Logical Form and LF

We can use sentences to present arguments, some of which are valid. This suggests that premises and conclusions, like sentences, have structure. This in turn raises questions about how logical structure is related to grammar, and how grammatical structure is related to thought and truth.

1. Patterns of Reason and Traditional Grammar

Consider the argument indicated with (1).

(1) Chris swam if Pat was asleep, and Pat was asleep; so Chris swam.

An ancient thought is that endlessly many such arguments have the following form: **Q** if **P**, and **P**; so **Q**. The conclusion is evidently part of the first premise, which has the second premise as another part. Let us say that the variables, represented in bold, range over *propositions*. This leaves it open what these potential premises/conclusions are: sentences, statements, states of affairs, or whatever. But presumably, propositions can be evaluated for truth or falsity; they can be endorsed or rejected. And sentences can be used to indicate (or "express") them. In ordinary conversation, the context partly determines which proposition (if any) is indicated. A speaker s might use 'I am tired' at time t to express one proposition, and use it at time t' to express another, while speaker s' uses the same sentence at t to express a third proposition. Context sensitivity, of various kinds, is ubiquitous in natural language. But if only for simplicity, let's assume that we can speak of the proposition indicated in a given context with a declarative sentence.[1]

Even given that propositions can be complex, it is not obvious that all valid inferences are valid by virtue of propositional structure. But this thought has served as an ideal for the study of logic, at least since Aristotle's treatment of syllogisms like (2).

(2) Every politician is deceitful, and every senator is a politician;
so every senator is deceitful.

The first premise—that every politician is deceitful—seems to have several parts, each of which is a part of the second premise or the conclusion. And conditionals of the form 'Every *P* is *D*, and every *S* is a *P*, then every *S* is *D*' are sure to be true. (So the corresponding argument schema is valid.) Similarly: if no *P* is *D*, and some *S* is a *P*, then some *S* is not *D*. The variables, represented here in italics, are intended to range over certain parts of propositions. Nouns like 'politician' and adjectives like 'deceitful' are general terms, since they can apply to more than one individual. If propositions contain corresponding predicates, then even "simple" propositions (with no propositional parts) exhibit logical structure. And the network of inferential relations revealed by syllogistic logic suggests that many propositions contain a quantificational element (indicated with words like 'every', 'some', or 'no') along with two predicates.

On some views, discussed below, the conclusion of (3) also has this form.

(3) Every planet is bright, and Venus is a planet; so Venus is bright.

Though one can describe the validity of (3), less tendentiously, in terms of the following schema: every *P* is *D*, and *n* is a *P*; so *n* is *D*; where the lower-case variable ranges over proposition-parts of the sort indicated by names. This highlights the intuitive division of declarative sentences into subjects and predicates: 'Every planet / is bright', 'Venus / is bright', 'Some politician / swam', etc. And on Aristotle's view, propositions are like sentences in this respect. With regard to the proposition that Venus is bright, he would have said that bright(ness) belongs to—or in modern terms, is predicated of—Venus; in the proposition that every politician is deceitful, deceitfulness is predicated of every politician. Using slightly different terminology, later theorists said that simple propositions have categorical form: subject-copula-predicate; where a copula, indicated with a word like 'is' or 'was', links a subject (which can consist of a quantifier and predicate) to a predicate.[2] A sentence like 'Every politician swam' can be paraphrased, as in 'Every politician was an individual who did some swimming', So perhaps the categorical form of the indicated proposition is not fully reflected with the first sentence. Maybe 'swam' abbreviates 'was one who did some swimming', much as 'bachelor' is arguably short for 'unmarried marriageable man'.

The proposition that *every planet is bright if Venus is bright* seems to be a compound of categorical propositions. And the proposition that *not only every planet is bright* apparently extends a categorical proposition, via elements indicated with 'not' and 'only'. Medieval logicians explored, with great ingenuity, the hypothesis that all propositions are composed of categorical propositions and a small number of so-called syncategorematic elements. Many viewed this project, in part, as an attempt to uncover principles of a mental language common to all thinkers. From this perspective, one expects a few differences between propositional structure and the manifest structure of spoken sentences. For example, Ockham held that a mental language would not need Latin's declensions. And the ancient Greeks were aware of sophisms like 'Since that dog is a father, and that dog is yours, that dog is your father', which contrasts with the superficially parallel but impeccable inference, 'Since that dog is a mutt, and that mutt is yours, that dog is your mutt'. Still, the assumption was that spoken sentences reflect the most important aspects of propositional form, including subject-predicate structure. The connection between logic and grammar was thought to run deep. But there were known problems.

2. Motivations for Revision

Some valid schemata, like (4), are reducible to others.

(4) Some *P* is not *D*, and every *S* is *D*; so not every *P* is an *S*.

If some *P* is not *D*, then trivially, not every *P* is *D*. So if it is also true that every *S* is *D*, it must be false that every *P* is an *S*; otherwise, every *P* is *D*. This fully general reasoning tells us that each instance of (4) is valid. And one suspects that there are relatively few *basic* inferential patterns. Perhaps '**Q** if **P**, and **P**; so **Q**' is so obvious that logicians should take it as axiomatic. But how many inference patterns are plausibly regarded as logically fundamental?

Medieval logicians made great strides in reducing syllogistic logic to two principles, *dictum de omni* and *dictum de nullo*. Often, perhaps even typically, replacing a predicate with a less restrictive predicate corresponds to a valid inference. Suppose that Rex is a brown dog. Then Rex is a dog. Replacing 'brown dog' with the less restrictive 'dog' yields a valid inference in environments like 'Rex is __'. (And if 'animal' is even less restrictive, then every dog is an animal, and it follows that Rex is an animal.) But sometimes, as in cases involving negation, the direction of valid inference is reversed. In the environment 'Rex is not __', replacing 'dog' with 'brown dog' yields a valid inference; if Rex is not a dog, then Rex is not a brown dog. It turns out that many valid inference forms, including Aristotle's original examples, can be captured in these simple terms. Nonetheless, traditional logic/grammar was inadequate.

Prima facie, propositions involving relations do not have categorical form. One can paraphrase 'Juliet kissed Romeo' with 'Juliet was a kisser of Romeo'. But 'kisser of Romeo' differs, in ways that matter to inference, from predicates like 'politician'. If some kisser of Romeo died, it follows that someone was kissed; whereas the proposition that some politician died has no parallel logical consequence to the effect that the someone was __-ed. Correlatively, if Juliet kissed Romeo, it follows that Juliet kissed someone. The proposition that Juliet kissed someone is of interest, even if we express it with 'Juliet was a kisser of someone', because a quantifier appears *within* the predicate. And complex predicates of this sort were problematic.

