \Omega, \tag{9}$$

$$0 \leq x_z \leq 1 \quad \forall z \in Z. \tag{10}$$

Constraint sets (2) and (3) limit the out-degree and in-degree of any node by the total number of transmitters and receivers, respectively. Constraint set (4) limits the number of lightpaths that can be carried over a fiber. (Multiple fibers between nodes are treated as a single fiber with the number of wavelengths available equal to the number of fibers multiplied by the number of wavelengths available at each fiber.) The next constraint set, (5), ensures that the traffic of any commodity can be sent on lightpath z only if that lightpath exists. Constraint set (6) ensures that the total traffic over lightpath z cannot exceed the total capacity of that lightpath. Constraint (5) is redundant in the presence of constraint (6); however, the use of this constraint strengthens the linear programming (LP) relaxation and, given that constraint (8) specifies the flow path variables are integral, allows the x_z variables to be defined as real variables (10). Finally, constraint set (7) ensures either that all the demand for a given commodity is satisfied or is entirely lost.

An important property of the MIP-PATH-LOCAL formulation is that both types of path-based variables used in this formulation (the local lightpath variables x_z and the flow path variables $f_p^{(s,d)}$) grow exponentially in number with the network size, making it impractical to include all the paths for either of these variables. Consequently, we propose a strategy that dynamically generates both types of variables using column generation.

3. Branch-and-Price Framework for the WDM Optical Network Design Problem

In this section we describe our branch-and-price framework for the WDM optical network design problem considered in this paper. In particular, we identify the challenges associated with column generation with two types of variables in row-incomplete formulations. We explain why developing an exact column generation procedure for the MIP-PATH-LOCAL formulation is a significant challenge that is difficult to solve. As a result, we propose an approximate column generation procedure and branch-and-price procedure for the MIP-PATH-LOCAL formulation that can be used as an upper bounding procedure. We also consider a weaker version of the MIP-PATH-LOCAL formulation that we refer to as MIP-PATH-LOCALw. For this latter formulation we propose a column generation and branch-and-price procedure that solve the problem to optimality.

3.1. Column Generation for the WDM Optical Network Design Problem

In general, having two types of variables in the column generation framework is not a problem, as long as the restricted master problem is row complete (that is, the number of constraints in the restricted master problem remains the same regardless of the number of variables dynamically added to the model). WDM optical network design problems that deal with the dynamic generation of two types of variables and have row-complete formulations are therefore relatively straightforward, as the applied column generation techniques do not significantly differ from the standard column generation techniques used for the formulations with a single type of variable dynamically added to a model (see Raghavan and Stanojević 2006 for an example).

However, the omission of any of the lightpath variables x_z from the restricted master problem of the MIP-PATH-LOCAL formulation makes the restricted master problem row incomplete (constraints (5) and (6) are specified only for those lightpaths that are included in the restricted master problem). This significantly complicates column generation because the standard pricing scheme used in column generation may end up being of little help when the master problem is row incomplete. The issue can be best explained by looking at the computation of reduced cost for the lightpath and flow path variables in the MIP-PATH-LOCAL formulation.

First, let the following dual variables correspond to the constraints of the MIP-PATH-LOCAL formulation:

- nonpositive α_i and β_j variables for constraints (2) and (3), respectively;
- nonpositive $\delta_{(l,m)}$ variables for constraint (4);
- nonnegative $r_z^{(s,d)}$ variables for constraint (5);
- nonegative v_z variables for constraint (6);
- unrestricted in sign $w^{(s,d)}$ variables for constraint (7); and
- nonpositive u_z variables for constraint (10).

Also, let Z^E denote the set of all lightpaths that are included in the restricted master problem, and let Z^N denote the set of all lightpaths that are not included in the restricted master problem. Similarly, let P^E denote the set of all flow paths that are included in the restricted master problem, and let P^N denote the set of all flow paths that are not included in the restricted master problem.

The reduced cost of any lightpath variable x_z with an origin at node $O(z)$ and destination at node $D(z)$ is then

$$RC_z = -\alpha_{O(z)} - \beta_{D(z)} - \sum_{(l,\,m)\in z} \delta_{(l,\,m)} - \sum_{(s,\,d)\in\Omega} r_z^{(s,\,d)} - v_z - u_z. \quad (11)$$

The reduced cost of any flow path variable $f_p^{(s,\,d)}$ for a commodity (s, d) is

$$RC_p^{(s,\,d)} = \sum_{z\in p,\, z\in Z^E} (r_z^{(s,\,d)} + T^{(s,\,d)} v_z) - w^{(s,\,d)}. \quad (12)$$

Proposition 1. *Column generation with independent pricing of lightpath variables x_z and flow path variables $f_p^{(s,\,d)}$ using reduced costs* (11) *and* (12) *does not provide a provably optimal solution for the linear relaxation of the MIP-PATH-LOCAL formulation.*

Proof. Note that if a lightpath z and the corresponding variable x_z are not already part of the restricted master model, dual variables $r_z^{(s,\,d)}$, v_z, and u_z can be set to zero, as the corresponding constraints are not in the restricted master model. Thus, the reduced cost RC_z becomes

$$RC_z = -\alpha_{O(z)} - \beta_{D(z)} - \sum_{(l,\,m)\in z} \delta_{(l,\,m)}. \quad (13)$$

Given that dual variables $\alpha_{O(z)}$, $\beta_{D(z)}$, and $\delta_{(l,\,m)}$ are nonpositive, it follows that the reduced cost of any new lightpath can never be negative, and therefore, it is never beneficial to add a new lightpath. While the last statement is obviously not correct (we may very well need to include many new lightpaths to the model), from a computational perspective this is not surprising, because the introduction of a new lightpath without the presence of a flow path that actually uses that lightpath has no effect on the objective function. This further implies that independent pricing of lightpath variables x_z and flow path variables $f_p^{(s,\,d)}$ using reduced costs (11) and (12) limits the linear relaxation of the MIP-PATH-LOCAL formulation to the initial set of lightpaths, therefore leading to a possible premature termination of the column generation algorithm. □

The fact that independent pricing of lightpath and flow path variables does not allow identification and introduction of new, potentially promising, lightpaths indicates that we need to define some form of a *combined reduced cost* (we use the term "combined reduced cost" to refer to the change in the objective value caused by the introduction of two or more nonbasic variables into the basis) that will determine the simultaneous effect of introduction of multiple lightpath and flow path variables. Unfortunately, finding the combined reduced cost for two or more nonbasic variables is a nontrivial task that usually does not have an efficient solution other than solving the problem with the new variables included in the restricted master model. One might incorrectly try to add up the reduced cost of the lightpath and flow path variables so that the cost of new lightpaths is directly accounted for when computing the reduced cost of flow paths. The resulting reduced cost of a flow path would have the following form:

$$RC_p^{(s,\,d)} = \sum_{z\in p,\, z\in Z^N} \left(-\alpha_{O(z)} - \beta_{D(z)} - \sum_{(l,\,m)\in z} \delta_{(l,\,m)}\right) + \sum_{z\in p,\, z\in Z^E} (r_z^{(s,\,d)} + T^{(s,\,d)} v_z) - w^{(s,\,d)}. \quad (14)$$

Note that, if a flow path variable $f_p^{(s,\,d)}$ has a negative reduced cost $RC_p^{(s,\,d)}$ specified by Equation (14), the flow path and any new lightpaths used by this flow path would be added to the model. The problem with this approach is that new lightpath(s) are introduced into the model only if their introduction can be justified by introduction of a single new flow path. This, however, is not always the case, as it is possible that the introduction of a new lightpath can be justified only if we introduce multiple new flow paths. As in the case of independent pricing of lightpath and flow path variables, this implies that some good lightpath and flow path variables may never get a chance to be introduced into the model, therefore leading to a possible premature termination of the column generation algorithm. Given this observation, the following proposition holds.

Proposition 2. *Column generation with simultaneous pricing of lightpath variables x_z and flow path variables $f_p^{(s,\,d)}$ using the combined reduced cost* (14) *does not provide a provably optimal solution for the linear relaxation of the MIP-PATH-LOCAL formulation.*

Given that straightforward approaches for simultaneous pricing of the lightpath and flow path variables do not guarantee optimality of column generation performed on a row incomplete restricted master model of the linear relaxation of the MIP-PATH-LOCAL formulation, we consider a relaxed version of the MIP-PATH-LOCAL formulation that does not include constraint set (5) (we will refer to this formulation as MIP-PATH-LOCALw) and propose an alternative column generation approach that eliminates the need for independent pricing of new lightpath variables altogether.

