To be a Value of a Plural Variable, You Don’t Have to be Plural (You Just Have to Be)
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Given a language with truth-evaluable sentences that contain variables, we say that such sentences are true or false relative to assignments of values to variables. Given a first-order language, with no plural variables, we say that each assignment A assigns exactly one value to each variable; where each value of a variable is an entity in the relevant domain. For example, the “open” sentence $Fx \& Gx$ has a singular variable, whose semantic role can be made explicit.

$$F( \ ) \& G( \ ) \text{ is true relative to } A \text{ iff the value that } A \text{ assigns to } x \text{ is such that: }$$ $\begin{align*} x \\ A(x) & \text{ satisfies } F, \text{ and } A(x) & \text{ satisfies } G \end{align*}$

Given plural variables, corresponding to ‘them’ (as opposed to ‘it’), we can

(a) retain the idea that each assignment assigns exactly one value to each variable, by saying that each value of a plural variable is a plural entity with elements that can be values of singular variables;

(b) retain the idea that each value of a variable (singular or plural) is an entity in the domain over which all variables range, by saying that a plural variable can have many values relative to a single assignment of values to variables.

$$\Phi( \ ) \& \Psi( \ ) \text{ is true relative to } A \text{ iff ... }$$ $\begin{align*} x_\text{pl} \\ (a) \text{ the plural entity that } A \text{ assigns to } x_\text{pl} \text{ is such that: } \text{ it satisfies } \Phi, \text{ and } \text{ it satisfies } \Psi \\
(b) \text{ the entities that } A \text{ assigns to } x_\text{pl} \text{ are such that: they satisfy } \Phi, \text{ and they satisfy } \Psi \end{align*}$

CLAIMS: the second option is coherent, not a notational variant of the first, and preferable; it can be part of a simple account of semantic composition that helps explain some otherwise puzzling facts, including the conservativity of determiners.

Assume a domain of exactly 5 things—a, b, c, d, e—and so 31 possible assignments to a variable

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It might seem that mereology is the only natural construal of the lattice. But we need not assume one value per variable.

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A Common Interpretation of Indices, Plural Demonstratives, and Verb Phrases

1. This trumps that
   \[ \forall i \forall A \forall x \forall y \exists z ([x \neq z] \land (y \in x) \land (z \in x)) \]

2. This trumps them
   \[ \forall i \forall A \forall x \forall y \exists z ([x \neq z] \land (y \in x) \land (z \in x)) \]

3. They trump it
   \[ \forall x \exists y \exists z ([x \neq z] \land (y \in x) \land (z \in x)) \]

4. They trump them
   \[ \forall x \exists y \exists z ([x \neq z] \land (y \in x) \land (z \in x)) \]

5. For each entity, it is Plural iff it has other entities as elements

6. \[ \forall x \{ \text{Plural}(x) \rightarrow \exists y \exists z ([y \neq z] \land (y \in x) \land (z \in x)) \} \]

7. For every plural entity X, plural entity Y, nonplural entity x, and nonplural entity y:
   - X trumps* Y iff every element of X trumps every element of Y,
   - X trumps* y iff every element of X trumps y,
   - x trumps* Y iff x trumps every element of Y, and
   - x trumps* y iff x trumps y

8. \[ \forall X \forall Y \forall x \forall y \{ \text{Trumps}(X, Y) \leftrightarrow \forall x' : x' \in X \forall y' : y' \in Y \{ \text{Trumps}(x', y') \} \} \]

9. This trumps that
   \[ \forall i \forall A \forall x \forall y \exists z ([x \neq z] \land (y \in x) \land (z \in x)) \]

10. \[ \forall i \forall A \forall x \forall y \exists z ([x \neq z] \land (y \in x) \land (z \in x)) \]

11. \[ \text{trump}(s) \land = \lambda \beta. \lambda \alpha. \text{Trumps}(\alpha, \beta) \]

12. Every heart trumps every club
   \[ \forall x : \text{Heart}(x) \{ \forall y : \text{Club}(y) \{ \text{Trumps}(x, y) \} \} \]

13. The hearts trump the clubs
   \[ \exists X : \{ \forall x : x \in X \rightarrow \text{Heart}(x) \} \{ \exists Y : \forall y : y \in Y \rightarrow \text{Club}(y) \{ \text{Trumps}(X, Y) \} \} \]

14. They trump them
   \[ \exists X : \{ \forall x : x \in X \rightarrow x \in A(1) \} \{ \exists Y : \forall y : y \in Y \rightarrow y \in A(2) \} \{ \text{Trumps}(X, Y) \} \]
Imagine a (team) game in which no one card trumps anything, but any 2 hearts trump any 2 clubs.