If 'respects some doctor' and 'respects some senator' indicate nonrelational proposition-parts, like ‘is tall’ and ‘is ugly’, then the argument indicated with (5)

(5) Some patient respects some doctor, and every doctor is a senator;
so some patient respects some senator

has the following form: Some *P* is *T*, and every *D* is an *S*; so some *P* is *U*. But this schema is not valid. Evidently, ‘respects some doctor’ and ‘respects some senator’ are logically related, in ways that ‘is tall’ and ‘is ugly’ are not. If we allow for propositions with relational components, introducing a variable ‘$\underline{\mathrm{R}}$’ ranging over relations, we can formulate valid schemata like the following: some *P* $\underline{\mathrm{R}}$ some *D*, and every *D* is an *S*; so some *P* $\underline{\mathrm{R}}$ some *S*. But this is a poor candidate for a basic inference pattern. And the problem remains. Inference (6) is valid.

(6) Every patient who met every doctor is tall, and
some patient who met every doctor respects every senator;
so some patient who respects every senator is tall.

But many inferences of the form ‘Every *P* is *T*, and some *P* $\underline{\mathrm{R}}$ every *S*; so some *U* is *T*’ are not. One can abstract a valid schema that covers (6), letting parentheses indicate a relative clause: every *P*($\underline{\mathrm{R1}}$ every *D*) is *T*, and some *P*($\underline{\mathrm{R1}}$ every *D*) $\underline{\mathrm{R2}}$ every *S*; so some *P*($\underline{\mathrm{R2}}$ every *S*) is *T*. But there can be still further quantificational structure within the predicates. And so on. It seems that quantifiers can be logically significant constituents of predicates, and not just devices for creating proposition-frames into which monadic predicates can be inserted.

Relative clauses posed further questions. If every patient respects some doctor, then every old patient respects some doctor. This is expected if the phrase ‘every (old) patient’ is governed by *dictum de nullo*: the direction of valid inference is from ‘patient’ to ‘old patient’. But in (7-8),

(7) No lawyer who saw every patient respects some doctor

(8) No lawyer who saw every old patient respects some doctor

the valid inference is from ‘old patient’ to ‘patient’. One can say that the typical direction of implication, from more to less restrictive predicates, has been “reversed twice” in (7-8). But one wants a detailed account of propositional structure that explains why and how this is so.[3]

3. Functions and Formal Languages

Frege (1879, 1892a) showed how to resolve these difficulties and more. But on his view, propositions have "function-argument" structure, as opposed to subject-predicate structure. Frege's system of logic, the single greatest contribution to the subject, required a substantial distinction between logical form and grammatical form as traditionally conceived. This had an enormous impact on subsequent discussions of thought and its relation to language.

We can represent the successor function, with a variable ranging over integers, as follows: $S(x) = x + 1$. This function takes integers as arguments; and the value of the function, given a certain argument, is the successor of that argument. Correspondingly, we can say that the arithmetic expression 'S(3)' exhibits function-argument structure, and that the "Semantic Value" (*Bedeutung*) of this complex expression is the number four—i.e., the value of the relevant function given the relevant argument. Likewise, the division function can be represented as a mapping from ordered pairs of numbers to quotients: $Q(x, y) = x/y$. And we can say that the Semantic Value of 'Q(8, 4)' is the number two. Functions can also be specified conditionally. Consider the function that maps every even integer onto itself, and every odd integer onto its successor: $C(x) = x$ if x is even, and $x + 1$ otherwise; $C(1) = 2$, $C(2) = 2$, $C(3) = 4$, *etc*. By itself, however, no function has a value. Frege's metaphor, encouraged by his claim that we can indicate functions with expressions like '$S(\ ) = (\ ) + 1$', is that a function is saturated by arguments of the right sort.[4]

On Frege's view, 'Mary sang' indicates a proposition with the following structure: Sang(Mary), with 'Mary' indicating the argument. Frege thought of the relevant function as a conditional mapping from individuals in a given domain to truth values: $\text{Sang}(x) = \mathbf{t}$ if x sang, and $\mathbf{f}$ otherwise; where '$\mathbf{t}$' and '$\mathbf{f}$' stand for values such that for each individual x, $\text{Sang}(x) = \mathbf{t}$ iff x sang, and $\text{Sang}(x) = \mathbf{f}$ iff x did not sing. The proposition that John admired Mary was said to have a functional component, indicated by the transitive verb, saturated by an ordered pair of arguments: Admired(John, Mary); where $\text{Admired}(x, y) = \mathbf{t}$ if x admired y, and $\mathbf{f}$ otherwise. According to Frege, the proposition that Mary was admired by John has the same function-argument structure, even though 'Mary' is the subject of the passive sentence. And his treatment of quantified propositions departs radically from previous conceptions of logical form.

Let F be the function indicated by 'sang', so that Mary sang iff $F(\text{Mary}) = \mathbf{t}$. Someone sang iff some individual x is such that $F(x) = \mathbf{t}$. Using a modern variant of Frege's notation, someone sang iff $\exists x[\text{Sang}(x)]$; where the quantifier '$\exists$' binds the variable '$x$'. Every individual in the domain sang iff F maps each individual onto **t**; in formal notation, $\forall x[\text{Sang}(x)]$. A quantifier binds each occurrence of its variable, as in '$\exists x[D(x) \ \& \ C(x)]$', which reflects the logical form of 'someone is deceitful and clever'. With regard to the proposition that some politician is deceitful, traditional grammar suggests the division 'Some politician / is deceitful'. But for Frege, the logically relevant division is between the existential quantifier and the rest: $\exists x[P(x) \ \& \ D(x)]$; someone is both a politician and deceitful. With regard to the proposition that every politician is deceitful, Frege again says that the logically important division is between the quantifier and its scope: $\forall x[P(x) \rightarrow D(x)]$; everyone is such that if he is a politician then he is deceitful. But in this case, the quantifier combines with a conditional predicate, suggesting that grammar is doubly misleading. The phrase 'every politician' does not indicate a constituent of the proposition. Grammar also masks a logical difference between the existential and universally quantified propositions: predicates are related conjunctively in the former, but conditionally in the latter.

Moreover, on Frege's view, two quantifiers can bind two unsaturated positions associated with a function that takes a pair of arguments. So the proposition that everyone trusts everyone has a very noncategorical form: $\forall x \forall y[T(x, y)]$. Given that 'John' and 'Mary' indicate arguments, it follows that John trusts everyone, and that everyone trusts Mary—$\forall y[T(j, y)]$ and $\forall x[T(x, m)]$. And it follows from all three propositions that John trusts Mary: $T(j, m)$. Frege's rules of inference capture this. A variable bound by a universal quantifier can be replaced with a name, and a name can be replaced with a variable bound by an existential quantifier: $\forall x(...x...)$, so $...n...$; and $...n...$, so $\exists x(....x....)$. Given that John trusts Mary, it follows that someone trusts Mary, and that John trusts someone: $T(j, m)$; so $\exists x[T(x, m)]$, and $\exists x[T(j, x)]$. And it follows from all three propositions that someone trusts someone: $\exists x \exists y[T(x, y)]$. A single quantifier can bind multiple argument positions, as in '$\exists x[T(x, x)]$'; but this means that someone trusts herself.