We will show that column generation with the reduced cost of flow path variables computed using the formula

$$RC_p^{(s,d)} = \sum_{z\in p,\, z\in Z^N} T^{(s,d)}\left(-\alpha_{O(z)} - \beta_{D(z)} - \sum_{(l,m)\in z} \delta_{(l,m)}\right) + \sum_{z\in p,\, z\in Z^E} T^{(s,d)} v_z - w^{(s,d)} \quad (15)$$

provides an optimal solution for the linear relaxation of the MIP-PATH-LOCALw formulation.

Proposition 3. *Column generation with indirect pricing of lightpath variables x_z using the combined reduced cost* (15) *for the flow path variables $f_p^{(s,d)}$ solves the linear relaxation of the MIP-PATH-LOCALw formulation.*

Proof. First, consider the formulation MIP-PATH-LOCALw and observe that the value of any new x_z variable used by any set of new flow paths $p \in P^N$ in the linear relaxation of the MIP-PATH-LOCALw formulation is

$$x_z = \sum_{(s,d)\in\Omega,\, p: z\in p} T^{(s,d)} f_p^{(s,d)} \quad \forall z \in Z^N. \quad (16)$$

If we substitute the new expression for x_z in the MIP-PATH-LOCALw formulation, only for those x_z variables not included in the master model, the linear relaxation of the formulation MIP-PATH-LOCALw becomes

$$\text{Min} \sum_{(s,d)\in\Omega} T^{(s,d)} h^{(s,d)} \quad (17)$$

subject to

$$\sum_{z: O(z)=i,\, z\in Z^E} x_z + \sum_{(s,d)\in\Omega,\, p: \exists z\in p: O(z)=i,\, z\in Z^N} T^{(s,d)} f_p^{(s,d)} \le \Delta_t^i \quad \forall i \in V, \quad (18)$$

$$\sum_{z: D(z)=j,\, z\in Z^E} x_z + \sum_{(s,d)\in\Omega,\, p: \exists z\in p: D(z)=j,\, z\in Z^N} T^{(s,d)} f_p^{(s,d)} \le \Delta_r^j \quad \forall j \in V, \quad (19)$$

$$\sum_{z: (l,m)\in z,\, z\in Z^E} x_z + \sum_{(s,d)\in\Omega,\, p: \exists z\in p: (l,m)\in z,\, z\in Z^N} T^{(s,d)} f_p^{(s,d)} \le L_{lm} \quad \forall (l,m) \in A, \quad (20)$$

$$x_z - \sum_{(s,d)\in\Omega,\, p: z\in p} T^{(s,d)} f_p^{(s,d)} \ge 0 \quad \forall z \in Z^E, \quad (21)$$

$$\sum_{p\in P^{(s,d)}} f_p^{(s,d)} + h^{(s,d)} = 1 \quad \forall (s,d) \in \Omega, \quad (22)$$

$$f_p^{(s,d)} \ge 0 \quad \forall p \in P^{(s,d)},\ (s,d) \in \Omega, \quad (23)$$

$$h^{(s,d)} \ge 0 \quad \forall (s,d) \in \Omega, \quad (24)$$

$$0 \le x_z \le 1 \quad \forall z \in Z. \quad (25)$$

Notice that there is no need for constraint (21) when it comes to new flow paths $p \in P^N$ given our condition (16). This allows the use of the combined reduced cost (15) for straightforward pricing of new $f_p^{(s,d)}$ variables that allows introduction of a single new flow path variable (and possibly multiple new lightpath variables). □

Although we can use the last proposition to find the optimal solution for the linear relaxation of the MIP-PATH-LOCALw formulation, we recognize that the linear relaxation of the MIP-PATH-LOCALw formulation is weaker than the linear relaxation of the MIP-PATH-LOCAL formulation (recall that MIP-PATH-LOCALw is obtained from MIP-PATH-LOCAL by removing constraint (5)). In addition, because we do not have constraint (5) in the MIP-PATH-LOCALw formulation, we need to define the x_z variables as integers, which adds to the complexity of the branching procedures for this formulation.

Naturally, it would be desirable to keep constraint set (5) and try to apply a column generation approach similar to the one proposed for the MIP-PATH-LOCALw. The problem with this approach is that the value of any x_z variable in the linear relaxation of the MIP-PATH-LOCAL formulation is a nonlinear term,

$$x_z = \max\left\{\max_{(s,d)\in\Omega} \sum_{p: z\in p} f_p^{(s,d)},\ \sum_{(s,d)\in\Omega,\, p:\ z\in p} T^{(s,d)} f_p^{(s,d)}\right\}, \quad (26)$$

and therefore cannot be directly implemented in the column generation framework for linear programs.

To avoid this nonlinearity and preserve the advantage of having a stronger linear relaxation, we propose the use of the following approximate value for new x_z variables within the column generation framework for the MIP-PATH-LOCAL formulation (to determine the impact of introducing a new flow path variable):

$$x_z = \sum_{(s,d)\in\Omega,\, p: z\in p} f_p^{(s,d)}. \quad (27)$$

Using arguments similar to those provided in the proof of Proposition 3, it follows that using expression (27) for new lightpath variables in the linear relaxation of the MIP-PATH-LOCAL formulation leads to column generation with simultaneous pricing of lightpath variables x_z and flow path variables $f_p^{(s,d)}$ using the combined reduced cost (14). As stated in Proposition 2, this approach may lead to overestimation of the reduced cost, which could cause omission of columns that might further improve the value of the objective function. However, the benefit of this type of approximate computation of the reduced cost is that, when used in combination with a valid branching strategy, this approach provides a valid upper bound for the original problem. We discuss one such strategy in the next section, where we first describe the details of our column generation approach for both MIP-PATH-LOCALw and MIP-PATH-LOCAL and then propose a branching procedure that can be used to obtain a provably optimal solution for MIP-PATH-LOCALw and valid upper bounds for MIP-PATH-LOCAL.

3.2. Column Generation Algorithm for the MIP-PATH-LOCALw and MIP-PATH-LOCAL Formulations

In each iteration of the column generation algorithm for the MIP-PATH-LOCALw formulation, we need to answer the following questions: *Do we need to add any new flow paths*? If we do need to add a new flow path, *should the new flow path solely use existing lightpaths, new lightpaths, or some combination of the two*? Moreover, if we do need to add (use) new lightpath(s) for the new flow path, *what is the best propagation path for the new lightpath(s)*?

These questions can be answered using several simple steps. First, recall that expression (15) was defined as a reduced cost that takes into account the combined effect of introduction of a new flow path variable $f_p^{(s,d)}$ and (possibly) multiple new lightpaths used by the same flow path variable. (We will refer to expression (15) as the reduced cost of the flow path variable, which is a slight abuse of terminology, because expression (15) captures the combined impact of introducing a new flow path variable and new lightpath variables, which this flow path uses, into the restricted master model.) Now, notice that the summation terms in (15) can be grouped by the type of lightpaths used for a given flow path as follows:

$$RC_p^{(s,d)} = \sum_{z\in p,\, z\in Z^E} \Pi^E_{z;(s,d)} + \sum_{z\in p,\, z\in Z^N} \Pi^N_{z;(s,d)} - w^{(s,d)}, \quad (28)$$

where

$$\Pi^E_{z;(s,d)} = T^{(s,d)} v_z \quad (29)$$

and

$$\Pi^N_{z;(s,d)} = -T^{(s,d)}\alpha_{O(z)} - T^{(s,d)}\beta_{D(z)} - T^{(s,d)} \sum_{(l,m)\in z} \delta_{(l,m)} \quad (30)$$

can be thought of as the *price* of an existing and a new lightpath, respectively, used by the new flow path $f_p^{(s,d)}$.

Finding a flow path with the minimum reduced cost for a given commodity (s,d) then requires solving the following pricing problem:

$$\min_{\forall p\in P:\, O(p)=s,\, D(p)=d} \left\{ \sum_{z\in p,\, z\in Z^E} \Pi^E_{z;(s,d)} + \sum_{z\in p,\, z\in Z^N} \Pi^N_{z;(s,d)} - w^{(s,d)} \right\},$$

or, equivalently,

$$\min_{\forall p\in P:\, O(p)=s,\, D(p)=d} \left\{ \sum_{z\in p,\, z\in Z^E} \Pi^E_{z;(s,d)} + \sum_{z\in p,\, z\in Z^N} \Pi^N_{z;(s,d)} \right\}. \quad (31)$$

The pricing problem (31) can be solved as a shortest path problem in an auxiliary graph $G^{(s,d)}$, where each lightpath already present in the restricted master problem represents an arc with the price $\Pi^E_{z;(s,d)}$, and each lightpath not present in the restricted master problem represents an arc with the price $\Pi^N_{z;(s,d)}$. Since the auxiliary graph $G^{(s,d)}$ may contain more than one lightpath between any two nodes (we may have many different ways to connect two nodes in the physical layer, and therefore many different lightpaths), we reduce the auxiliary graph $G^{(s,d)}$ to include only one arc for a given pair of nodes by selecting either a new or an existing lightpath with the minimum price (recall that in any shortest path, only the cheapest arc between two pairs of nodes will be selected). Now, note that for a given lightpath with origin i and destination j, a lightpath of the minimum price is the one that satisfies the following relation:

$$\Pi_{z;(s,d)} = \min\left\{ \min_{\forall z\in Z^E:\, O(z)=i,\, D(z)=j} \Pi^E_{z;(s,d)},\; \min_{\forall z\in Z^N:\, O(z)=i,\, D(z)=j} \Pi^N_{z;(s,d)} \right\}. \quad (32)$$