15. They_{1} trump them_{2} 
   collective: Plural[A(1)] & Plural[A(2)] & A(1) trumps^{o} A(2)
   7<h 9<h Q<h J<h 
   distributive: Plural[A(1)] & Plural[A(2)] & A(1) trumps* A(2)

16. for every plural entity X, and every plural entity Y, X trumps^{o} Y iff
   the elements of X (together) trump the elements of Y

QUESTIONS: is this just to say that X trumps^{o} Y iff X trumps Y?
   Does \{7<h, 9<h\} trump \{Q<h, J<h\}? Or is ‘trumps^{o}’ a theoretical term we must define?
   If we adopt the hypothesis that each plural demonstrative has a plural entity as its value,
   relative to each assignment of values to variables, what else do we need to say?
   Are we forced to introduce multiple type-shifting principles for verb meanings?
   [see Landman]

17. They_{1} wrote them_{2} 
   Plural[A(1)] & Plural[A(2)] & A(1) wrote^{o} A(2)
   \(\bigotimes\emptyset \otimes \emptyset\) \(\emptyset\) 
   a b c d e f g h i j k

18. Five professors wrote six papers

19. \(\exists X \forall Y [\text{FiveProfessors}(X) & \text{SixPapers}(Y) & \text{Wrote}^{o}(X, Y)]\)
   FiveProfessors(X) \iff FiveMembered(X) & \forall x: x \in X[\text{Professor}(x)]
   SixPapers(X) \iff SixMembered(X) & \forall x: x \in X[\text{Paper}(x)]

   total autonomy         semi-cooperation       total cooperation
   a: f,g                   a,b,c: f,g,h             a,b: f                a,b,c,d,e: f,g,h,i,j,k
   b: h                     d,e: i,j,k               c,b: g
   c: i                     d,c: h
   d: j                     a,c,e: i,j
   e: k                     b,d: k
[see Gillon, Schein]

20. \text{Wrote}^{o}([\{a, b, c, d, e\}, \{f, g, h, i, j, k\}]
   if this allows for less than total cooperation, then what does it mean, if not:
   the elements of \{a, b, c, d, e\} were, somehow, the writers of the elements of \{f, g, h, i, j, k\}?

21. Five professors wrote six papers in March (quickly, under pressure, and inelegantly)

22. \(\exists X \forall Y \forall e [\text{Agent}(e, X) & |X| = 5 & \forall x: x \in X[\text{Professor}(x)] & \text{PastWriting}(e) & \text{Theme}(e, Y) & |Y| = 6 & \forall y: y \in Y[\text{Paper}(y)] & \text{In}(e, March) & \ldots ]\)
   one big event?

23. \(\exists X \forall Y \forall e [\text{Agent}(E, X) & \ldots & \forall e: e \in E[\text{PastWriting}(e)] & \text{Theme}(E, Y) & \ldots ]\)

   each-of-X was-an-Agent-of some-of-E
   each-of-E was-done-by some-of-X
   each-of-E was-a-production-of some-of-Y
   each-of-Y was-a-Theme-of some-of-E
(24) Five professors wrote six papers in March

\[ \exists E \left( \exists X : \text{FIVE}(X) \& \text{Professors}(X)[\text{Agent}(E, X)] \& \text{PastWriting}(E) \& \exists X : \text{SIX}(X) \& \text{Papers}(X)[\text{Theme}(E, X)] \& \text{In}(E, \text{March}) \right) \]