Mixed quantification introduces an interesting wrinkle. The propositions indicated with '$\exists x \forall y[T(x, y)]$' and '$\forall y \exists x[T(x, y)]$' differ. We can paraphrase the first as 'there is someone who trusts everyone' and the second as 'everyone is trusted by someone or other'. The second follows from the first, but not *vice versa*. This suggests that 'Someone trusts everyone' can be used to indicate two different propositions. According to Frege, this is further evidence that natural language is not suited to the task of representing propositions perspicuously. Natural language is good for efficient human communication. But he suggested that natural language is like the eye, while a good formal language can be like a microscope that reveals structure not otherwise observable. On this view, propositional form is revealed by the structure of a sentence in an ideal formal language, a *Begriffsschrift* (Concept-Script); where the sentences of such a language exhibit function-argument structures, as opposed to subject-predicate structures.

The real power of Frege's logic is most evident in his discussion of the Dedekind-Peano axioms for arithmetic—and in particular, how the proposition that every number has a successor is logically related to more basic truths.[5] But here, it will be enough to consider (9-10) and the corresponding Fregean analyses.

(9) Every patient respects some doctor

(9a) $\forall x\{P(x) \dashrightarrow \exists y[D(y) \;\&\; R(x,y)]\}$

(10) Every old patient respects some doctor

(10a) $\forall x\{[O(x) \;\&\; P(x)] \dashrightarrow \exists y[D(y) \;\&\; R(x,y)]\}$

In Frege's logic, (10a) follows from (9a), as desired. But one can also account for why the proposition indicated with (7) follows from the one indicated with (8).

(7) No lawyer who saw every patient respects some doctor

(7a) $\neg\exists x\{Lx \;\&\; \forall y[P(y) \dashrightarrow S(x,y)] \;\&\; \exists z[D(z) \;\&\; R(x,z)]\}$

(8) No lawyer who saw every old patient respects some doctor

(8a) $\neg\exists x\{Lx \;\&\; \forall y\{[O(y) \;\&\; P(y)] \dashrightarrow S(x,y)\} \;\&\; \exists z[D(z) \;\&\; R(x,z)]\}$

In this way, one can handle a wide range of inferences that had puzzled logicians since Aristotle.

Frege originally spoke as though propositional constituents just were the relevant functions and (ordered n-tuples of) entities that such functions map to truth-values. But he refined this view in light of his distinction between *Sinn* and *Bedeutung*: the *Sinn* of an expression was said to be a "way of presenting" the corresponding *Bedeutung*, which would be an entity, truth-value, or function. We can think of 'Hesperus' as an expression that presents the evening star (Venus) as such, while 'Phosphorus' presents the morning star (also Venus) in a different way. Likewise, we can think of 'is bright' as an expression that presents a certain function in a certain way, and 'Hesperus is bright' as a sentence that presents its truth-value in a certain way—i.e., as the value of the function in question given the argument in question (**t** if Hesperus is bright, and **f** otherwise). From this perspective, propositions are sentential ways of presenting truth-values. Frege could thus distinguish the proposition that Hesperus is bright from the proposition that Phosphorus is bright, even though these propositions are alike with regard to the relevant function and argument.[6] Likewise, he could distinguish the trivial claim that Hesperus is Hesperus from the nontrivial claim that Hesperus is Phosphorus. This is an attractive view. For intuitively, ancient astronomers were correct not to regard the inference 'Hesperus is Hesperus, so Hesperus is Phosphorus' as an instance of the valid schema '**P**, so **P**'. But this raised questions about what the *Sinn* of an expression really is, what "presentation" could amount to, and what to say about a name with no *Bedeutung*.

4. Descriptions and Analysis

It can seem obvious that names and descriptions, like 'John' and 'the tall boy from Canada', indicate arguments as opposed to functions. So one might think that the logical form of any proposition indicated with 'The tall boy from Canada sang' is simply 'Sang(b)', where 'b' stands for the individual in question. But this makes the linguistic elements of the description logically irrelevant. And if the tall boy from Canada sang, then a boy from Canada sang; hence, a boy sang. Moreover, 'the' apparently implies uniqueness in a way that 'some' does not.[7]

Russell (1919) held that such implications reflect logical form. On his view, a proposition expressed with 'The boy sang' has the following structure: $\exists x\{\text{Boy}(x)\ \&\ \forall y[\text{Boy}(y) \dashrightarrow y = x]\ \&\ \text{Sang}(x)\}$. As we'll see, the middle conjunct is just a way of expressing uniqueness with Fregean tools, and it can be rewritten without affecting the main point. According to Russell, even if a speaker refers to a certain boy when saying 'The boy sang', that boy is not a constituent of the proposition indicated: the proposition has the form of an existential quantification, not the form of a function saturated by (an argument that is) the boy referred to; and in this respect, 'the boy' is like 'some boy'. Though on Russell's view, not even 'the' indicates a propositional constituent. This extended Frege's idea that natural language is misleading.

As Russell stressed, a description can be meaningful without describing any thing. While France is kingless, 'The present king of France is bald' can be used to indicate a proposition. Call this proposition '**Frank**'. If **Frank** consists of the function indicated with 'Bald( )', saturated by an entity indicated with 'The present king of France', there must *be* such an entity. But appeal to nonexistent kings is, or ways of presenting them, is dubious at best. Russell held instead that **Frank** is a quantificational proposition of the form '$\exists x\{K(x)$ & $\forall$y$[K(y)$ --> $y = x]$ & $B(x)\}$'. In which case, the following reasoning is spurious: since **Frank** is true or false, the present king of France is bald or not; so there is a king of France, who is either bald or not. On Russell's view, **Frank** is false, given that $\neg\exists x[K(x)]$. It hardly follows that $\exists x\{K(x)$ & $[B(x)$ v $\neg B(x)]\}$. But the ambiguity of natural language may lead us to *confuse* the true negation of **Frank** with the following false claim: $\exists x\{K(x)$ & $\forall y[K(y)$ --> $y = x]$ & $\neg B(x)\}$. According to Russell, puzzles about "nonexistence" can be resolved without dubious metaphysics, given the right views about logical form. (This invited the thought, developed by Wittgenstein [1921] and others, that other philosophical puzzles might dissolve if we properly understood the logical forms of our claims.)