Finding the minimum price existing lightpath for a given pair of origin and destination nodes i and j is straightforward, and is determined by finding an existing lightpath (with $O(z)=i$ and $D(z)=j$) with the minimum $\Pi^E_{z;(s,d)} = T^{(s,d)} v_z$ value. Finding the minimum price new lightpath for a given pair of origin and destination nodes i and j is more involved because it requires finding a new lightpath that satisfies the following relation:

$$\Pi^N_{z;(s,d)} = \min_{\forall z\in Z^N:\, O(z)=i,\, D(z)=j} -T^{(s,d)}\alpha_i - T^{(s,d)}\beta_j - T^{(s,d)} \sum_{(l,m)\in z} \delta_{(l,m)}.$$

This can effectively be determined by solving the following problem:

$$\min_{\forall z\in Z^N:\, O(z)=i,\, D(z)=j} -\sum_{(l,m)\in z} \delta_{(l,m)}. \quad (33)$$

The minimum price new lightpath for a given pair of nodes i and j can therefore be found by solving a shortest path problem in a graph with a set of arcs identical to the one in the original graph G, with the cost of any single arc defined by the negation of the value of its corresponding dual variable $\delta_{(l,m)}$.

The steps of the procedure for finding the minimum price flow path for a given commodity (s,d) can be summarized as follows:

Step 1. For each pair of nodes, determine the lowest price lightpath included in the restricted master problem.

Step 2. For each pair of nodes, determine the lowest price lightpath not included in the restricted master problem.

Step 3. Construct the auxiliary graph $G^{(s,d)}$ so that there is exactly one arc between each pair of nodes (given the lightpaths selected in Steps 1 and 2, an arc added to the graph $G^{(s,d)}$ is the one with lower price).

Step 4. Solve the shortest path problem between nodes s and d in the auxiliary graph $G^{(s,d)}$.

Step 5. Compute the reduced cost (28) for commodity (s,d). If the reduced cost is negative, check whether the entering flow path variable is using any new lightpaths. If it is, add these lightpaths and the corresponding constraints (21) and (25) to the restricted master model. Finally, add the new flow path variable to the restricted master model.

This five-step procedure is repeated for all commodities, so in one iteration of the column generation algorithm we may add one new flow path variable for each commodity. The column generation algorithm continues until we cannot reduce the objective value any further—that is, when the reduced cost for each commodity (s,d) is nonnegative.

The column generation algorithm described for the MIP-PATH-LOCALw formulation can be directly applied to the MIP-PATH-LOCAL formulation as an approximate column generation procedure with three minor modifications. First, the price for using an existing lightpath z is

$$\Pi^{E}_{z;(s,d)} = r_z^{(s,d)} + T^{(s,d)} v_z. \tag{34}$$

Second, the price for using a new lightpath z with origin at node i and destination at node j is

$$\Pi^{N}_{z;(s,d)} = -\alpha_i - \beta_j - \sum_{(l,m)\in z} \delta_{(l,m)}. \tag{35}$$

Finally, if for a given commodity (s,d), the reduced cost turns out to be negative, and a new lightpath needs to be introduced, then we add the variable corresponding to that lightpath and the corresponding constraints (5), (6), and (10) to the model. (Recall that in the MIP-PATH-LOCALw formulation we never use constraint (5).) As we explained previously, this approach provides an upper bound for the linear relaxation of the MIP-PATH-LOCAL formulation and, combined with a valid branching strategy, can be used to derive valid upper bounds for the WDM optical network design problem.

Figure 1 summarizes the main steps of our pricing procedure for the formulations MIP-PATH-LOCALw and MIP-PATH-LOCAL.

3.3. Branching Strategy

One of the challenging aspects of the branch-and-price framework lies in the relationship between the branching rules and the pricing problem used to solve the LP relaxation at each node of the branch-and-bound tree. The main issue in this framework is that the pricing problem has to be modified at each branch of the branch-and-bound tree so that all applicable branching rules are taken into account. We would ideally like to have branching rules that do not require significant modifications of the pricing problem. Unfortunately, such branching rules sometimes do not exist or are difficult to identify. As we will show next, the approximate MIP-PATH-LOCAL formulation allows use of branching rules that maintain the structure of the pricing problem, whereas the exact MIP-PATH-LOCALw formulation requires branching rules that significantly increase the complexity of the pricing problem.

For the MIP-PATH-LOCAL formulation, we propose a hierarchical three-layer branching strategy for the flow path variables, with branching in two of the layers similar to the one of Barnhart et al. (2000). The hierarchical approach is key to this branching procedure, because WDM networks have two layers, and it is possible that a single flow path visits the same node more than once in the physical layer. However, for WDM optical networks, it also holds that a flow path never revisits a node in the logical topology, and a lightpath never revisits a node in the physical topology. Thus, we can design a branching strategy where we first make sure that a flow path of a given commodity has a unique path in the logical topology. Once we make sure that flow paths for all commodities satisfy this condition, we can move on to branching in the physical layer.

In addition to three levels of branching on the flow path variables, we also branch on the lost traffic variables, creating a four-level hierarchical branching approach. If the branching criteria are met at any level of branching, we perform the branching and solve the linear relaxations on the newly created branches. This is followed by selection of the next branch for column generation and subsequent branching. The procedure is repeated until the optimal solution is found.

Branching Level 1. *Lost traffic variables*: We first check whether commodities with partially lost traffic exist. If no such commodities exist, we check for any fractional flow in the logical layer (branching level 2). Otherwise, we identify all commodities with partially lost traffic and select the commodity with the greatest demand. We perform branching on the lost traffic variable $h^{(s,d)}$ of this commodity by creating two branches: on one branch, we enforce the use of this commodity (by setting variable $h^{(s,d)}$ to one); and on the other branch, we prohibit the use of this commodity (by setting variable $h^{(s,d)}$ to zero).

Branching Level 2. *Fractional flow in the logical layer*: At this level of branching, we first check whether