There are one or more things \( E \) (the Es) such that:
(i) five professors were their \( E \) Agents—i.e.,
their \( E \) Agents were some things \( X \) such that: they \( X \) are five and they \( X \) are professors
and (ii) they \( E \) were events of writing—i.e.,
each of them \( E \) was an event of writing \( \forall e : E [\text{PastWriting}(e)] \)
and (iii) six papers were their \( E \) Themes—i.e.,
their \( E \) Themes were some things \( X \) such that: they \( X \) are six and they \( X \) are papers
and (iv) they \( E \) were in March—i.e.,
each of them \( E \) occurred in March

‘Xx’ means that x is an X—i.e., x is one of the Xs—not that x is an element of X

| Professors(X) | the Xs are professors | \( \forall x : Xx[\text{Professor}(x)] \) |
| Papers(X) | the Xs are papers | \( \forall x : Xx[\text{Paper}(x)] \) |
| SIX(X) | the Xs are six \( \text{EssPl} \) | \( \exists Y \exists z \{ \text{Five}(Y) \& \neg Yz \& \forall x : Xx <\rightarrow Yx v (x = z) \} \) |
| FIVE(X) | the Xs are five \( \text{EssPl} \) | \( \exists Y \exists z \{ \text{Four}(Y) \& \neg Yz \& \forall x : Xx <\rightarrow Yx v (x = z) \} \) |
| Agent(E, X) | the Xs are the Agents of the Es | \( \forall x : Xx \{ \exists e : E [\text{Agent}(e, x)] \} \) |
| Theme(E, X) | the Xs are the Themes of the Es | \( \forall e : E [\exists x : Xx[\text{Theme}(e, x)] \} \) |

(17) They \( _1 \) wrote them \( _2 \) TRUE relative to an assignment \( A \) iff
the things that \( A \) assigns to the first index wrote
(i.e., were the Agents of some PastWritings whose Themes were)
the things that \( A \) assigns to the second index

\[ \exists E \left( \exists X : \forall x : (Xx <\rightarrow \text{Assigns}(A, x, '1'))[\text{Agent}(E, X)] \& \text{PastWriting}(E) \& \exists X : \forall x : (Xx <\rightarrow \text{Assigns}(A, x, '2'))[\text{Theme}(E, X)] \right) \]

(18) Five professors wrote six papers
\[ \exists E \left( \exists X : \text{Five}(X) \& \text{Professors}(X)[\text{Agent}(E, X)] \& \text{PastWriting}(E) \& \exists X : \text{Six}(X) \& \text{Papers}(X)[\text{Theme}(E, X)] \right) \]

| a,b: 1 | a,c,e: 45 | \( \varepsilon_1 \) | \( \varepsilon_2 \) | \( \varepsilon_3 \) | \( \varepsilon_4 \) | \( \varepsilon_5 \) | \( \varepsilon_6 \) |
| c,b: 2 | b,d: 6 | / \ | / \ | / \ | / \ | / \ | / \ |
| d,c: 3 | \( \varepsilon_{1a} \) | \( \varepsilon_{1b} \) | \( \varepsilon_{2c} \) | \( \varepsilon_{2b} \) | \( \varepsilon_{3d} \) | \( \varepsilon_{3c} \) | \( \varepsilon_{4a} \) | \( \varepsilon_{4c} \) | \( \varepsilon_{4e} \) | \( \varepsilon_{5a} \) | \( \varepsilon_{5c} \) | \( \varepsilon_{5e} \) | \( \varepsilon_{6b} \) | \( \varepsilon_{6d} \) |

(25) The rocks rained down on the huts clustered near the lakes in which our ancestors fished
Another Familiar Theory and Some Further Familiar Questions

(26) Every bottle fell \( \{ z: \text{Fell}(z) \} \supset \{ z: \text{Bottle}(z) \} \)

\[
\| \text{every} \| = \lambda Y. \lambda X. \{ z: Y(z) \} \supset \{ z: X(z) \} = 0 \quad \| \text{fallen} \| = \lambda x. \text{Fell}(x)
\]

\[
\| \text{include} \| = \lambda x. \text{Bottle}(x)
\]

\( \{ z: \text{Bottle}(z) \} \supset \{ z: \text{Fell}(z) \} = 0 \)

the fallen include the bottles

the bottles are among the fallen

(27) Most bottles fell

\[
\| \{ z: \text{Bottle}(z) \} \cap \{ z: \text{Fell}(z) \} \| > \| \{ z: \text{Bottle}(z) \} - \{ z: \text{Fell}(z) \} \|
\]