Russell also held that we are directly acquainted with the constituents of propositions we entertain. But at least typically, we are not directly acquainted with the mind-independent bearers of proper names. This led Russell to say that typical names are disguised descriptions, not labels for propositional constituents. On this view, 'Hesperus' is semantically associated with a complex predicate—say, for illustration, a predicate of the form '$E(x)$ & $S(x)$'. In which case, 'Hesperus is bright' indicates a proposition of the form '$\exists x\{[E(x)$ & $S(x)]$ & $\forall y\{[E(y)$ & $S(y)]$ --> $y = x]\}$ & $B(x)\}$'. It follows that Hesperus exists iff $\exists x[E(x)$ & $S(x)]$; and this would be challenged by Kripke (1980). But Russell offered an attractive account of why the proposition that Hesperus is bright differs from the proposition that Phosphorus is bright. He could say that 'Phosphorus is bright' indicates a proposition of the form '$\exists x\{[M(x)$ & $S(x)]$ & $\forall y\{[M(y)$ & $S(y)]$ --> $y = x]\}$ & $B(x)$'; where '$E(x)$' and '$M(x)$' indicate different functions, specified (respectively) in terms of evenings and mornings. This leaves room for the discovery that '$E(x)$ & $S(x)$' and '$M(x)$ & $S(x)$' both indicate functions that map Venus and nothing else to the truth-value **t**.

5. Regimentation and Quantification

Positing unexpected logical forms thus seemed to have explanatory payoffs. This invited attempts to provide analyses of propositions, and accounts of the "conventions" governing natural language, with the aim of saying how sentences could be used to indicate propositions. The logical positivists held that the conventional meaning of a declarative sentence is (ideally) a procedure for determining the truth or falsity of that sentence. But they had little success in formulating rules that were plausible both as descriptions of how ordinary speakers understand natural language, and bases for the envisioned analyses. And until Montague (1970), discussed briefly below, there was no real progress in showing how to systematically associate quantificational constructions of natural language with Fregean logical forms.

Carnap (1950) developed a sophisticated position according to which philosophers could (and should) articulate alternative sets of conventions for associating sentences of a language with propositions. Within each such language, the conventions would determine what follows from what. But one would have to decide, on broadly pragmatic grounds, which interpreted language was best for certain purposes. On this view, questions about "the" logical form of an ordinary sentence are in part questions about which conventions one should adopt. This was, in many ways, an attractive view. But it also raised a worry. Perhaps the structural mismatches between sentences of a natural language and sentences of a *Begriffsschrift* are so severe that we cannot systematically associate the former with the latter.

Quine (1951, 1960) combined behaviorist psychology with a conception of logical form similar to Carnap's. The result was an influential view according to which: there is no fact of the matter about which proposition a speaker indicates with a sentence of natural language, because talk of propositions is at best a way of talking about how we should regiment our verbal behavior for purposes of scientific inquiry; claims about logical form are in this sense evaluative; and such claims are not determined by the totality of facts concerning our dispositions to use language. From this perspective, mismatches between logical and grammatical form are expected. Quine also held that decisions about how to associate natural and formal sentences should be made holistically. As he sometimes put it, the "unit of translation" is an entire language, not a particular sentence. On this view, one can regiment a sentence S of natural language with a structurally mismatching sentence μ of a formal language—even if it seems (locally) implausible that S is used to indicate the proposition associated with μ—so long as the association between S and μ is part of a more general system of regimentation that is at least as good as any alternative.

For present purposes, we can abstract from debates about whether this is plausible. But one aspect of Quine's thought, about the kind of regimented language we should use, proved esepcially important for discussions of logical form. Recall that Frege's *Begriffsschrift* was designed to capture the Dedekind-Peano axioms for arithmetic, including the axiom of induction.

This required quantification into positions occupiable by predicates. In current notation, Frege allowed for formulae like ‘$(Fa \ \&\ Fb) \dashrightarrow \exists X(Xa \ \&\ Xb)$’ and ‘$\forall x \forall y[x = y \leftarrow\!\!\dashrightarrow \forall X(Xx \leftarrow\!\!\dashrightarrow Xy)]$’. And he took second-order quantification to be quantification over functions. On this construal, ‘$\exists X(Xa \ \&\ Xb)$’ is true iff: there is a function that maps the individual *a* and the individual *b* onto the truth-value **t**. Frege also assumed that each predicate indicates a function such that for each individual *x*, the function maps *x* to **t** iff *x* satisfies the predicate. This generated Russell’s Paradox, given predicates like ‘is not an element of itself’. And for various reasons, Quine and others advocated restriction to the *first-order* fragment of Frege’s logic, disallowing quantification into positions occupied by predicates.[8] From this perspective, we should replace ‘$(Fa \ \&\ Fb) \dashrightarrow \exists X(Xa \ \&\ Xb)$’ with first-order quantification over sets, as in ‘$(Fa \ \&\ Fb) \dashrightarrow \exists s \forall x\{[x \in s \longleftrightarrow Fx] \ \&\ a \in s \ \&\ b \in s)\}$’; where this conditional is a nonlogical hypothesis. Insisting on first-order regimentation now seems tendentious; see Boolos (1998). But it fueled the idea that logical form can diverge wildly from grammatical form, since first-order regimentations of natural sentences are often highly artificial (and in some cases, unavailable).

Another strand of thought in analytic philosophy—pressed by Wittgenstein (1953) and developed by others, including Strawson and Austin—also suggested that a single sentence could be used (on different occasions) to express different kinds of propositions. Strawson (1950) argued that *pace* Russell, a speaker could use an instance of ‘The F is G’ to express a singular proposition about a specific individual: namely, the F in the context at hand. According to Strawson, sentences themselves do not indicate propositions; and speakers can use ‘The boy is tall’ to express a proposition with the contextually relevant boy as a constituent. Donnellan (1966) went on to argue that a speaker could even use an instance of ‘The F is G’ to express a singular proposition about an individual that is not an F. Such considerations suggested that relations between natural language sentences and propositions are (at best) very complex and mediated by speakers’ intentions.[9] This bolstered the Quine-Carnap idea that questions about the structure of premises and conclusions are really questions about how we *should* talk, when trying to describe the world, much as logic itself seems to be concerned with how we *should* reason. From this perspective, the connections between logic and grammar seemed rather shallow.

On the other hand, more recent work on quantifiers suggests that the divergence had been exaggerated, in part because of how Frege’s idea of variable-binding was originally

implemented. Consider again the proposition that some boy sang, and the proposed logical division:
$\exists x[\mathrm{Boy}(x)\ \&\ \mathrm{Sang}(x)]$. This is one way to regiment the English sentence. But one can also offer a "logical paraphrase" that parallels the grammatical division between 'some boy' and 'sang': for some individual x such that x is a boy, x sang. One can formalize this by using restricted quantifiers, which incorporate restrictions on the domain over which bound variables range. For example, '$\exists x{:}\mathrm{Boy}(x)$' is an existential quantifier that binds a variable ranging over boys in the relevant domain. So '$\exists x{:}\mathrm{Boy}(x)[\mathrm{Sang}(x)]$' means that some boy sang. And logic provides no reason for preferring '$\exists x[\mathrm{Boy}(x)\ \&\ \mathrm{Sang}(x)]$'.