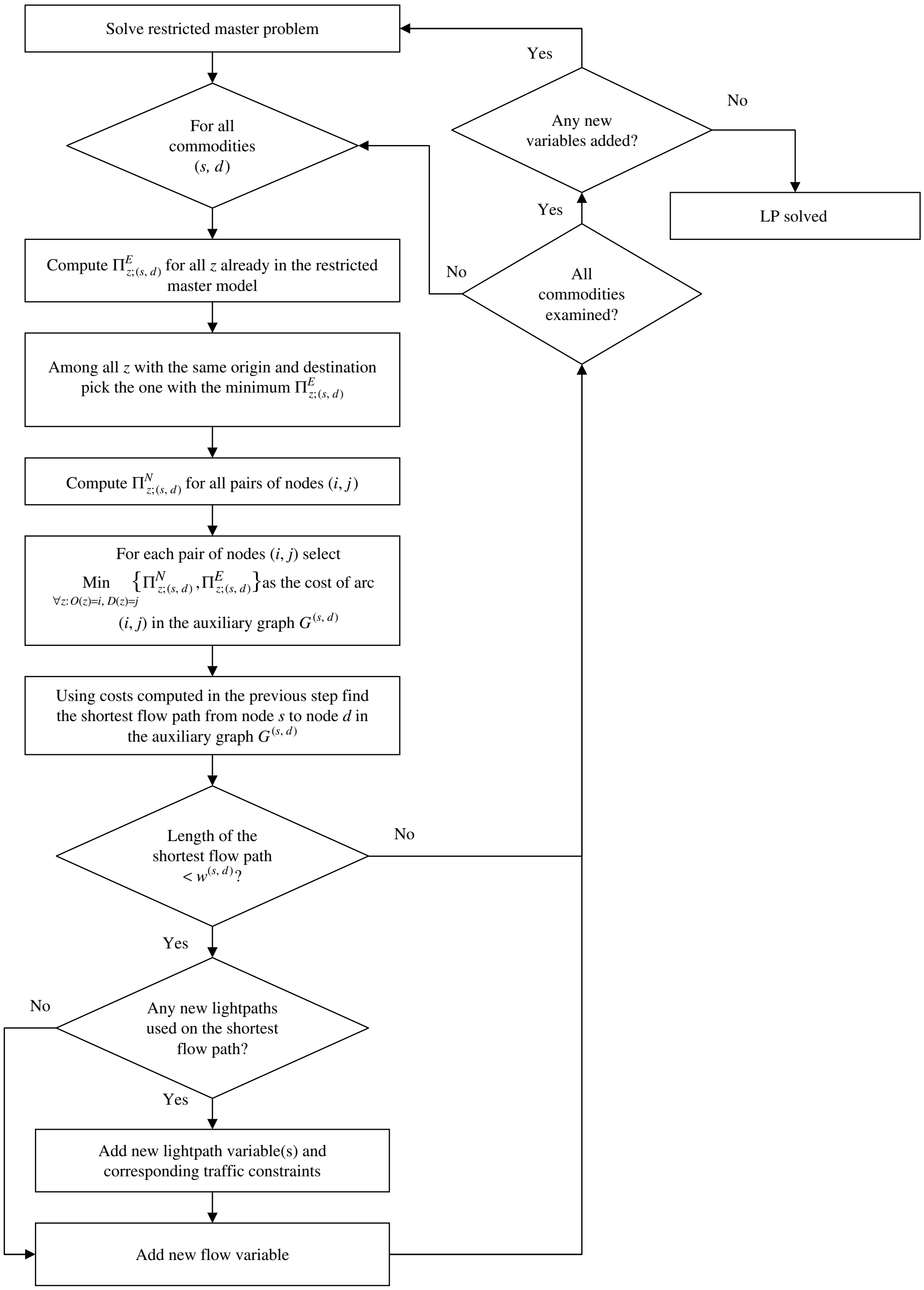

Figure 1 **Steps of Column Generation Algorithm for MIP-PATH-LOCALw and MIP-PATH-LOCAL Formulations**

commodities with fractional traffic that use paths differing in the logical layer exist. If no such commodities exist, we check for any fractional flow in the physical layer (branching level 3). Otherwise, we identify a commodity with fractional traffic that uses paths differing in the logical layer. We then locate the first divergence node in the logical layer and define two subsets of lightpaths originating at the divergence node such that one set contains one of the fractional paths of the given commodity and the second set contains the other fractional path of the same commodity. (In other words, we use the branching strategy developed in Barnhart et al. 2000, but only in the logical layer.) Branching is performed on the flow path variables of this commodity by creating two branches: on one branch, we do not allow lightpaths from the first set to be used by the selected commodity; and on the other branch, we do not allow lightpaths from the second set to be used by the selected commodity. (To ensure proper implementation of the branching rules at this branching level, we never generate lightpaths for origin–destination pairs that are forbidden for a given commodity.)

Branching Level 3. *Fractional flow in the physical layer, considering lightpaths with different propagation paths in the physical layer*: At the third level of branching, we check whether any commodities have fractional flow while using identical paths in terms of the lightpath origins and destinations but differing in propagation paths in the physical layer. If no such commodities exist, we proceed to the lowest level of branching (branching level 4). Otherwise, we select one such commodity. We then select two lightpaths carrying fractional traffic of this commodity that have identical origin and destination but have different propagation paths in the physical topology. Next, we locate the divergence node of these two lightpaths in the physical layer and define two sets of arcs such that one set contains the arc originating at the divergence node belonging to the first fractional flow path and the second one contains the arc originating at the divergence node belonging to the second fractional flow path. The branching is performed on the flow path variables of this commodity by creating two branches: on one branch, we do not allow lightpaths for a selected origin and destination and commodity to use any of the arcs from the first set of arcs; on the other branch, we do not allow lightpaths for a selected origin and destination and commodity to use any of the arcs from the second set of arcs. (The rules defined at this level of branching are enforced by setting the cost $-\delta_{(l,m)}$ to high values in the pricing part of the column generation algorithm for those arcs that are prohibited by the proposed branching rule.)

Branching Level 4. *Fractional flow in the physical layer, considering lightpaths with identical propagation paths in the physical layer*: At this branching level, we first select a commodity with fractional flow. Next, we identify two lightpaths with the same origin i and destination j that are used by this commodity (notice that these lightpaths will have the same propagation path in the physical layer). This is followed by identifying two (exhaustive and mutually exclusive) sets of lightpaths originating and terminating at nodes i and j, respectively. The branching is performed on the flow path variables of this commodity by creating two branches: on one branch, we prohibit the selected commodity from using any of the lightpaths in the first set; and on the other branch, we prohibit the selected commodity from using any of the lightpaths in the second set. Note that branching rule 4 may cause the generation of many identical lightpaths with the same origin and destination (i.e., once we prohibit the use of a certain set of lightpaths, there is nothing to stop the generation of a new, identical lightpath in the next iteration of our column generation algorithm). We resolve this issue and guarantee termination of the algorithm by imposing an additional rule. That is, for each commodity, we keep track of the number of forbidden lightpaths that have the same origin and destination and that use the same arc in the physical topology. Once this number reaches the maximum possible number of lightpaths that can be established over a given arc in the physical topology (this is either the number of transmitters and receivers, or the maximum number of lightpaths that can be established on any physical link), we do not allow the generation of any additional new lightpaths with the same origin and destination that require use of the same arc. (This rule is used separately for each commodity, so we may not allow use of certain lightpaths for one commodity, but we would still allow use of the same lightpaths for other commodities.)

Because the x_z variables in the MIP-PATH-LOCAL formulation do not have to be defined as binary, the described branching strategy provides an integer solution for this formulation, and it therefore provides a valid upper bound for the original WDM optical network design problem.

In the case of the MIP-PATH-LOCALw formulation, however, we need to perform additional branching to ensure that the x_z variables are binary. It turns out that this is a nontrivial task (short of prespecifying all possible lightpath variables beforehand) that requires the solution of an NP-complete pricing problem.

First, note that branching on the x_z variables based on variable dichotomy is not convenient. The reason is that this type of branching requires that, for a given fractional variable x_z and its current fractional value x_z^*, we create two new nodes by adding the restriction $x_z \le \lfloor x_z^* \rfloor$ on one branch and the restriction $x_z \ge \lceil x_z^* \rceil$ on the other branch. Although enforcing the second restriction is not a problem, the first restriction requires that we do not generate any new lightpaths that would have the same propagation path as the lightpath z corresponding to the variable x_z. Because the first restriction is hard to enforce, we use a different branching strategy that indirectly forces all lightpath variables to be binary.

The branching strategy that we use to ensure integrality of the x_z variables is based on the observation that if the sum of all lightpath variables with the same origin and destination is integer over every arc used in the physical layer, then either

(a) all lightpath variables are binary, or

(b) all lightpath variables do not have binary values, but the solution can be converted to a binary one. (In this case we solve a single commodity flow problem between the origin and the destination using the sum of the lightpath variables with the origin and destination as the capacity of the arc. Because the capacities have integer values, the resulting flows between the origin and destination have integer values as well, which then allows us to recover binary values for each lightpath variable; see Raghavan and Stanojević 2006.)

The specific branching step that we apply to ensure integrality of the x_z variables performs a check on whether any arcs exist with a fractional sum of all lightpath variables with the same origin and destination and using that arc. If there is no such arc, we proceed to branching level 1. Otherwise, we create two new branches:

Branch 1. Add restriction

$$\sum_{z:(m,n)\in z,\, O(z)=i,\, D(z)=j} x_z \le \left\lfloor \sum_{z:(m,n)\in z,\, O(z)=i,\, D(z)=j} x_z^* \right\rfloor.$$

Branch 2. Add restriction

$$\sum_{z:(m,n)\in z,\, O(z)=i,\, D(z)=j} x_z \ge \left\lceil \sum_{z:(m,n)\in z,\, O(z)=i,\, D(z)=j} x_z^* \right\rceil,$$

where (m, n) and (i, j) are the arc and the lightpath origin–destination pair identified in the previous check, and x_z^* are the actual values of lightpath variables in the current solution.

The problem with this approach is that the dual variable associated with branch 2 is positive and thus may lead to the creation of negative cost arcs that in turn could result in a negative cost cycle. In the presence of a negative cost cycle the shortest (loopless) path problem is NP-complete and thus is no longer polynomially solvable. In our computational experiments, however, we did not encounter the negative cost arc problem. Thus, we use a standard shortest path algorithm to solve the pricing problem. When a negative cost cycle is encountered, we resort to solving the pricing problem via integer programming. (The shortest loopless path problem between a source (s) and a destination (d) may be formulated as a minimum cost network flow problem where we wish to send one unit of flow from s to d with the additional restrictions that the flow into any intermediate node is at most one, and all flow is integer). We note that negative cost cycles rarely appear in our pricing problem. The reason for this is threefold. First, the dual variable associated with a branch 2 constraint is only positive when constraint (4) associated with the same arc is near its capacity. Second, the cost of arcs in the corresponding graph is comprised of both nonnegative and nonpositive dual variables, meaning that the arc cost will not necessarily have a negative value even if the dual variable associated with a branch 2 constraint is positive. Third, the existence of a negative cost arc by itself does not necessarily imply the existence of a negative cost cycle.