\( \text{the bottles that fell outnumber the bottles that didn’t fall} \)

(28) Every bottle fell iff every bottle is a bottle that fell

Most bottles fell iff most bottles are bottles that fell

Some/No/The bottle(s) fell iff some/no/the bottle(s) izza/are bottle(s) that fell

Between five and eleven bottles fell iff between five and eleven bottles are bottles that fell

(29) \[ \text{DET NOUN} \ \text{PREDICATE} \] iff \[ \text{DET NOUN} \ \text{copula} \ [\text{NOUN that PREDICATE}] \]

(30) \| \text{bottle(s) that fell} \| = \lambda x. \text{Bottle}(x) \& \text{Fell}(x)

(31) Most bottles are bottles that fell

\[ \| \{ z: \text{Bottle}(z) \} \cap \{ z: \text{Bottle}(z) \& \text{Fell}(z) \} \| > \| \{ z: \text{Bottle}(z) \} - \{ z: \text{Bottle}(z) \& \text{Fell}(z) \} \|
\]

the bottles that are bottles that fell outnumber the bottles that are not bottles that fell

(32) The bottles are equinumerous with the things that fell

The bottles “samenummer” (correspond 1-to-1 with) the things that fell

(33) Equi bottles fell

\[ \| \text{Equi} \| = \lambda Y. \lambda X. \| \{ z: Y(z) \} \| = \| \{ z: X(z) \} \|
\]

\( \text{the bottles samenumber the fallen} \)

(34) Equi bottles are bottles that fell

\[ \| \{ z: \text{Bottle}(z) \} \| = \| \{ z: \text{Bottle}(z) \& \text{Fell}(z) \} \|
\]

\( \text{the bottles samenumber the bottles that fell} \)

(35) Equi bottles fell iff equi bottles are bottles that fell

\[ \| \{ z: \text{Bottle}(z) \} \| = \| \{ z: \text{Fell}(z) \} \| \iff \| \{ z: \text{Bottle}(z) \}\| = \| \{ z: \text{Bottle}(z) \& \text{Fell}(z) \} \|
\]

FALSE!

(36) The bottle fell

\[ \| \{ z: \text{Bottle}(z) \} \| = 1 \& \| \{ z: \text{Bottle}(z) \} \cap \{ z: \text{Fell}(z) \} \| > 0
\]

A bottle, and there was only one, fell

(37) Gre bottle fell

\[ \| \{ z: \text{Fell}(z) \} \| = 1 \& \| \{ z: \text{Bottle}(z) \} \cap \{ z: \text{Fell}(z) \} \| > 0
\]

A bottle was the only thing that fell

(38) Gre bottle is a bottle that fell

\[ \| \{ z: \text{Bottle}(z) \& \text{Fell}(z) \} \| = 1 \& \| \{ z: \text{Bottle}(z) \} \cap \{ z: \text{Fell}(z) \} \| > 0
\]

A bottle was the only bottle that fell

bottle-1 bottle-2 cup-1
fell didn’t fall fell

(37) can be false, while (38) is true
Using Barwise and Cooper’s terminology: determiners “live on” their internal arguments; some but not all relations between functions (from individuals to truth values) are conservative:

\[ X \mathbb{R} Y \quad \text{iff} \quad (X \cap_{x,t} Y) \mathbb{R} Y \]

But many otherwise “natural” second-order relations, like equinumerosity, are nonconservative. So why don’t we find determiners that—like the invented term ‘Equi’—express such relations? Keenan and Stavi suggest that all determiner meanings are constructible, in a conservativity-preserving way, from “basic” determiner meanings that are conservative. But even if this is right: why is the ‘Equi’-relation, which lies near the heart of arithmetic, not a basic determiner meaning? Why is ‘Most’ constructible, while ‘Equi’ is not? Why is ‘The’ more natural than ‘Gre’? If determiners are of type \( \langle x, t \rangle \), \( \langle x, t \rangle, \langle t \rangle \), why are certain functions of this type not possible determiner meanings?