Universal quantifiers can be restricted, as in '$\forall x{:}\mathrm{Boy}(x)[\mathrm{Sang}(x)]$', interpreted as follows: for every individual x such that $\mathrm{Boy}(x)$, x sang; that is, every boy sang. Restrictors can also be complex, as in 'Some tall boy sang' or 'Every boy who respects Mary sang', rendered as '$\exists x{:}\mathrm{Tall}(x)\ \&\ \mathrm{Boy}(x)[\mathrm{Sang}(x)]$' and '$\forall x{:}\mathrm{Boy}(x)\ \&\ \mathrm{Respects}(x, m)[\mathrm{Sang}(x)]$'. So it seems that the inferential difference between 'Some boy sang' and 'Every boy sang' lies entirely with the propositional contributions of 'Some' and 'Every' after all—not with the different contributions of '&' and '-->'. Words like 'someone', and the grammatical requirement that 'every' be followed by a noun (phrase), reflect the fact that natural language employs restricted quantifiers. Expressions like 'every boy' are composed of a determiner and a noun. So one can think of determiners like 'every' as words that can combine with an ordered pair of predicates to form a sentence, much as transitive verbs can combine with an ordered pair of names to form a sentence. And this analogy, between determiners and transitive verbs, has a semantic correlate.

On Frege's view, the function indicated by 'loves' maps the ordered pair $\langle x, y\rangle$ to the truth value **t** iff x loves y. Here, 'y' corresponds to the verb's *internal* argument (or direct object), which combines with the verb to form a phrase, as in 'loves Juliet'; 'x' corresponds to the verb's *external* argument. In 'Every boy sang', 'boy' is the internal argument of 'Every', since 'Every boy' is a phrase, and we can think of 'sang' as the external argument. So following Frege, let 'X' and 'Y' be second-order variables ranging over functions, from individuals to truth values. Then we can say that the function indicated by 'Every' maps the ordered pair $\langle X, Y\rangle$ to **t** iff the extension of X *includes* the extension of Y. Likewise, the function indicated by 'Some' maps $\langle X, Y\rangle$ to **t** iff the extension of X *intersects with* the extension of Y.[10]

This suggests an alternative to Russell's treatment of 'The'; see Montague (1970). We can rewrite '$\exists x\{\text{Boy}(x)$ & $\forall y[\text{Boy}(y) \dashrightarrow x = y]$ & $\text{Sang}(x)\}$' as '$\exists x{:}\text{Boy}(x)[\text{Sang}(x)]$ & $|\text{Boy}| = 1$', interpreted as follows: for some individual x such that x is a boy, x sang, and exactly one (relevant) individual is a boy. Neither 'the boy' nor 'the' corresponds to a constituent of this formalism. But one can depart farther from Russell's notation, while stressing that 'The' is relevantly like 'Some'. One can analyze 'The boy sang' as '$!x{:}\text{Boy}(x)[\text{Sang}(x)]$', specifying the propositional contribution of '!' as follows: $!x{:}Y(x)[X(x)] = \mathbf{t}$ iff the extensions of X and Y intersect, and Y maps exactly one (relevant) individual to **t**. This preserves Russell's central claim. Even if a speaker refers to a boy in saying 'The boy sang', that boy is not a constituent of the quantificational proposition indicated with '$!x{:}\text{Boy}(x)[\text{Sang}(x)]$'; see Neale (1990) for discussion. But far from showing that logical form diverges from grammatical form, the second-order restricted-quantifier notation suggests that in this case, propositional structure parallels sentential structure.

6. Transformational Grammar

Still, the subject/predicate structure of 'Mary / trusts every doctor' diverges from the restricted quantifier formula '$\forall y{:}\text{Doctor}(y)[\text{Trusts}(\text{Mary}, y)]$'. We can rewrite '$\text{Trusts}(\text{Mary}, y)$' as '$[\text{Trusts}(y)](\text{Mary})$', reflecting the fact that 'trusts' combines with a direct object. But this does not affect the main point. Grammatically,'trusts' and 'every doctor' form a phrase. Though with respect to logical form, 'trusts' combines with 'Mary' and a variable to form a complex predicate that is in turn an external argument of the higher-order predicate 'every'. Similar remarks apply to 'Some boy trusts every doctor' and '$[\exists x{:}\text{Boy}(x)][\forall y{:}\text{Doctor}(y)]\{\text{Trusts}(x, y)\}$'. So it seems that mismatches remain, in the very places that troubled medieval logicians—quantificational direct objects, and other examples of complex predicates with quantificational constituents.

Montague (1970) showed that these mismatches do not preclude systematic association of natural language sentences with the corresponding propositional structures. He specified an algorithm that pairs each natural language sentence containing one or more quantificational expressions with appropriate sentences of a *Begriffsschrift*. This was a significant advance, establishing that one can fruitfully employ Frege's formal tools in the study of natural language. Montague still held that the syntax of natural language was misleading for purposes of (what he took to be) real semantics. But even this was becoming less clear.

In thinking about the relation of logic to grammar, one must not assume a naive conception of the latter. For example, the grammatical form of a sentence need not be determined by the linear order of its words. Using brackets to indicate phrasal structure, we can distinguish sentence (11) from the homophonous sentence (12).

(11) {Mary [saw [the [boy [with binoculars]]]]}

(12) {Mary [[saw [the boy]] [with binoculars]]}

The direct object of (11) is ‘the boy with binoculars’, while in (12), ‘saw the boy’ is modified by an adverbial phrase. Presumably, only (11) implies that the boy had binoculars, and only (12) implies that Mary used binoculars to see the boy.

More generally, the study of natural language suggests a rich, nonobvious conception of grammatical form; see especially Chomsky (1957, 1965, 1981, 1986, 1995). A leading idea of modern linguistics is that at least some grammatical structures are transformations of others. Expressions often appear to be *displaced* from positions canonically associated with certain grammatical relations. For example, the word ‘who’ in (13) seems to be associated with the internal (direct object) argument position of ‘saw’.

(13) Mary wondered who John saw

Correspondingly, (13) can be glossed as ‘Mary wondered which person is such that John saw that person’. This invites the hypothesis that (13) reflects a transformation of the “Deep Structure” (13D) into the “Surface Structure” (13S),

(13D) {Mary [wondered {John [saw who]}]}

(13S) {Mary [wondered [who$_i$ {John [saw (_)$_i$]}]]}

with indices indicating a structural relation between the coindexed positions. In (13D), the embedded clause has the same form as ‘John saw Bill’. But in (13S), ‘who’ occupies another position. Similar remarks apply to ‘Who did John see’ and other question-words (like ‘why’, ‘what’, ‘when’, and ‘how’).

One might also explain the synonymy of (14) and (15) by positing a common deep structure, (14D):

(14) John seems to like Mary

(15) It seems John likes Mary

(14D) {Seems {John [likes Mary]}]}

(14S) {John$_i$ [seems { (_)$_i$ [to like Mary]}]}

If every English sentence needs a grammatical subject, (14D) must be modified: either by displacing 'John', as in (14S); or by inserting a pleonastic subject, as in (15). Note that in (15), 'It' does not indicate any thing; compare 'There' in 'There is something in the garden'. Appeal to displacement also lets one distinguish the superficially parallel sentences (16) and (17).