In our implementation of the branching strategy for the MIP-PATH-LOCALw, we first perform the step described above and then the previously described four-level branching procedure. In other words, we first branch on the sum of lightpath variables, then we branch on the lost traffic variables, and finally, we branch on the flow path variables.

To guarantee feasibility of the restricted master model on each branch of the branch-and-bound tree, we apply the necessary modifications to our branch-and-price procedures. In the case of the MIP-PATH-LOCAL formulation, the only situation when our branching strategy can cause infeasibility in one of the branches is when we impose the restriction that all the traffic must be served (that is, when on one branch we set $h^{(s,d)} = 0$). In this case, we may not have a sufficient number of lightpath or flow path variables to satisfy additional demand for a given commodity (s, d) (recall that we performed branching on the lost traffic variables only if the lost traffic variable has a fractional value in a given solution). To ensure that the branch is not pruned in such situations, we add an artificial variable to constraint (7) in the restricted master model, which accounts for all traffic that cannot be served. We also set a high cost for this artificial variable in the objective function so that this variable does not assume a positive value when a feasible solution to the original problem exists. If at the end of the column generation stage for a given branch in the branch-and-bound tree this variable still has a positive value, we fathom the branch, because this means that, given the restrictions in the branch-and-bound tree, a feasible solution for the original problem does not exist (if there were a feasible solution, the artificial variable would have to be equal to zero, given its high cost in the objective function).

In the case of the MIP-PATH-LOCALw formulation, we use the same logic, except that the artificial variable is added to all constraints of the type $\sum_{z:(m,n)\in z} x_z \ge \lceil \sum_{z:(m,n)\in z} x_z^* \rceil$ (this is necessary because on a given branch of the branch-and-bound tree, the lightpaths that are present in the restricted master model may be such that there is insufficient capacity in the network to accommodate the new requirement $\sum_{z:(m,n)\in z} x_z \ge \lceil \sum_{z:(m,n)\in z} x_z^* \rceil$).

3.4. Alternative Design Objectives

The algorithm proposed in the previous section solves the WDM optical network design problem with the objective of minimizing the total lost traffic. Although this is the most common objective that a telecommunication service provider faces in practice, it is possible that different design objectives need to be considered when solving this problem. We discuss several objectives that have been studied in the literature and show that our algorithm can be easily modified to solve the WDM optical network design for different objective functions. In all cases that we will discuss next, the lost traffic variables can be dropped from our path formulation without any effect on the pricing procedure. The branching strategy changes only in that we do not use lost traffic variables for branching any longer.

3.4.1. Minimizing the Network-Wide Average Packet Delay. Given the propagation delay $\xi_{(l,m)}$ for each physical arc (l, m), this objective function becomes

$$\text{Min} \sum_{(s,d)\in\Omega,\, p\in P^{(s,d)}} \sum_{(l,m)\in p} \xi_{(l,m)} f_p^{(s,d)}.$$

Notice that if we were to set $\xi_{(i,j)} = 1$, for all arcs (i, j), this objective function would minimize the average hop distance measured by the number of physical edges traversed by a flow of a given commodity.

The use of this objective function in our branch-and-price algorithms would only change the expression for the reduced cost used in the pricing part as follows:

(MIP-PATH-LOCAL)

$$RC_p^{(s,d)} = \sum_{(l,m)\in p} \xi_{(l,m)} + \sum_{z\in p,\, z\in Z^E} (r_z^{(s,d)} + T^{(s,d)} v_z) - w^{(s,d)} + \sum_{z\in p,\, z\in Z^N} \left(-\alpha_{O(z)} - \beta_{D(z)} - \sum_{(l,m)\in z} \delta_{(l,m)}\right),$$

(MIP-PATH-LOCALw)

$$RC_p^{(s,d)} = \sum_{(l,m)\in p} \xi_{(l,m)} + \sum_{z\in p,\, z\in Z^E} T^{(s,d)} v_z - w^{(s,d)} + \sum_{z\in p,\, z\in Z^N} T^{(s,d)} \left(-\alpha_{O(z)} - \beta_{D(z)} - \sum_{(l,m)\in z} \delta_{(l,m)}\right),$$

which can easily be used in the computation of the cost of using existing and new lightpaths. The other parts of our branch-and-price algorithms remain the same.

3.4.2. Minimizing the Congestion. In this case the objective function becomes

$$\text{Min}\, \lambda_{\max},$$

where $\lambda_{\max}$ is a variable that represents the maximum flow on any given lightpath. To use this objective function, it is necessary to add a new set of constraints to the MIP-PATH-LOCAL and MIP-PATH-LOCALw formulations:

$$\lambda_{\max} - \sum_{(s,d)\in\Omega,\, p:\, z\in p} T^{(s,d)} f_p^{(s,d)} \geq 0 \quad \forall z.$$

This also requires modification of the expression for the reduced cost used in the pricing part of our branch-and-price algorithm in the following way:

(MIP-PATH-LOCAL)

$$RC_p^{(s,d)} = \sum_{z\in p,\, z\in Z^E} (r_z^{(s,d)} + T^{(s,d)} v_z + T^{(s,d)} t_z) + \sum_{z\in p,\, z\in Z^N} \left(-\alpha_{O(z)} - \beta_{D(z)} - \sum_{(l,m)\in z} \delta_{(l,m)}\right) - w^{(s,d)},$$

(MIP-PATH-LOCALw)

$$RC_p^{(s,d)} = \sum_{z\in p,\, z\in Z^E} (T^{(s,d)} v_z + T^{(s,d)} t_z) + \sum_{z\in p,\, z\in Z^N} T^{(s,d)} \left(-\alpha_{O(z)} - \beta_{D(z)} - \sum_{(l,m)\in z} \delta_{(l,m)}\right) - w^{(s,d)},$$

where t_z is a nonnegative dual variable associated with the new set of constraints. Again, as in the case of the minimization of propagation delay, the new objective function requires only a minor change in computation of the reduced cost, and the other parts of our branch-and-price algorithms remain the same.

3.4.3. Minimizing the Number of Transponders. In this case the objective function becomes

$$\text{Min} \quad \sum_i (\Delta_t^i + \Delta_r^i).$$

In our branch-and-price algorithm, the use of this objective function would only mean that Δ_t^i and Δ_r^i would be variables, not given constants. This has *no* effect on the algorithmic steps of our branch-and-price algorithms, because lightpath variables x_z are integer, which immediately implies that Δ_t^i and Δ_r^i will be integer as well; i.e., there is no need to branch on these variables to get integer values.

4. Lower Bound Procedure

One of the problems related to the linear relaxations of two-layer multicommodity flow formulations is that these formulations are not easy to strengthen, especially when it is not necessary to serve all the commodities in the network. Stanojević (2005) showed that adding cover inequalities for constraint (6) in the MIP-PATH-LOCAL formulation is not an efficient way to improve the lower bounds. Consequently, we propose the use of an independent lower bounding procedure for the WDM optical network design problem that we now describe.

An alternative way of formulating the WDM optical network design problem is to define an additional type of lightpath variables, *global lightpath variables*, that would provide information only on the number of lightpaths established between any two nodes in the network. The advantage of this approach is that the number of global lightpath variables is relatively small, and we can use these variables to specify flow paths in the logical layer. This approach eliminates the challenging issues we encountered with the MIP-PATH-LOCAL formulation, but it also introduces additional complexity when it comes to ensuring no bifurcation of flow in the physical layer. In this section, we present a formulation that utilizes global lightpath variables (we refer to this formulation as MIP-PATH-GLOBAL) and propose a solution approach that can be used to obtain valid lower bounds for the WDM optical network design problem.

The notation that we use for the MIP-PATH-GLOBAL formulation is identical to the one that we used for the MIP-PATH-LOCAL formulation, with a few additional precomputed terms and slightly redefined decision variables.

Predetermined input

Ψ—The set that includes all possible subsets of the commodities.

$\Theta_{\psi}^{(i,j)}$—The number of lightpaths needed to carry the traffic of all commodities $(s,d) \in \psi$ $(\psi \in \Psi)$ without bifurcation.

C_{ψ}—The cardinality of the set ψ.

Variables

$y^{(i,j)}$—global lightpath variable that indicates the number of lightpaths established between nodes i and j. Note that these variables do not provide information on how the lightpaths of a given origin and destination are routed over the physical topology.

$x_z^{(i,j)}$—local lightpath variable that indicates the number of lightpaths z used in a given solution.

$f_p^{(s,d)}$—flow path indicator variable that indicates whether flow path p is used to carry traffic demand for commodity (s,d). In the MIP-PATH-GLOBAL formulation, flow paths are defined over the global lightpaths (i.e., we do not determine the exact propagation path of a given commodity in the physical topology).