Why don’t we lexicalize ‘only’ as a determiner that is the nonconservative converse of ‘every’? Compare the absence of “thematically inverted” verbs: \( \| \text{grote} \| = \lambda y. \lambda x. y \text{ wrote } x \).

The invented phrases ‘Equi bottles’ and ‘Gre bottle’ would not be restricted quantifiers. But this is another form of the question: if determiners express second-order relations, why do they express relations corresponding to restricted quantifiers (with the noun as restrictor)?

A related question, assuming that determiner phrases like ‘every bottle’ and ‘most bottles’ raise.

Suppose that the lexical meaning of ‘every’ would not be properly expressed if ‘every bottle’ was interpreted as an argument of ‘fell’. If a determiner takes an internal and external argument, like a transitive verb, maybe ‘every’ raises to “see” its external argument and “express itself.” If so, this lexical requirement is satisfied in the configuration above—with the determiner taking a sentential external argument, whose value is TRUE or FALSE, relative to any assignment of values to variables. But does this fit with the idea that ‘every’ indicates a relation between sets? A sentence with one variable is, in many ways, like the corresponding predicate of type \( \langle x, t \rangle \). But if ‘every’ raises to a position in which its lexical requirements are met, why do we still have to “cheat” by construing the open sentence as a device for expressing a function of type \( \langle x, t \rangle \)?

\[ \langle \text{fell} \rangle_{S} \text{ TRUE, relative to any assignment A, iff A(1) fell} \]

\[ \langle \lambda x. (\text{true iff x fell}) \rangle_{S} \text{ relative to any assignment A} \]

\[ \langle \text{dropped} \rangle_{S} \text{ TRUE, relative to any assignment A, iff A(1) dropped A(2)} \]

\[ \langle \lambda x. (\text{true iff A(1) dropped x}) \rangle_{S} \text{ relative to any assignment A} \]

\[ \text{Every bottle [with \( \langle \text{he dropped t} \rangle_{S} \)] has no truth-evaluable reading: why not?} \]
(42) They wrote them
\[\exists E \{\text{Agent}(E, \text{They}) \land \text{PastWriting}(E) \land \text{Theme}(E, \text{them})\}\]
\[\text{PastWriting}(E) \rightarrow \forall e: E \{\text{Event}(e)\}\]
\[\text{Event}(e) \rightarrow \forall x \{[\text{External}(e, x) \leftrightarrow \text{Agent}(e, x)] \land [\text{Internal}(e, x) \leftrightarrow \text{Theme}(e, x)]\}\]

(43) They wrote them
\[\exists E \{\text{External}(E, \text{They}) \land \text{PastWriting}(E) \land \text{Internal}(E, \text{them})\}\]

(44) External(E, They) \leftrightarrow \exists X: \forall x \{[\text{Assigns}(A, x, '1')] \land [\text{External}(E, X)]\}

(45) Internal(E, them) \leftrightarrow \exists X: \forall x \{[\text{Assigns}(A, x, '2')] \land [\text{Internal}(E, X)]\}

(46) There were one or more things such that
their ExternalParticipants (Agents) were the things Assigned to the first index, and
they were events of writing, and
their InternalParticipants (Themes) were the things Assigned to the second index

(47) Every bottle fell
\[\exists \{\text{PastFalling}(\_) \land \text{Theme}[, A(1)]\}\]
\[\text{Every}\text{.INT}...\]
\[\text{fell int}\]
\[t_{1: \sim \text{pl}} \text{ (singular variable)}\]

(48) \exists \{\text{Every}(\_) \land \text{Internal}[, \text{Bottle}(\_) \land 1^\text{External}[, \text{TRUE relative to A iff A(1) fell}]\}\]

PROPOSAL: determiners are predicates of “FregePairs,” ordered pairs of the form \(<v, x>\);
where the external element \(v\) is a truth value (\text{TRUE} or \text{FALSE}, \top or \bot), and
the internal element \(x\) is one of the things over which (singular and plural) variables range

Every(F) \text{ the Fs are all of the form } \langle \top, x \rangle
\[\forall f: F \{[\text{External}(f, \top)]\}\]