(16) John is easy to please

(17) John is eager to please

If (16) is true, John is easily pleased; using a pleonastic subject, it is easy (for someone) to please John. But if (17) is true, John is eager that he please someone or other. This asymmetry is effaced by representations like 'Easy-to-please(John)' and 'Eager-to-please(John)'. The contrast is made manifest, however, with (16S) and (17S);

(16S) {John$_i$ [is easy { *e* [to please (_)$_i$]}]}

(17S) {John$_i$ [is eager { (_)$_i$ [to please *e*]}]}

where '*e*' indicates an unpronounced argument position. This reflects the idea that the "surface subject" of a sentence may be understood as the direct object of a verb embedded within the main predicate, as in (16S). Such hypotheses about grammatical structure require defense. But Chomsky and others have long argued that such hypotheses are needed to account for many facts. As an illustration of the *kind* of data that is relevant, note that (18-20) are perfectly fine expressions of English, while (21) is not.

(18) The boy who sang was happy

(19) Was the boy who sang happy

(20) The boy who was happy sang

(21) *Was the boy who happy sang

This suggests that an auxiliary verb cannot be displaced from some positions. We can encode this hypothesis by saying that (19S) is the result of a permissible transformation, while (21S) is not.

(19S) Was$_i$ {[the [boy [who sang]]] [(_)$_i$ happy]}

(21S) *Was$_i$ {[the [boy [who [(_)$_i$ happy]]]] sang}

The ill-formedness of (21) is striking, since one can sensibly ask whether or not the boy who was happy sang. Likewise, one can also ask whether or not (22) is true. But (23) is not the yes/no question corresponding to (22).

(22) The boy who was lost kept crying

(23) Was the boy who lost kept crying

Rather, (23) is the yes/no question corresponding to ‘The boy who lost was kept crying’. We can explain this “negative fact,” concerning what (23) *cannot* mean, assuming that ‘was’ *cannot* be displaced from the relative clause in (22): *Was$_i$ {[the [boy [who [(_)$_i$ lost]]]] [kept crying]}. For in that case, (23) must be understood as structured in (23S).

(23S) Was$_i$ {[the [boy [who lost]]][(_)$_i$ kept crying]}

Such explanations appeal to substantive constraints on transformations. The idea was that a sentence has a deep structure (DS), which reflects semantically relevant relations between verbs and their arguments, and a surface structure (SS) that may include displaced (or pleonastic) elements; and in some cases, pronunciation might depend on still further transformations of SS, resulting in a distinct “phonological form” (PF). Linguists posited various constraints on these levels of grammatical structure, and the transformations that relate them. Though as the theory was elaborated and refined under empirical pressure, various apparently relevant facts still went unexplained. This suggested another level of grammatical structure, called ‘LF’ (intimating ‘logical form’), obtained by a different kind of transformation on SS.

The hypothesized transformation, which targeted the kinds of expressions that indicate (restricted) quantifiers, mapped structures like (24S) onto structures like (24L).

(24S) {Pat [trusts [every doctor]]}

(24L) {[every doctor]$_i$ {Pat [trusts (_)$_i$]}}

Clearly, (24L) does not reflect the pronounced word order in English. But the idea was that PF determines pronunication, while LF was said to be the level at which the scope of a natural language quantifier is determined; see May (1985). If we think of ‘every’ as a second-order transitive predicate, which can combine with two predicates like ‘doctor’ and ‘Pat trusts __’ to form a complete sentence, we should expect that at some level of analysis, the sentence ‘Pat trusts every doctor’ has the structure indicated in (24L). And mapping (24L) to the logical form ‘[∀x:Doctor(x)]{Trusts(Pat, x)}’ is trivial. Likewise, one can hypothesize that (25S) may be mapped onto (25L) or (25L'),

(25S) {[some boy] [trusts [every doctor]]}

(25L) $\{[\text{some boy}]_i \{[\text{every doctor}]_j \{(\ \_\ )_i [\text{ trusts } (\ \_\ )_j]\}\}$

(25L') $\{[\text{every doctor}]_j \{[\text{some boy}]_i \{ (\ \_\ )_i [\text{ trusts } (\ \_\ )_j]\}\}\}$

which are easily mapped onto '$[\exists x{:}\text{Boy}(x)][\forall y{:}\text{Doctor}(y)]\{\text{Trusts}(x, y)\}$' and '$[\forall y{:}\text{Doctor}(y)][\exists x{:}\text{Boy}(x)]\{\text{Trusts}(x, y)\}$'. This assimilates quantifier scope ambiguity to the structural ambiguity of examples like 'Mary saw the boy with binoculars'.

More generally, many apparent examples of grammar/logic mismatches were rediagnosed as mismatches between different aspects of *grammatical* structure—between those aspects that determine pronunciation, and those that determine interpretation. In one sense, this is fully in keeping with the idea that in natural language, "surface appearances" are often misleading with regard to propositional structure. But it makes room for the idea that grammatical and logical form converge, in ways that can be discovered through investigation, once we move beyond traditional subject-predicate conceptions of structure with regard to both logic and grammar.

There is independent evidence for "covert quantifier raising"—displacement of quantificational expressions from their audible positions, as in (24L); see Huang (1995), Hornstein (1995). Consider the French translation of 'Who did John see', 'Jean a vu qui'. If we assume that qui ('who') is displaced at LF, we can explain why the question-word is understood in both French and English like a quantifier binding a variable: which person x is such that John saw x? Similarly, example (26) from Chinese is transliterated as in (27).

(26) Zhangsan zhidao Lisi mai-te sheme

(27) Zhangsan know Lisi bought what

But (26) is ambiguous, between the interrogative (27a) and the complex declarative (27b).

(27a) Which thing is such that Zhangsan knows Lisi bought it

(27b) Zhangsan knows which thing (is such that) Lisi bought (it)

This suggests covert displacement of the quantificational question-word in Chinese; see Huang (1982, 1995). And note that (28) has the reading indicated in (28a) but not the reading indicated in (28b), suggesting that 'every patient' gets displaced, but only so far.

(28) It is false that Chris saw every patient

(28a) $\neg\forall x{:}\text{Patient}(x)[\text{Saw}(\text{Chris}, x)]$

(28b) $\forall x{:}\text{Patient}(x)\neg[\text{Saw}(\text{Chris}, x)]$

Likewise, (13) cannot mean that for every patient *x*, no lawyer who saw x respects some doctor.