$h^{(s,d)}$—lost traffic indicator variable that indicates whether demand of commodity (s,d) is lost or satisfied.

$\kappa_{\psi}^{(i,j)}$—indicator variable equal to one, only if all commodities that belong to set ψ use a global lightpath connecting nodes i and j.

MIP-PATH-GLOBAL Formulation

$$\text{Min} \sum_{(s,d)\in\Omega} T^{(s,d)} h^{(s,d)} \tag{36}$$

$$\text{subject to} \sum_{j:(i,j)\in\Lambda} y^{(i,j)} \le \Delta_t^i \quad \forall\, i \in V, \tag{37}$$

$$\sum_{i:(i,j)\in\Lambda} y^{(i,j)} \le \Delta_r^j \quad \forall\, j \in V, \tag{38}$$

$$y^{(i,j)} - \sum_{z:O(z)=i,\,D(z)=j} x_z^{(i,j)} = 0 \quad \forall\,(i,j)\in\Lambda, \tag{39}$$

$$\sum_{(i,j)\in\Lambda,\,z:(l,m)\in z} x_z^{(i,j)} \le L_{lm} \quad \forall\,(l,m)\in A, \tag{40}$$

$$y^{(i,j)} - \sum_{p:(i,j)\in p} f_p^{(s,d)} \ge 0 \quad \forall\,(i,j)\in\Lambda,\,(s,d)\in\Omega, \tag{41}$$

$$y^{(i,j)} - \sum_{(s,d)\in\Omega,\,p:(i,j)\in p} T^{(s,d)} f_p^{(s,d)} \ge 0 \quad \forall (i,j)\in\Lambda, \tag{42}$$

$$\sum_{p\in P^{(s,d)}} f_p^{(s,d)} + h^{(s,d)} = 1 \quad \forall\,(s,d)\in\Omega, \tag{43}$$

$$y^{(i,j)} - \Theta_{\psi}^{(i,j)} \kappa_{\psi}^{(i,j)} \ge 0 \quad \forall\,(i,j)\in\Omega,\ \psi\in\Psi, \tag{44}$$

$$C_{\psi} - \sum_{(s,d)\in\psi,\,p:(i,j)\in p} f_p^{(s,d)} + \kappa_{\psi}^{(i,j)} \ge 1 \quad \forall\,(i,j)\in\Lambda,\ \psi\in\Psi, \tag{45}$$

$$\kappa_{\psi}^{(i,j)} - f_p^{(s,d)} \le 0 \quad \forall\,(i,j)\in p,\ (s,d)\in\psi,\ \psi\in\Psi, \tag{46}$$

$$\kappa_{\psi}^{(i,j)} \in \{0,1\} \quad \forall\,(i,j)\in\Lambda,\ \psi\in\Psi, \tag{47}$$

$$f_p^{(s,d)} \in \{0,1\} \quad \forall\, p\in P^{(s,d)},\,(s,d)\in\Omega, \tag{48}$$

$$y^{(i,j)} \ge 0 \quad \forall\,(i,j)\in\Lambda, \tag{49}$$

$$x_z^{(i,j)} \ge 0 \ \text{and integer} \ \forall\, z\in Z,\ (i,j)\in\Lambda. \tag{50}$$

Constraints in the MIP-PATH-GLOBAL formulation have a similar interpretation to those in the MIP-PATH-LOCAL formulation. Constraint sets (37) and (38) limit the out-degree and in-degree of any node by the total number of transmitters and receivers, respectively. Constraint set (39) ensures that all global lightpaths are defined in the physical topology through an adequate number of the local lightpaths in the physical topology. Constraint set (40) represents a limit on the number of lightpaths that can be established on any physical edge. Constraint set (41) ensures that the flow path of any given commodity (s,d) can use global lightpath (i,j) only if that lightpath is included in the logical topology. Constraint set (42) are capacity constraints limiting total flow over all global lightpaths established between two nodes. Constraint set (43) ensures that either all demand for a given commodity is satisfied, or is entirely lost. Constraints (44)–(46) ensure no bifurcation of flow among lightpaths with the same origin and destination. These constraints are bin-packing constraints, where the number of lightpaths represents the number of bins, the capacity of each lightpath represents the size of each bin, and demands of commodities to be carried by a given set of lightpaths represent the size of individual items that need to be placed in the bins. Constraints (45) and (46) ensure proper definition of the indicator variable $\kappa_{\psi}^{(i,j)}$ by guaranteeing that variable $\kappa_{\psi}^{(i,j)}$ will be equal to one if all commodities from set ψ use global arc (i,j) (constraint (45)) and that variable $\kappa_{\psi}^{(i,j)}$ will be equal to zero if at least one of the commodities from set ψ is not using global arc (i,j) (constraint (46)).

In addition to the prohibitively large number of packing constraints, the MIP-PATH-GLOBAL formulation has several other drawbacks. First, to add the packing constraints, it is necessary to know the number of possible lightpaths between all pairs of nodes in advance. Second, this formulation is less versatile in terms of its adaptation to different WDM optical network settings. For example, because the flow path variables are not mapped to the local lightpath variables, there is no way to determine the number of physical hops used by each individual flow path. This means that the MIP-PATH-GLOBAL formulation cannot be used for networks where we need to place a limit on the number of physical hops for each individual flow path.

Although the MIP-PATH-GLOBAL formulation in the form (36)–(50) does not appear to be a good choice for the WDM optical networks without bifurcation of flow, this formulation has important advantages over MIP-PATH-LOCAL formulation in optical networks where bifurcation of flow is allowed. The reason is that MIP-PATH-GLOBAL formulation remains row-complete in the column generation framework. Specifically, the pricing procedure is straightforward, and we can perform direct and independent pricing of x_z and $f_p^{(s,d)}$ variables. In a related paper (Raghavan and Stanojević 2006), we have developed such a procedure, and in this paper, we adapt that procedure as a lower bounding procedure for use in the optical networks without bifurcation of flow. (Note that we can allow bifurcation of flow in the MIP-PATH-GLOBAL formulation by dropping constraints (44)–(47) and allowing noninteger values of the flow path variables. The objective value determined by solving this relaxed form of the MIP-PATH-GLOBAL formulation represents a lower bound for the optimal objective value of the original MIP-PATH-GLOBAL formulation.) The details of the branch-and-price algorithm for MIP-PATH-GLOBAL formulation with bifurcation of flow can be found in Raghavan and Stanojević (2006). In this branching strategy, we first branch on the global lightpath variables, then on the local lightpath variables, followed by branching on the lost traffic variables, and finally branching on the flow path variables. For the global lightpath variables and the lost traffic variables, variable dichotomy is used. Branching on the local lightpath variables is based on the sums of local lightpath variables with the same origins and destinations crossing the same physical arcs. To strengthen the lower bound that can be obtained by the MIP-PATH-GLOBAL relaxation with fractional variables, we impose a restriction on the flow path variables to have binary values. (Branching on the fractional flow path variables can be performed using the branching strategy defined in Barnhart et al. 2000 by looking at the divergence node of the fractional flow variables in the logical topology.) This integrality requirement imposed on the flow path variables guarantees that there will be no bifurcation of flow for a given commodity over lightpaths with different origins and destinations, but it does not guarantee that bifurcation of flow will not occur over the lightpaths with the same origin and destination (we refer to this relaxation of the MIP-PATH-GLOBAL formulation as MIP-PATH-GLOBALr).

In our computational experiments with four different network settings, this procedure found the optimal solution in all instances where the optimal solution was known.

5. Computational Experiments

The procedures presented in this paper were coded using Microsoft Visual C++, ILOG Maestro (ILOG 2003), and CPLEX 9.0. All computations were performed on a workstation with 2.66 GHz Xeon processor with 2 GB of RAM.

We used four different sets of problems in our computational tests (the test problems are identical to those described in Raghavan and Stanojević 2006). These sets of problems include two sets of randomly defined problems that use complete and incomplete graphs in the physical layer (defined in Raghavan and Stanojević 2006) with low (L) and high (H) demand levels D, and a simple six-node network and NSFNET network used in the study of Prathombutr et al. (2005) with varying number of transmitters and receivers (T/R Nbr) and varying number of wavelengths (Wav. Nbr) that can be established on each fiber. The low demand networks had traffic requests for each commodity in the range $[0.1, 0.5]$, whereas the high demand networks had traffic requests in the range $[0.1, 1]$.

In all our computational experiments, we have used the following settings. The initial set of lightpaths used in column generation is determined using the all-pairs shortest path algorithm (where the length of the path is defined by the number of physical hops used by the lightpath). The initial set of flow paths is determined using the shortest path algorithm with a single flow path defined for each commodity (where the length of the flow path is defined by the number of lightpaths used).