Internal[F, bottle(\_) ] \text{ the InternalParticipants of the Fs are the bottles}
\[tX: \text{Bottle}(X) [\text{Internal}(F, X)]\]

\(1^\text{External}[F, \text{TRUE relative to A iff A(1) fell}]\) \text{ the Fs conform to the following rule: } \top \text{ iff x fell}
\text{for each}_{f}, \text{its}_{f} \text{ ExternalParticipant is } \top \text{ iff}
\text{its}_{f} \text{ InternalParticipant fell}
If the noun ‘bottle’ appears as the internal argument of a determiner, then (relative to any assignment A) some FregePairs$_F$ (the Fs) are semantic values of that internal argument iff their$_F$ InternalParticipants are the bottles.

If the open sentence $\langle \text{fell } t_1 \rangle_S$ appears as the external argument of an indexed determiner, then relative to any assignment A, some FregePairs$_F$ (the Fs) are semantic values of that external argument iff for each$_F$ of them$_F$: its$_F$ ExternalParticipant is $\tau$ iff the open sentence is true relative to the (minimal) variant of A that assigns its$_F$ InternalParticipant to the indexed variable.

To illustrate, suppose a domain of exactly three bottles and two cups: $b_1, b_2, b_3, c_1, c_2$

\[
\text{[Every bottle]}_D \quad \langle \text{fell } \_ \rangle_S \quad i
\]

(49) $\exists F\{\text{Every}(F) \& \text{Internal}(F, \text{bottle}( \_ )) \& i^\text{\text{External}}(F, \text{true relative to A iff A}(i) \text{ fell})\}$

The phrase ‘Every bottle’ imposes two conditions on semantic values$_F$ (the Fs): Every one of them$_F$ must be of the form $\langle \tau, x \rangle$, and their$_F$ InternalParticipants must be (all and only) the bottles.
So there is only one “choice” of FregePairs that will satisfy the determiner phrase:
$\langle \tau, b_1 \rangle, \langle \tau, b_2 \rangle, \langle \tau, b_3 \rangle$

These three FregePairs are (together) the semantic values of ‘Every bottle’.
Every bottle fell iff these FregePairs conform to the following rule: $\tau$ iff $x$ fell.

\begin{align*}
\text{Some}(F) & \quad \exists f:F[\text{External}(f, \tau)] \\
\text{No}(F) & \quad \neg \exists f:F[\text{External}(f, \tau)] \\
\text{Most}(F) & \quad \exists Y \forall f\{\text{Outnumber}(Y, N) \& [Y \leftrightarrow Ff \& \text{External}(f, \tau)] \& [N \leftrightarrow Ff \& \text{External}(f, \bot)]\}
\end{align*}

(50) Every bottle fell
$\exists F\{\text{EVERY}(F) \& \forall X:\text{Bottle}(X)[\text{Internal}(F, X)] \&$
for each$_F$: its$_F$ ExternalParticipant is $\tau$ iff its$_F$ InternalParticipant fell

(51) Every bottle is a bottle that fell
$\exists F\{\text{EVERY}(F) \& \forall X:\text{Bottle}(X)[\text{Internal}(F, X)] \&$
for each$_F$: its$_F$ ExternalParticipant is $\tau$ iff its$_F$ InternalParticipant is a bottle that fell

If the bottles are the InternalParticipants of the Fs, and f is an F whose InternalParticipant is $x$, then trivially: $[\text{External}(f, \tau) \leftrightarrow \text{Fell}(x)] \quad \text{iff} \quad [\text{External}(f, \tau) \leftrightarrow \text{Bottle}(x) \& \text{Fell}(x)]$

The “trick” is to not ignore the syntax: external arguments of determiners are sentential—(assignment-relative) expressions of type $<t>$, and not disguised predicates of type $<x, t>$. We don’t need variables ranging over functions to capture Frege’s insights about quantification. We don’t need to associate arguments of determiners with extensions, and maybe we shouldn’t.

(52) $\langle \text{he dropped } \_ \rangle_S$  (53) $^\text{[
\text{Every bottle}]}_D$  $\langle \text{he dropped } \_ \rangle_S$

(54) There are many sets. None of them are selfelemental. But all of them are selfidentical.
Appendix: Everybody Needs Event Variables

(1) Plum stabbed Green quickly with a knife
(2) Plum stabbed Green with a knife quickly
(3) Plum stabbed Green quickly
(4) Plum stabbed Green with a knife
(5) Plum stabbed Green

see Davidson (1967, 1985), Taylor (1985), Parsons (1990), etc.