(13) No lawyer who saw every patient respects some doctor

As we have already seen, English seems to abhor "fronting" certain elements from within an embedded relative clause. This invites the hypothesis that quantifier displacement is subject to a similar constraint, and hence, that quantifiers are often displaced. Indeed, many linguists (following Chomsky [1995, 2000]) would now posit only two levels of grammatical structure, PF and LF—the thought being that constraints on DS and SS can be eschewed in favor of a simpler theory that only posits constraints on how expressions can be combined in the course of constructing complex expressions that can be pronounced and interpreted. If this development of earlier theories proves correct, then (some future analog of) LF may be *the* semantically relevant level of grammatical structure. But in any case, there is a large body of work suggesting that many logical properties of quantifiers, names, and pronouns are reflected in properties of LF.

For example, linguists have discovered modern grammatical correlates of *dictum de nullo* environments. The word 'ever' can be used in sentences like (29-31). But there is something wrong with (32-34).

(29) No senator ever lied

(30) No senator who ever lied got away with it

(31) Every senator who ever lied got away with it

(32) *Every senator ever lied

(33) *Some senator ever lied

(34) *Some senator who ever lied got away with it

To a first approximation, certain expressions like 'ever' can appear only in phrases that licence inferences from more restrictive to less restrictive predicates. (Idiomatic alternatives to 'any'—like 'a plug nickel', roughly synonymous with 'any money'—exhibit this pattern: Nobody/*Somebody would pay a plug nickel for that horse.) Such discoveries, of which there have been many, confirm the Aristotelian and medieval suspicion that logical properties and grammatical properties are deeply related after all.[11]

There is, to be sure, an important conceptual distinction between the theoretical notion of LF and the traditional notion of logical form. There is no guarantee that structural features of natural language sentences will mirror the structural features of propositions. But this leaves room for a range of empirical hypothesis about how grammar is related to logic. For example, even if the LF of a sentence S underdetermines the logical form of the proposition a speaker expresses with S (on a given occasion of use), perhaps the LF provides a "scaffolding" that is somehow elaborated in particular contexts—with little or no *mismatch* between sentential and propositional architecture. If some such view is correct, it would avoid some unpleasant questions prompted by earlier Fregean views: how can sentences be used (reliably) to indicate propositions with very different structures; and if grammar is deeply misleading, why think that our intuitions concerning impeccability of inferences provide good evidence for which propositions follow from which? These are, however, issues that remain very much unsettled.

7. Semantic Structure and Events

Prima facie, 'Every tall sailor respects some doctor' and 'Some short boy likes every politician' exhibit common modes of linguistic combination. So especially in light of transformational grammars, a natural hypothesis is that the meaning of each sentence is somehow fixed by these modes of combination, given the word meanings. It may be hard to see how this hypothesis could be true, given pervasive mismatches between logical and grammatical form. But it is also hard to see how the hypothesis could be false, given that children typically acquire the capacity to understand the endlessly many expressions of the languages spoken around them. A great deal of recent work has focussed on these issues, concering the connections between logical form and apparent compositionality of natural language.

It was implicit in Frege that each sentence of an ideal language has a compositionally determined truth-condition. Frege did not specify an algorithm that would associate each sentence of his *Begriffsschrift* with its truth-condition. But Tarski (1933) showed how to do this for the first-order predicate calculus, focussing on the interesting cases of multiple quantification. This made it possible to capture, with precision, the idea that an inference is valid in the predicate calculus iff: every interpretation that makes the premises true makes the conclusion true, holding fixed the interpretations of symbols like '∃' and '¬'. Davidson (1967a) conjectured that there are similar "theories of truth" for natural languages; see Higginbotham (1985) for development within an explicitly Chomskyan framework. And Montague, also inspired by Tarski, showed how to start dealing with quantificational predicates. Sentences like 'Pat thinks that Hesperus is Phosphorus' present difficulties; though Davidson (1968) offered an influential suggestion. And while many apparent objections to the conjecture remain, Davidson's (1967b) proposal concerning examples like (35-38) proved especially fruitful.

(35) Juliet kissed Romeo quickly at midnight.

(36) Juliet kissed Romeo quickly.

(37) Juliet kissed Romeo at midnight.

(38) Juliet kissed Romeo.

If (35) is true, so are (36-38); and if (36) or (37) is true, so is (38). The inferences are impeccable. But if we treat 'kissed quickly at midnight' as an unstructured transitive predicate like 'kissed', we treat the inference from (35) to (38) as having the form '$K^*(x, y)$, so $K(x, y)$'. And invalid inferences, like 'Juliet kicked Romeo, so Juliet kissed Romeo', share this form. Put another way, one wants to know *why* conditionals like following are tautologous: if Juliet kissed Romeo in a certain manner at a certain time, then Juliet kissed Romeo. Davidson argued that sentences like (35-38) mask important semantic structure. He proposed that such sentences are understood in terms of quantification over events, as suggested by paraphrases like 'There was a kissing of Romeo by Juliet' and 'There was a quick kissing of Romeo by Juliet, and it happened at midnight'. The details are less important here than the idea that a sentence like (35) might be understood as a quantificational claim, structured along the following lines: $\exists$e[Agent(e, Juliet) & Kissing(e) & Patient(e, Romeo) & Quick(e) & At(e, midnight)].

This raises the possibility that theories of meaning/understanding for natural languages will associate sentences (whose grammatical structures are not obvious) with "semantic structures" that are not obvious.[12] Perhaps in the end, talk of logical forms is best construed as talk of the structure(s) that speakers impose on words in order to understand natural language systematically; see Lepore and Ludwig (2002), Ludwig (2003). From this perspective, which remains tendentious, the phenomenon of valid inference is at least largely a reflection of semantic compositionality.

At this point, many issues become relevant to discussions of logical form. Given any sentence of natural language, one can ask interesting questions about its grammatical structure and what it can(not) be used to say. More generally, how should we characterize sentential meanings? (In terms of truth theories? In first-order terms?) What should we say about the various paradoxes? Are claims about the "semantic structure" of a sentence fundamentally descriptive claims about speakers (or their communities, or their languages)? Or is there an important sense in which claims about semantic structure are normative claims? Are facts about language acquisition germane to hypotheses about propositional structure? But it seems clear that the traditional questions—what kinds of structures do propositions and sentences exhibit, and how do human beings relate thought to speech—must be addressed in terms of increasingly sophisticated conceptions of logic and grammar.

References

Boolos, G. 1998: *Logic, Logic, and Logic*. Cambridge, MA: Harvard University Press.

Carnap, R. 1950: Empiricism, Semantics, and Ontology. Reprinted in *Meaning and Necessity*, second edition (Chicago: University of Chicago Press, 1956).

Chomsky, N. 1957: *Syntactic Structures*. The Hague: Mouton.

—1965: *Aspects of the Theory of Syntax*. Cambridge, MA: MIT Press.

—1981: *Lectures on Government and Binding*. Dordrecht: Foris.

—1986: *Knowledge of Language*.(New York: Praeger).

—1995: *The Minimalist Program*. Cambridge, MA: MIT Press.

Davidson, D. 1967a: Truth and Meaning. *Synthese* 17: 304-23.

—1967b: The Logical Form of Action Sentences. In N. Rescher (ed.) *The Logic of Decision and Action* (Pittsburgh: University of Pittsburgh Press).