We included the use of the CPLEX MIP optimizer for the initial LP in the branch-and-bound tree with a 600-second CPU time limit. Also, in the branch-and-price algorithm for the MIP-PATH-GLOBALr formulation, we included the use of the CPLEX MIP optimizer at all nodes of the branch-and-bound tree at which both global and local lightpath variables are integer, but fractional flow path variables exist. Finally, in each subsequent iteration of our branch-and-price algorithms, we select the branch that has a minimum parent lower bound in the branch-and-bound tree at that point, perform column generation, and (if the solution is fractional) perform further branching steps.

Table 1 provides the results for the randomly defined networks that use a complete graph in the physical layer. These results indicate that, as expected, the branch-and-price algorithm for (B&P) the MIP-PATH-LOCALw formulation requires longer computational times to solve the problems. This procedure actually solves to optimality only one test instance within the 3,600-second CPU time limit. The other two branch-and-price algorithms (for the formulations MIP-PATH-LOCAL and MIP-PATH-GLOBALr), on the other hand, solved six out of eight instances in the networks with five and seven nodes. We can see that in all solved instances the bound provided by the branch-and-price algorithms for MIP-PATH-LOCAL and MIP-PATH-GLOBALr are the same, indicating that the solutions found by these procedures are optimal. (Recall that solutions provided by the branch-and-price algorithm for MIP-PATH-LOCAL represent upper bounds for the original problem, whereas the solutions provided by the branch-and-price algorithm for MIP-PATH-GLOBALr represent lower bounds for the original problem. This means that we have an optimal solution for the original problem if the upper bound obtained from MIP-PATH-LOCAL is equal to the lower bound obtained from MIP-PATH-GLOBALr.) We can also see that in terms of upper bounds the branch-and-price algorithm for the MIP-PATH-LOCAL formulation outperforms the branch-and-price algorithm for the MIP-PATH-GLOBALr formulation. This may appear surprising considering the fact that the (final) solutions provided by the branch-and-price algorithm for the MIP-PATH-GLOBALr formulation represent a lower bound for the original problem. However, these results may indicate that a better set of lightpaths was generated for the initial LP solution of the algorithm using the MIP-PATH-LOCAL formulation. This is likely because the local lightpath variables in the MIP-PATH-LOCAL formulation are defined as binary variables, whereas in the MIP-PATH-GLOBALr formulation these variables are defined as general integers. Consequently, we can expect to have a larger number of lightpaths (that differ from the lightpaths in the initial set) generated for the initial LP solution of the branch-and-price algorithm for the MIP-PATH-LOCAL formulation. As a result, in the case of MIP-PATH-LOCAL formulation, the search space corresponding to the problem solved by CPLEX optimizer has a higher chance to include larger number of good candidate lightpaths.

Table 1 **Minimizing the Lost Traffic in the Network: Complete Physical Network with Two Fibers (Fiber Capacity of Two Lightpaths) Between All Pairs of Nodes**

			B&P (LOCALw)			B&P (LOCAL)			B&P (GLOBALr)		
$\|V\|$	$\|\Omega\|$	D	LB	UB	CPU (sec)	LB	UB	CPU (sec)	LB	UB	CPU (sec)
5	20	H	0.11	0.88	3,602.16	0.62	0.62	2.31	0.62	0.62	2,283.58
5	20	L	0.00	0.89	3,602.58	0.00	0.00	0.34	0.00	0.00	0.13
5	10	H	0.00	0.16	3,600.58	0.00	0.00	0.17	0.00	0.00	0.06
5	10	L	0.00	0.00	3.22	0.00	0.00	0.19	0.00	0.00	0.05
7	42	H	2.28	6.97	3,600.58	4.32	5.32	3,601.36	4.29	5.29	3,602.26
7	42	L	0.00	4.25	3,600.33	0.00	0.15	3,600.88	0.00	0.00	312.05
7	21	H	0.19	1.90	3,600.73	0.70	0.70	11.23	0.70	0.70	3,617.31
7	21	L	0.00	1.23	3,600.39	0.00	0.00	2.06	0.00	0.00	0.67
10	90	H	17.65	24.50	3,600.20	21.14	22.89	3,602.56	21.15	23.14	3,601.14
10	90	L	0.21	14.18	3,600.17	5.18	7.14	3,602.33	5.20	6.92	3,879.33
10	45	H	1.47	5.88	3,600.23	2.41	4.33	3,600.66	2.55	4.17	3,603.84
10	45	L	0.00	3.81	3,600.20	0.00	0.43	3,601.83	0.00	0.14	3,602.39
20	380	H	149.39	156.67	3,607.74	153.68	155.44	3,601.36	152.44	155.04	4,137.05
20	380	L	54.13	86.84	3,610.52	74.87	86.84	3,601.99	70.49	74.50	3,653.72
20	190	H	48.07	57.09	3,702.78	54.11	57.29	3,601.80	52.76	57.01	3,626.23
20	190	L	7.19	31.17	3,602.48	18.97	32.80	3,600.38	16.60	22.26	3,672.38

Table 2 Minimizing the Lost Traffic in the Network: Incomplete Physical Network with Two Fibers (Fiber Capacity of 20 Lightpaths) on All Arcs

				B&P (LOCALw)			B&P (LOCAL)			B&P (GLOBALr)		
$\|V\|$	$\|A\|$	$\|\Omega\|$	D	LB	UB	CPU (sec)	LB	UB	CPU (sec)	LB	UB	CPU (sec)
5	10	15	H	0.38	0.89	3,604.70	0.69	0.69	1.06	0.69	0.69	1,953.45
5	10	15	L	0.00	0.00	53.97	0.00	0.00	0.22	0.00	0.00	0.09
7	15	30	H	1.71	4.61	3,601.27	2.40	3.44	3,601.81	2.04	3.10	3,607.97
7	15	30	L	0.00	2.60	3,600.49	0.00	0.16	3,600.72	0.00	0.00	9.39
10	20	40	H	1.71	4.25	3,600.51	2.41	2.95	3,601.33	1.86	3.05	3,605.39
10	20	40	L	0.00	2.61	3,600.28	0.00	0.16	3,601.72	0.00	0.19	3,601.51
20	30	60	H	4.68	6.55	3,600.33	4.68	6.19	3,605.47	4.68	6.42	3,609.92
20	30	60	L	0.00	4.18	3,600.33	0.00	1.20	3,619.41	0.00	1.32	3,628.48

Table 2 provides results for the randomly defined networks that use an incomplete graph in the physical layer. These results indicate the same pattern in the performance of our branch-and-price algorithms as in the case of the first set of instances defined over the complete physical networks.

The results for test instances defined in the study of Prathombutr et al. (2005) are shown in Tables 3 and 4. In this case, our branch-and-price algorithms did not find optimal solutions for any of the instances within a specified CPU time limit. However, the upper bounds provided by the branch-and-price algorithm for the MIP-PATH-GLOBALr formulation were, in most instances, better than the upper bounds provided by the branch-and-price algorithm for the MIP-PATH-LOCAL formulation. The most likely reason is that in most of these instances, the branch-and-price algorithm for the MIP-PATH-LOCAL formulation did not complete column generation for the initial LP, and the algorithm did not get a chance to have any benefits from the local optimizer otherwise applied to the initial LP solution. An analysis of the progress of our column generation indicated that the long time spent at the root is not due to degeneracy or stalling of the solution procedure at a specific lower bound, in which case common stabilization procedures could be used. Instead, the problem was mostly related to small incremental improvements achieved at each iteration of the column generation algorithm. We believe that one good venue for further improvements would be to try to generate a pool of good lightpaths that could be considered for introduction into the restricted master problem at any given branch of the branch-and-bound tree (see Chabrier 2003 for a similar strategy applied to the multicommodity multifacility network design problem). We leave this for future research.