(1a) At least one stabbing of Green by Plum was done quickly and with a knife
(1b) $\exists e \{\text{PastStabOfGreenByPlum}(e) \land \text{Quick}(e) \land \text{With}(e, \text{a knife})\}$
(1c) $\exists e \{\text{Agent}(e, \text{Plum}) \land \text{PastStab}(e) \land \text{Theme}(e, \text{Green}) \land \text{Quick}(e) \land \exists x: \text{Knife}(x)[\text{With}(e, x)]\}$

(6) $\exists x[\text{Red}(x) \land \text{Ball}(x)] \rightarrow \exists x[\text{Red}(x)] \land \exists x[\text{Ball}(x)]$
(7) $\exists x[\text{Red}(x)] \land \exists x[\text{Ball}(x)] \rightarrow \exists x[\text{Red}(x) \land \text{Ball}(x)]$

see Castañeda (1967), Carlson (1985), Higginbotham (1985)

(8) Plum kicked Green
(9) Green was kicked by Plum
(10) Green was kicked
(11) Plum kicked
(12) Plum kicked the ball
(13) Plum kicked the ball to Green
(14) Plum kicked Green the ball
(15) The ball was kicked
(16) Plum kicked to Green

(8a) $\exists e \{\text{Agent(Plum)} \land \text{PastKick}(e) \land \text{Theme}(e, \text{Green})\}$
(9a) $\exists e \{\text{Theme}(e, \text{Green}) \land \text{PastKick}(e) \land \text{Agent(Plum)}\}$
(10a) $\exists e \{\text{Theme}(e, \text{Green}) \land \text{PastKick}(e)\}$
(11a) $\exists e \{\text{Agent}(e, \text{Plum}) \land \text{PastKick}(e)\}$
(12a) $\exists e \{\text{Agent}(\text{Plum}) \land \text{PastKick}(e) \land \text{Theme}(e, \text{the ball})\}$
(13a) $\exists e \{\text{Agent}(\text{Plum}) \land \text{PastKick}(e) \land \text{Theme}(e, \text{the ball}) \land \text{Goal}(e, \text{Green})\}$
(14a) $\exists e \{\text{Agent}(\text{Plum}) \land \text{PastKick}(e) \land \text{Goal}(e, \text{Green}) \land \text{Theme}(e, \text{the ball})\}$
(15a) $\exists e \{\text{Theme}(e, \text{the ball}) \land \text{PastKick}(e)\}$
(16a) $\exists e \{\text{Agent}(e, \text{Plum}) \land \text{PastKick}(e) \land \text{Goal}(e, \text{Green})\}$

(17) On Monday, Plum hit Green the ball with a red stick
(18) On Tuesday, Plum hit the ball to Green with a blue stick
(19) On Wednesday, Plum hit the balls to Green with red sticks
(20) On Thursday, they hit twenty balls to them with blue sticks

(21) The senator called the millionaire from Texas

(a) The senator called the millionaire from Texas, and the millionaire was from Texas
(b) The senator called the millionaire from Texas, and the call was from Texas
(c) The senator called the millionaire from Texas, and the senator was from Texas

(G) $[\text{The senator} \ [\text{called} \ [\text{the} \ [\text{millionaire} \ [\text{from Texas}]]]]]$
(M) $\exists e \{\text{tx:Senator}(x)[\text{Agent}(e, x)] \land \text{PastCall}(e) \land \text{tx:Mill}(x)\text{&From}(x, \text{Texas})[\text{Theme}(e, x)]\}$
(G') $[\text{The senator} \ [[\text{called} \ [\text{the millionaire}]] \ [\text{from Texas}]]]$
(M') $\exists e \{\text{tx:Senator}(x)[\text{Agent}(e, x)] \land \text{PastCall}(e) \land \text{tx:Mill}(x)[\text{Theme}(e, x)] \land \text{From}(e, \text{Texas})\}$