—1968: On Saying That. *Synthese* 19: 130-46.

Demopolous, W, (ed.) 1994: *Frege's Philosophy of Mathematics*. Cambridge, MA: Harvard.

Donnellan, K. 1966: Reference and Definite Descriptions. *Philosophical Review* 75: 281-304.

Frege, G. 1879: *Begriffsschrift*. Halle: Louis Nebert. English translation in J. van Heijenoort (ed.), *From Frege to Gödel: A Source Book in Mathematical Logic, 1879-1931* (Cambridge, MA: Harvard University Press, 1967).

—1892a: Function and Concept. In Geach and Black (1980).

—1892b: Sense and Reference. In Geach and Black (1980).

Fodor, J. 1975. *The Language of Thought*. New York: Crowell.

Fodor, J. 1978: *Propositional Attitudes*. The Monist 61:501-23.

Geach, P. and Black, M. 1980: *Translations from the Philosophical Writings of Gottlob Frege* (Oxford: Blackwell).

Grice, H. Paul. Logic and Conversation. In *Syntax and Semantics, vol. 3*, eds. P. Cole and J. Morgan. New York: Academic Press, 1975.

Higginbotham, J. 1985: On Semantics. *Linguistic Inquiry*: 16 547-93.

Hornstein, N. 1995: *Logical Form: From GB to Minimalism*. Oxford: Blackwell.

Huang, J. 1995: Logical Form. In Webelhuth, G. (ed.) *Government and Binding Theory and the Minimalist Program: Principles and parameters in syntactic theory* (Oxford: Blackwell).

Kneale, W. and Kneale, M. 1962: *The Development of Logic*. Oxford: Oxford University Press, 1962 (reprinted 1984).

Kripke, Saul. *Naming and Necessity*. Cambridge, MA: Harvard University Press, 1980.

Ladusaw, W. 1981: On the Notion *Affective* in the Analysis of Negative Polarity Items. *Journal of Linguistic Research* 1:1-16.

Lepore, E. and Ludwig, K.. 2002: What is Logical Form. In Preyer and Peters 2002.

Ludwig, K. 2003: *Contemporary Philosophy in Focus: on Davidson* (Cambridge: Cambridge University Press).

Ludlow, P. 2002: LF and Natural Logic. In Preyer and Peters 2002.

May, R. 1985: *Logical Form: Its Structure and Derivation*. Cambridge, MA: MIT Press.

Montague, R. 1970: English as a Formal Language. Reprinted in his collected esssays, *Formal Philosophy* (New Haven: Yale University Press, 1974).

Pietroski, P. 2003: Quantification and Second-Order Monadicity. *Philosophical Perspectives*: 17:259-98.

Preyer, G. and G. Peters, G. (eds.) 2002: *Logical Form and Language*. Oxford, OUP.

Quine, W.V.O. 1950: Methods of Logic. New York: Henry Holt.

—1951: Two Dogmas of Empiricism. *Philosophical Review* 60:20-43.

—1960: *Word and Object*. Cambridge MA: MIT Press.

—1970: *Philosophy of Logic*. Englewood Cliffs, NJ: Prentice Hall.\

Russell, Bertrand. *Introduction to Mathematical Philosophy*. London: George Allen and Unwin, 1919.

Soames, S. 1987: Direct Reference, Propositional Attititudes, and Semantic Content. *Philosophical Topics* 15:47-87.

—1995: Beyond Singular Propositions. *Canadian Journal of Philosophy* 25: 515-50.

Strawson, P. 1950: On Referring. *Mind* 59: 320-44.

Tarski, A. 1933: The Concept of Truth in Formalized Languages. Reprinted in *Logic, Semantics, Metamathematics* (tr. J.H. Woodger, 2nd edition, ed. J Corcoran). Indianapolis: Hacket.

Wittgenstein, G. *Tractatus Logico-Philosophicus*, tr. D. Pears and B. McGuinness London: Routledge and Kegan Paul, 1921.

—*Philosophical Investigations*. New York: Macmillan, 1953.

Zalta, Edward. Frege (logic, theorem, and foundations for arithmetic). *The Stanford Encyclopedia of Philosophy* (Fall 2003 Edition). Available from http://plato.stanford.edu/archives/fall2003/entries/frege-logic.

Notes

1. Eventually, we may have to relax this assumption. Is any *one* proposition indicated with a sentence containing a vague predicate? Is *any* proposition indicated with a sentence containing demonstrative if nothing is demonstrated? Another complication is that in speaking of an argument or inference, one might be talking about a verbal or mental *process*. But it will be simpler to characterize episodes of (in)valid reasoning in terms of propositions, even if talk of propositions should ultimately be understood in terms of intentional activities.

2. In English, an article is often required, as in 'Every senator is *a* politician'. But let's assume that this particular feature of English does not reflect propositional structure.

3. For further discussion, see Ludlow (2002), Kneale and Kneale (1962).

4. Variable letters like '*x*' and '*y*' are typographically convenient. But we can index "gaps" as follows: $Q[(\ )_i , (\ )_j] = (\ )_i / (\ )_j$. We could also replace the subscripts with lines that link gaps. But the idea, however we encode it, is that a proposition has at least one constituent saturated by the requisite number of arguments. One can think of an unsaturated proposition-part as the result of abstracting away from the arguments in a particular proposition. Frege was here influenced by Kant's discussion of judgment, and the ancient observation that merely combining two things does not make the combination truth-evaluable; predicates evidently play a special role in "unifying" propositions.

5. Zalta (2003) provides a helpful introduction and preparation for Demopolous (1994).

6. This in turn led Frege (1892b) to say that psychological reports, like 'Mary thinks that Venus is bright', are also misleading with respect to the forms of the indicated propositions; cf. Soames (1987, 1995).

7. Of course, one can say 'The boy sang' without denying that universe contains more than one boy. But likewise, in ordinary conversation, one can say 'Everything is in the trunk' without denying that the universe contains some things not in the trunk. And intuitively, a speaker who uses 'the boy' does imply that there is exactly one contextually relevant boy.

8. Gödel had proved the completeness of first-order predicate calculus, thus providing a purely formal criterion for what followed from what in that language. Quine (150, 1970) also held that second-order quantification illicitly treated predicates as names for sets, thereby spoiling Frege's conception of propositions as unified by virtue of having unsaturated predicational constituents that are satisfied by things denoted by names.

9. See also Grice (1975). Fodor (1975, 1978) combines a version of this view with the idea that propositions are sentences of a mental language that may well differ structurally from the languages humans use to communicate.

10. See Pietroski (2003) for a variant, drawing on Boolos (1998) and Schein (1993), that avoids Russell's Paradox.

11. See Ladusaw (1981), and Ludlow (2002) for further discussion and references.

12. Whether or not such theories take the form of an algorithm for *transforming* sentences of natural language into sentences of a mental (or invented ideal) language.