In Table 5 we provide a comparison of the branch-and-price algorithm for MIP-PATH-LOCALw and

Table 3 Minimizing the Lost Traffic in the Network: The Six-Node Network with a Single Fiber on Each Arc

		B&P (LOCALw)			B&P (LOCAL)			B&P (GLOBALr)		
T/R Nbr	Wav. Nbr	LB	UB	CPU (sec)	LB	UB	CPU (sec)	LB	UB	CPU (sec)
3	3	3.47	6.46	3,602.08	5.10	6.02	3,606.42	5.09	6.02	3,614.95
4	3	0.28	3.14	3,601.84	0.77	2.78	3,601.02	1.12	2.68	3,605.81
5	3	0.28	2.81	3,601.64	0.44	2.43	3,600.94	0.28	1.29	3,616.30
7	3	0.28	2.81	3,601.78	0.44	2.45	3,600.75	0.28	0.92	3,618.39
3	4	3.60	6.46	3,602.02	5.10	6.02	3,605.81	5.03	6.02	3,615.16
4	4	0.17	3.02	3,601.80	0.75	2.63	3,602.53	0.95	2.48	3,608.02
5	4	0.00	1.31	3,601.08	0.00	0.35	3,600.69	0.00	0.91	3,611.61

Table 4 Minimizing the Lost Traffic in the Network: The NSFNET Network with a Single Fiber on Each Arc

		B&P (LOCALw)			B&P (LOCAL)			B&P (GLOBALr)		
T/R Nbr	Wav. Nbr	LB	UB	CPU (sec)	LB	UB	CPU (sec)	LB	UB	CPU (sec)
3	3	0.00	19.26	3,600.35	4.13	8.89	3,606.41	3.99	8.99	3,655.20
4	3	0.00	16.82	3,600.32	0.00	8.98	3,626.69	0.00	6.71	3,641.36
5	3	0.00	15.39	3,600.35	0.00	5.32	3,641.06	0.00	2.62	3,612.26
4	4	0.00	17.04	3,600.32	0.00	8.46	3,614.05	0.00	9.30	3,618.47
5	4	0.00	14.91	3,601.21	0.00	4.93	3,609.97	0.00	2.39	3,614.84
6	4	0.00	13.51	3,600.32	0.00	3.26	3,620.81	0.00	0.21	3,917.52

Table 5 Summary: Minimizing the Lost Traffic in the Network

	B&P (LOCALw)		Combined LOCAL and GLOBALr	
Network type	Avg. gap (%)	Avg. CPU (sec)	Avg. gap (%)	Avg. CPU (sec)
Complete	24.04	3,383.43	7.94	4,726.48
Incomplete	14.49	3,157.73	3.65	5,205.99
Six nodes	13.46	3,601.75	7.53	7,215.49
NSFNET	54.03	3,600.48	20.40	7,296.44
Overall	24.84	3,411.13	8.96	5,717.80

Table 6 Summary: Minimizing the Total Number of Transmitters and Receivers in the Network

	B&P (LOCALw)		Combined LOCAL and GLOBALr	
Network type	Avg. gap (%)	Avg. CPU (sec)	Avg. gap (%)	Avg. CPU (sec)
Complete	131.85	3,608.85	29.36	6,879.83
Incomplete	93.49	3,602.56	14.97	7,561.20
Overall	119.06	3,606.75	24.56	7,106.95

Note. We have assumed an upper bound of 380 for one instance of LOCALw where we did not have an integer solution.

the combined use of the branch-and-price algorithms for MIP-PATH-GLOBALr and MIP-PATH-LOCAL. (In the combined use of the branch-and-price algorithms for MIP-PATH-GLOBALr and MIP-PATH-LOCAL, the lower bounds are those of the branch-and-price algorithms for MIP-PATH-GLOBALr, and the upper bounds are those of branch-and-price algorithms for MIP-PATH-LOCAL.) The results in this table include the average percentage gaps and CPU times for each type of network used in our test instances. The percentage gaps in this case were calculated as $((\text{UB}-\text{LB})/(\text{total demand}-\text{LB}))*100\%$ (the straightforward computation of the gap in the form $(\text{UB}-\text{LB})/\text{LB}$ was not practical for these problems because the lower bound was equal to zero in many instances). In other words, we provide the percentage gaps with the respect to the total traffic served instead of the total traffic lost. These results clearly indicate superior performance of the combined branch-and-price algorithms for MIP-PATH-LOCAL and MIP-PATH-GLOBALr.

In addition to the computational experiments described, we have tested the performance of our branch-and-price procedures for the objective function that minimizes the number of transmitters and receivers in the network. We have found that, in general, our procedures do not provide good upper bounds for this objective, which is likely because our search for integer solutions (upper bounds) is left to the CPLEX optimizer. We suspect that integration of custom upper bounding heuristics within our branch and price procedures would provide better results. The summary of the results of our computational experiments for the objective of minimizing the number of transmitters and receivers is shown in Table 6, with the percentage gaps calculated as $(\text{UB}-\text{LB}')/\text{LB}'$, and the lower bound LB' calculated as the smallest even integer number greater or equal to the actual LB obtained with our branch-and-price algorithm.

In our final set of experiments, we tested the application of our approximate branch-and-price procedures for the design of WDM optical networks where design requirements differ from the problem defined and studied in this paper. The requirements that we considered include networks where flow bifurcation is allowed and networks where there is ample fiber capacity, so there is no constraint on the number of lightpaths that can be established on any physical link. We also looked at WDM optical networks where we can make both assumptions. That is, we allowed bifurcation of flow and assumed that an infinite number of wavelengths is available in the network. The proposed branch-and-price procedures required only minor modifications in each alternative network design setting.

Specifically, for the case of WDM optical networks with the bifurcation of flow allowed, we only relaxed the integrality constraints on the flow path variables, and in the case of the MIP-PATH-LOCAL formulation, we imposed an integrality constraint on the lightpath variables x_z (recall that the constraint $x_z - \sum_{p:z\in p} f_p^{(s,d)} \geq 0 \; \forall z \in Z, (s,d) \in \Omega$ was ensuring integrality of the variables x_z as long as the flow path variables $f_p^{(s,d)}$ had integer values), which can be accomplished by using the same branching strategy that we used for the x_z variables in the branch-and-price algorithm for the MIP-PATH-LOCALw formulation. (The branch-and-price procedure for the MIP-PATH-GLOBALr formulation in this case is an exact procedure and is described in Raghavan and Stanojević 2006.)

For the case of WDM optical networks with an infinite number of wavelengths, we eliminated the $x_z^{(i,j)}$ variables in the MIP-PATH-GLOBALr formulation and applied our branch-and-price procedure without any other modifications. As for the MIP-PATH-LOCAL formulation, we only eliminated the constraint on the number of lightpaths on physical links.

The summary of our computational results for different types of network restrictions and the two objective functions used in our computational experiments are shown in Tables 7 and 8 (the abbreviations BA and BNA refer to networks where bifurcation of flow is allowed and not allowed, respectively, whereas FW and IW refers to networks with finite and infinite number of wavelengths, respectively). The results

Table 7 Alternative Network Settings: Minimizing the Total Lost Traffic in the Network

	B&P (LOCALw)		B&P (LOCALw)		Combined LOCAL & GLOBALr	
Network setting	Avg. gap (%)	Avg. CPU	Avg. gap (%)	Avg. CPU	Avg. gap (%)	Avg. CPU
BNA & FW	4.88	2,602.52	3.73	2,334.37	5.64	4,936.89
BA & FW	3.48	2,033.29	0.77	1,472.31	3.51	3,505.60
BNA & IW	3.92	2,488.12	3.89	2,351.08	4.52	4,839.19
BA & IW	0.74	1,613.19	0.88	1,446.29	0.73	3,059.48

Table 8 Alternative Network Settings: Minimizing the Total Number of Transmitters and Receivers

	B&P (LOCALw)		B&P (LOCALw)		Combined LOCAL & GLOBALr	
Network setting	Avg. gap (%)	Avg. CPU	Avg. gap (%)	Avg. CPU	Avg. gap (%)	Avg. CPU
BNA & FW	19.13	3,160.84	18.34	2,643.77	21.59	5,804.61
BA & FW	16.33	3,623.87	16.92	2,971.01	16.82	6,594.87
BNA & IW	12.87	3,191.54	10.60	2,651.05	12.41	5,842.59
BA & IW	18.53	2,979.76	8.67	3,134.47	17.25	6,114.23

presented in Tables 7 and 8 are a summary of the results for a selected set of eight instances in complete physical networks and eight instances in incomplete physical networks. For the second network design objective, minimization of the number of transmitters and receivers, we have also used node degree inequalities to strengthen the lower bounds (see Stanojević 2005 for details). As expected, the gaps significantly decrease in the less restricted WDM optical network settings. The only exception seem to be the networks where bifurcation of flow is allowed and an infinite number of wavelengths is available when the objective is minimizing the total number of transmitters and receivers.

6. Conclusion

In this paper, we discussed the design of branch-and-price algorithms for the WDM optical network design problem with no bifurcation of flow. The unique feature of the proposed algorithms is the use of column generation for the row-incomplete formulations. We explained how the WDM optical network design problem can be addressed in this situation both exactly and approximately by proposing one exact and two approximate branch-and-price algorithms that can be used to obtain valid lower and upper bounds for this problem.

Our computational experiments with the exact and approximate lower bounding and upper bounding branch-and-price algorithms indicate that the combined use of the approximate branch-and-price algorithms (one of which provides a lower bound and the other an upper bound) provides significantly better results than the proposed exact branch-and-price procedure.

We have also performed computational tests with our approximate branch-and-price algorithm for WDM optical networks with different network restrictions. Specifically, we looked at (i) WDM optical networks where bifurcation of flow is allowed, (ii) networks with ample fiber capacity, and (iii) networks that both have ample fiber capacity and allow bifurcation of flow. Our computational experiments indicate that the performance of our approximate branch-and-price algorithms improves as we relax restrictions in the network settings.

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